



Bielefeld-Edinburgh-Swansea Stochastic Spring

26–28 March 2018

Bielefeld University
Lecture Room: V2-210/216

This workshop is part of the DFG-funded CRC 1283
*Taming uncertainty and profiting from randomness and low regularity
in analysis, stochastics and their applications*
at Bielefeld University

Organisers: Michael Röckner and Katharina von der Lühe

https://www.sfb1283.uni-bielefeld.de/2018_BES/

Schedule

	Monday, 26 March	Tuesday, 27 March	Wednesday, 28 March
09:10–09:55		Sotirios Sabanis	Niels Jacob
10:00–10:45	Benjamin Gess	Chenggui Yuan	Sigurd Assing
		Coffee Break	
11:15–12:00	David Šiška	Jiang-Lun Wu	Eugene Lytvynov
12:05–12:50	Gonçalo dos Reis	Vassili Kolokoltsov	Dmitri Finkelshtein
		Lunch Break	
14:20–15:05	Zeev Sobol	Martina Hofmanová	Michael Hinz
15:10–15:55	Miryana Grigorova	Michele Coghi	Jamil Chaker
		Coffee Break	
16:25–17:10	Hanwu Li	Huanyu Yang	
17:15–17:40	Patrick Schuhmann	Neelima	
17:45–18:10	Ifan Johnston	Ying Zhang	

Schedule: Monday March 26

Lecture Room: **V2-210/216**

09:30 – 09:55 **Registration**

10:00 – 10:45 **Benjamin Gess** (Bielefeld University)
Optimal regularity for the porous medium equation

10:45 – 11:15 **Coffee Break** (Room V3-201)

11:15 – 12:00 **David Šiška** (University of Edinburgh)
McKean–Vlasov SDEs under Measure Dependent Lyapunov Conditions

12:05 – 12:50 **Gonçalo dos Reis** (University of Edinburgh)
Large Deviations for McKean Vlasov Equations in path space

12:50 – 14:20 **Lunch Break**

14:20 – 15:05 **Zeev Sobol** (Swansea University)
Pricing on Security Markets allowing for an Arbitrage

15:10 – 15:55 **Miryana Grigороva** (Bielefeld University)
Reflected BSDEs and non-linear optimal stopping: beyond right-continuity

15:55 – 16:25 **Coffee Break** (Room V3-201)

16:25 – 17:10 **Hanwu Li** (Bielefeld University)
Reflected BSDEs driven by G-Brownian motion and their applications

17:15 – 17:40 **Patrick Schuhmann** (Bielefeld University)
An Optimal Dividend Problem with Capital Injections over a Finite Horizon

17:45 – 18:10 **Ifan Johnston** (University of Warwick)
Caputo-type fractional derivatives and Feller processes on bounded domains

Schedule: Tuesday March 27

Lecture Room: **V2-210/216**

- 09:10 – 09:55 **Sotirios Sabanis** (University of Edinburgh)
Tamed Unadjusted Langevin Algorithm
- 10:00 – 10:45 **Chenggui Yuan** (Swansea University)
Invariant measures for stochastic differential equations and their time discretizations
- 10:45 – 11:15 **Coffee Break** (Room V3-201)
- 11:15 – 12:00 **Jiang-Lun Wu** (Swansea University)
On weak solutions of stochastic differential equations with sharp irregular drifts
- 12:05 – 12:50 **Vassili Kolokoltsov** (University of Warwick)
Recent advances in analytic and probabilistic methods for solving fractional PDEs
- 12:50 – 14:20 **Lunch Break**
- 14:20 – 15:05 **Martina Hofmanová** (Bielefeld University)
Stationary solutions to stochastic compressible Navier-Stokes system
- 15:10 – 15:55 **Michele Coghi** (WIAS Berlin, Bielefeld University)
Mean field limit of interacting filaments for 3D Euler equations
- 15:55 – 16:25 **Coffee Break** (Room V3-201)
- 16:25 – 17:10 **Huanyu Yang** (Bielefeld University)
Conservative stochastic 2-dimensional Cahn-Hilliard equation
- 17:15 – 17:40 **Neelima** (University of Edinburgh)
Regularity for a class of Semilinear SPDEs on bounded domains
- 17:45 – 18:10 **Ying Zhang** (University of Edinburgh)
Towards a Wagner-Platen expansion approach for numerical schemes of SDEs with superlinear coefficients

Schedule: Wednesday March 28

Lecture Room: **V2-210/216**

- 09:10 – 09:55 **Niels Jacob** (Swansea University)
Negative definite symbols as Hamilton functions
- 10:00 – 10:45 **Sigurd Assing** (University of Warwick)
Extension Technique for Trace Processes: a stochastic approach
- 10:45 – 11:15 **Coffee Break** (Room V3-201)
- 11:15 – 12:00 **Eugene Lytvynov** (Swansea University)
Particle-hole duality in the continuum and determinantal point processes
- 12:05 – 12:50 **Dmitri Finkelshtein** (Swansea University)
Stochastic dynamics of complex systems: mesoscopic description and beyond
- 12:50 – 14:20 **Lunch Break**
- 14:20 – 15:05 **Michael Hinz** (Bielefeld University)
Canonical diffusions on the pattern spaces of aperiodic Delone sets
- 15:10 – 15:55 **Jamil Chaker** (Bielefeld University)
Nonlocal operators with singular anisotropic kernels

Abstracts

Sigurd Assing (University of Warwick)

Extension Technique for Trace Processes: a stochastic approach

Caffarelli & Silvestre showed in 2007 that any d -dimensional fractional Laplacian $(-\Delta)^{\alpha/2}$, $\alpha \in (0,2)$, can be described as a Dirichlet-to-Neumann operator with respect to the operator $\nabla_{x,y}(y^{1-\alpha}\nabla_{x,y})$ in half-space $R^d \times (0,\infty)$. In 2017, Kwasnicki & Mucha aimed to give a similar result for all complete Bernstein functions of $-\Delta$. Using a different but stochastic method, we cover the results obtained by Kwasnicki & Mucha and extend them to a class of generators of trace processes in half-space.

Jamil Chaker (Bielefeld University)

Nonlocal operators with singular anisotropic kernels

In this talk we study a class of anisotropic nonlocal operators. The operators under consideration generate anisotropic jump processes, e.g., a jump process that behaves like a stable process in each direction but with a different index of stability. Its generator is the sum of one-dimensional fractional Laplace operators with different orders of differentiability. We study such operators in the general framework of bounded measurable coefficients. We prove a weak Harnack inequality and Hölder regularity results for solutions to corresponding integro-differential equations. This talk is based on a joint work with Moritz Kassmann.

Michele Coghi (WIAS Berlin, Bielefeld University)

Mean field limit of interacting filaments for 3D Euler equations

Solutions to the 3D Euler equations are obtained as a mean field limit of finite families of interacting curves, the so called vortex filaments. Families of N interacting curves are considered, with long range, mean field type interaction. A family of curves defines a 1-current, concentrated on the curves, analog of the empirical measure of interacting point particles. This current is proved to converge, as N goes to infinity, to a solution of the 3D Euler equation. In the limit, each curve interacts with the mean field current and two different curves have an independence property if they are independent at time zero.

Gonçalo dos Reis (University of Edinburgh)

Large Deviations for McKean-Vlasov Equations in path space

We discuss two Freidlin-Wentzell large deviation principles for McKean-Vlasov equations in certain path space topologies. The equations have a drift of polynomial growth and an existence/uniqueness result is provided. Lastly, we discuss the Functional Strassen theorem for this class of equations.

Dmitri Finkelshtein (Swansea University)

Stochastic dynamics of complex systems: mesoscopic description and beyond

We discuss statistical description to the study of stochastic dynamics of complex systems in continuum. We give an overview of methods to show the existence and uniqueness of the corresponding evolutions of correlation functions, and consider mesoscopic (kinetic) description as a way to describe the quantitative and qualitative behavior of these functions. We consider examples of the appearing mesoscopic (kinetic) equations, which are nonlocal nonlinear PDE, and discuss variety of effects their solution have, and briefly the methods they require. Finally, we describe the method which allows us to find the next term of the approximation for the density of a complex system beyond the mesoscopic description.

Benjamin Gess (Bielefeld University)

Optimal regularity for the porous medium equation

The regularity of solutions to porous media equations has been intensively studied in the literature. Nevertheless, optimal regularity in Sobolev spaces could not be obtained so far. In particular, the available estimates are inconsistent with the linear case and scaling arguments suggest that they are not optimal. We will first recall elementary properties of the porous medium equation and some basic estimates. We then present a proof of optimal regularity for solutions to porous media equations in Sobolev spaces.

Miryana Grigorova (Bielefeld University)

Reflected BSDEs and non-linear optimal stopping: beyond right-continuity

Reflected backward stochastic differential equations (RBSDEs) can be seen as a variant of Backward Stochastic Differential Equations (BSDEs) in which the (first component of the) solution is constrained to remain greater than or equal to a given process called the obstacle. Compared to the case of (non-reflected) BSDEs, there is an additional nondecreasing predictable process which keeps the (first component of the) solution above the obstacle. RBSDEs have been introduced by El Karoui et al. (1997) in the case of a continuous obstacle and have proved useful, for instance, in the study of American options in financial mathematics. There have been several extensions of this work to the case of a discontinuous obstacle in all of which an assumption of right-continuity on the obstacle is made.

In this talk we present a further extension of the theory of RBSDEs to the case where the obstacle does not satisfy any regularity assumption. Compared to the right-continuous case, the additional nondecreasing process, which "pushes" the (first component of the) solution to stay above the obstacle, is no longer right-continuous. We establish existence and uniqueness of the solution in appropriate Banach spaces. We characterize the solution in terms of the value process of an optimal stopping problem with non-linear g -expectation (where g is the driver of the RBSDE). Our results use some tools from the general theory of processes (Mertens decomposition, Gal'chouk-Lenglart's formula, ...) and some tools from the "classical" optimal stopping theory (with linear expectations). The talk is based on joint works with P. Imkeller, E. Offen, Y. Ouknine, and M.C. Quenez.

Michael Hinz (Bielefeld University)

Canonical diffusions on the pattern spaces of aperiodic Delone sets

In this talk we consider differential operators and diffusion processes on pattern spaces of aperiodic Delone sets. Such spaces arise naturally in tiling theory and diffraction theory, and they have features of both manifolds and fractals. We first discuss Feller properties. Assuming unique ergodicity we then study items L^2 -theory, such as properties of self-adjoint Laplacians and Dirichlet forms, the non-existence of heat kernels or Liouville theorems. For pattern spaces with one-dimensional orbits we finally discuss a Helmholtz-Hodge decomposition and the non-validity of the Hodge-deRham theorem. The results are joint with P. Alonso-Ruiz, A. Teplyaev and R. Trevino.

Martina Hofmanová (Bielefeld University)

Stationary solutions to stochastic compressible Navier-Stokes system

We study the long-time behavior of solutions to a stochastically driven Navier-Stokes system describing the motion of a compressible viscous fluid driven by a temporal multiplicative white noise perturbation. The existence of stationary solutions is established in the framework of Lebesgue–Sobolev spaces pertinent to the class of weak martingale solutions. The methods are based on new global-in-time estimates and a combination of deterministic and stochastic compactness arguments. In contrast with the deterministic case, where related results were obtained only under rather restrictive constitutive assumptions for the pressure, the stochastic case is tractable in the full range of constitutive relations allowed by the available existence theory.

Niels Jacob (Swansea University)

Negative definite symbols as Hamilton functions

A refined study of spectral properties of generators of Lévy - type processes needs an understanding of the associated Hamiltonian mechanics. This is however non-trivial due to a lack of regularity of the symbol which needs to be neither twice continuously differential nor convex. In the talk we start will discuss some first observations and results for generators of type potential plus generator of a Lévy process.

Ifan Johnston (University of Warwick)

Caputo-type fractional derivatives and Feller processes on bounded domains

From a probabilistic point of view, Caputo fractional derivatives of order $\beta \in (0, 2)$ can be viewed as generators of β -stable Lévy motions interrupted on crossing a boundary. We extend this idea to Feller processes on bounded domains generated by a generalized Caputo-type operator. These ideas result in a useful probabilistic way of viewing problems involving fractional derivatives, and allows for some stochastic representations of solutions to a range of problems involving different fractional derivatives. I will also discuss the extension of these ideas to higher dimension.

Vassili Kolokoltsov (University of Warwick)

Recent advances in analytic and probabilistic methods for solving fractional PDEs

New approaches developed by the author to solving fractional PDEs and their extensions will be presented. These approaches are based on three main ideas: (1) Markov processes interrupted at the attempt to cross the boundary, (2) Integral representations for the operator-valued generalized Mittag-Leffler functions, (3) Operator-valued chronological Feynman-Kac formula.

Hanwu Li (Bielefeld University)

Reflected BSDEs driven by G-Brownian motion and their applications

In this talk, we introduce the reflected solutions of one-dimensional backward stochastic differential equations driven by G-Brownian motion. The reflection means the solution is required to be above (or below) a given stochastic process. We apply a “martingale condition” instead of the Skorohod condition to formulate this problem. Similar to the classical case, we prove the existence by approximation via penalization. We then give some applications such as a generalized Feynman-Kac formula of an obstacle problem for fully nonlinear partial differential equation.

Eugene Lytvynov (Swansea University)

Particle-hole duality in the continuum and determinantal point processes

Let X be an underlying locally compact Polish space equipped with a Borel measure σ . Let $K(x,y):X^2 \rightarrow \mathbb{C}$ and let K denote the integral operator in $L^2(X,\sigma)$ with integral kernel $K(x,y)$. A point process μ on X is called determinantal with the correlation operator K if the correlation functions of μ are given by $k^{(n)}(x_1, \dots, x_n) = \det[K(x_i, x_j)]_{i,j=1, \dots, n}$. If the operator K is self-adjoint, a determinantal point process with correlation operator K exists if and only if K is locally trace-class and $0 \leq K \leq 1$. Each determinantal point process with a Hermitian correlation kernel can be understood as the (spectral measure of) the particle density $\rho(x) = \partial_x^\dagger \partial_x$ ($x \in X$), where the operator-valued distributions $\partial_x^\dagger, \partial_x$ come from a gauge-invariant quasi-free representation of the canonical anticommutation relations (CAR). If the space X is discrete and divided into two disjoint parts, X_1 and X_2 , by exchanging particles and holes on the X_2 part of the space, one obtains from a determinantal point process with correlation kernel K a determinantal point process with correlation kernel $\widehat{K} = KP_1 + (1-K)P_2$, where P_i is the orthogonal projection onto $L^2(X_i, \sigma)$. In particular, the operator \widehat{K} is J -self-adjoint. In the case where the space X is continuous, a direct procedure of swapping particles and holes makes no sense. Nevertheless, we prove that it is possible to carry out such a procedure by swapping creation operators ∂_x^\dagger with annihilation operators ∂_x on the X_2 part of the space. This leads to a quasi-free representation of CAR and the corresponding particle density is a determinantal point process with correlation operator \widehat{K} , which is J -self-adjoint. (This is a joint work with Maryam Alsheri.)

Neelima (University of Edinburgh)

Regularity for a class of Semilinear SPDEs on bounded domains

Regularity results for solutions to semilinear stochastic partial differential equations (on a bounded domain) with monotone semilinear term are obtained.

This is done by using the monotonicity property of the semilinear term and a cutting argument to obtain some L^p -estimates. These estimates are then used along with the results from L^p -theory for linear SPDEs to obtain new spatial and time regularity results in weighted Sobolev spaces.

Sotirios Sabanis (University of Edinburgh)

Tamed Unadjusted Langevin Algorithm

We will consider the problem of sampling from a probability measure π having a density on \mathbb{R}^d proportional to $e^{-U(x)}$. The Euler discretization of the associated Langevin stochastic differential equation (SDE) is known to be unstable, when the potential U is superlinear. Based on recent progress of explicit numerical methods for SDEs with superlinear coefficients, the Tamed Unadjusted Langevin Algorithm (TULA) is introduced. Non-asymptotic bounds in V -total variation norm and Wasserstein distance of order 2 are obtained between the iterates of TULA and π .

Patrick Schuhmann (Bielefeld University)

An Optimal Dividend Problem with Capital Injections over a Finite Horizon

In this talk we propose and solve an optimal dividend problem with capital injections over a finite time horizon. The surplus dynamics obeys a linearly controlled drifted Brownian motion that is reflected at zero, dividends give rise to time-dependent instantaneous marginal profits, whereas capital injections are subject to time-dependent instantaneous marginal costs. The aim is to maximize the sum of a liquidation value at terminal time and of the total expected profits from dividends, net of the total expected costs for capital injections. Inspired by the study in [1] on reflected follower problems, we relate the optimal dividend problem with capital injections to an optimal stopping problem for a drifted Brownian motion that is absorbed at zero, and we show that the value function of the latter gives the derivative of the value function of the optimal dividend problem. The optimal dividends' distribution strategy is then shown to be triggered by the time-dependent free boundary of the associated stopping problem. The properties of this boundary are investigated in a case study in which instantaneous marginal profits and costs from dividends and capital injections are constants discounted at a constant rate.

References

- [1] ELKAROUI, N., KARATZAS, I. (1989). *Integration of the optimal risk in a stopping problem with absorption*, Séminaire de probabilités, tome 23, pp. 405–420.

David Šiška (University of Edinburgh)

McKean–Vlasov SDEs under Measure Dependent Lyapunov Conditions

We prove the existence of weak solutions to McKean–Vlasov SDEs defined on a domain $D \subseteq \mathbb{R}^d$ with continuous and unbounded coefficients that satisfy Lyapunov type conditions, where the Lyapunov function may depend on measure. We propose a new type of *integrated* Lyapunov condition, where the inequality is only required to hold when integrated against the measure on which the Lyapunov function depends, and we show that this is sufficient for the existence of weak solutions to McKean–Vlasov SDEs defined on D . The main tool used in the proofs is the concept of a measure derivative due to Lions. We prove results on uniqueness under weaker assumptions than that of global Lipschitz continuity of the coefficients.

This is joint work with W. Hammersley and L. Szpruch.

Zeev Sobol (Swansea University)

Pricing on Security Markets allowing for an Arbitrage

In contrast to a classic no arbitrage model, we present security pricing for markets allowing for no arbitrage, including an American option (optimal stopping).

Jiang-Lun Wu (Swansea University)

On weak solutions of stochastic differential equations with sharp irregular drifts

In this talk we will discuss the following Brownian motion driven SDE

$$X_t = x + \int_0^t b(s, X_s) ds + W_t, \quad t \in [0, T], \quad x \in \mathbb{R}^d$$

for an irregular drift $b: [0, T] \times \mathbb{R}^d \rightarrow \mathbb{R}^d$. We are aiming to extend Krylov and Röckner [Strong solutions to stochastic equations with singular time dependent drift. *Probab. Theory Relat. Fields* **131** (2005) 154-196] to the case with a sharp critical b . To be more precise, if $b := b_1 + b_2$ such that $b_1(T - \cdot) \in \mathcal{C}_q^0((0, T]; L^p(\mathbb{R}^d))$ with the critical condition $2/q + d/p = 1$ for $p, q \geq 1$ and $\|b_1(T - \cdot)\|_{\mathcal{C}_q((0, T]; L^p(\mathbb{R}^d))}$ is sufficiently small, and that b_2 is bounded and Borel measurable, then there exists a unique weak solution to the above equation. Moreover, we derive the strong Feller property of the semi-group and existence of density associated with the above SDE. As an application, we extend classical regularity results for parabolic PDEs with $L^q(0, T; L^p(\mathbb{R}^d))$ coefficients to equations with $L_q^\infty(0, T; L^p(\mathbb{R}^d))$ coefficients, and further derive the Lipschitz regularity.

The talk is based on joint work [arXiv:1711.05058] with Jinlong Wei (Zhongnan University of Economics and Law, Wuhan, China) and Guangying Lv (Henan University, Kaifeng, China).

Huanyu Yang (Bielefeld University)

Conservative stochastic 2-dimensional Cahn-Hilliard equation

We consider the conservative stochastic 2-dimensional Cahn-Hilliard equation:

$$\begin{cases} dX_t = -\frac{1}{2} \Delta (\Delta X - :X^3:) dt + \nabla \cdot dW_t, \\ X(0) = z \in H^{-1}, \end{cases} \quad (1)$$

We use two different approaches to study this equation. First we prove that there exists a unique solution Y to the "shifted equation". Then $X := Y + Z$ is the unique solution to the stochastic Cahn-Hilliard equation, where Z is the corresponding O-U-process. Moreover, we use the Dirichlet form approach to construct the probabilistically weak solution the original equation (1) above. By clarifying the precise relation between the two solutions, we also get the restricted Markov uniqueness of the generator and the uniqueness of the martingale solutions to the equation (1).

Chenggui Yuan (Swansea University)

Invariant measures for stochastic differential equations and their time discretizations

In this talk, we are concerned with existence and uniqueness of invariant measures for stochastic differential equations and stochastic functional differential equations with Markovian switching, and their time discretizations. Under certain ergodic conditions, we show that the solutions enjoys a unique invariant probability measure and converges to its equilibrium under the Wasserstein distance. Also, we demonstrate that the time discretization of the SDEs involved admits a unique invariant probability measure and shares the corresponding ergodic property when the stepsize is sufficiently small. During this procedure, the difficulty arose from the time-discretization of continuous time Markov chain has to be deal with, for which an estimate on its exponential functional is presented.

Ying Zhang (University of Edinburgh)

Towards a Wagner-Platen expansion approach for numerical schemes of SDEs with superlinear coefficients

Due to recent research, new explicit Euler (Milstein) type schemes have been developed to approximate SDEs with superlinearly growing coefficients. The techniques used in the previous works can be extended to higher order Wagner-Platen type schemes. In this talk, I will introduce a tamed order 1.5 scheme, which is obtained using the model suggested in [Kumar and Sabanis (2016). [arXiv:1601.02695\[math.PR\]](https://arxiv.org/abs/1601.02695)]. One of the important results is that the new order 1.5 scheme converges strongly to the true solution of the superlinear SDE.

Registered Participants

Sigurd Assing	(University of Warwick)
Lubomir Banas	(Bielefeld University)
Jamil Chaker	(Bielefeld University)
Michele Coghi	(Bielefeld University)
Martin Dieckmann	(Bielefeld University)
Gonçalo dos Reis	(University of Edinburgh)
Dmitri Finkelshtein	(Swansea University)
Benjamin Gess	(Bielefeld University)
Miryana Grigorova	(Bielefeld University)
Emanuela Gussetti	(Bielefeld University)
Michael Hinz	(Bielefeld University)
Martina Hofmanová	(Bielefeld University)
Walter Hoh	(Bielefeld University)
Niels Jacob	(Swansea University)
Ifan Johnston	(University of Warwick)
Vassili Kolokoltsov	(University of Warwick)
Shilei Kong	(Bielefeld University)
Oleksandr Kutovyi	(Bielefeld University)
Hanwu Li	(Bielefeld University)
Eugene Lytvynov	(Swansea University)
Nora Müller	(Bielefeld University)
Neelima	(University of Edinburgh)
Tetyana Pasurek	(Bielefeld University)
Michael Röckner	(Bielefeld University)
Tim Rofsmeier	(Bielefeld University)
Sotirios Sabanis	(University of Edinburgh)
Waldemar Schefer	(Bielefeld University)
Patrick Schuhmann	(Bielefeld University)
Arthur Sinulis	(Bielefeld University)
David Šiška	(University of Edinburgh)
Zeev Sobol	(Swansea University)
Christian Vieth	(Bielefeld University)
Katharina von der Lüche	(Bielefeld University)
Lukas Wresch	(Bielefeld University)
Jiang-Lun Wu	(Swansea University)
HuanYu Yang	(Bielefeld University)
Chenggui Yuan	(Swansea University)
Ying Zhang	(University of Edinburgh)

Tram map



- 1 Senne – Jahnplatz – Schildesche
- 2 Sieker – Jahnplatz – Milse
- 3 Stieghorst – Jahnplatz – Babenhausen Süd
- 4 Rathaus – Jahnplatz – Universität – Lohmannshof

Campus map

