Nonlinear Dirac equations and related problems

28–30 May 2018

Bielefeld University
Lecture Room: V2-210/216

This workshop is part of the DFG-funded CRC 1283
Taming uncertainty and profiting from randomness and low regularity in analysis, stochastics and their applications
at Bielefeld University

Organisers: Timothy Candy and Sebastian Herr

https://www.sfb1283.uni-bielefeld.de/2018_NDE/

Schedule

	Monday, 28 May	Tuesday, 29 May	Wednesday, 30 May
09:00–09:40	Registration	Nabile Boussaïd	Eric Séré
	Opening at 9:30 am		
09:45–10:25	Piero D'Ancona	Roland Donninger	William Borrelli
	Break		
11:00–11:40	Herbert Koch	Cristian Gavrus	Xian Liao
11:45–12:25	Naiara Arrizabalaga	Joachim Krieger	Daniel Tataru
	Lunch Break		
14:00–14:40	Anne-Sophie de Suzzoni	Irfan Glogic	
14:45–15:25	Federico Cacciafesta	Birgit Schörkhuber	
15:30–16:10	Ruijun Wu		
19:00	Conference Dinner		

Schedule: Monday May 28

Lecture Room: **V2-210/216**

09:00 – 09:30 **Registration**

09:30 – 09:40 **Opening**

09:45 – 10:25 **Piero D'Ancona** (Sapienza Università di Roma)
On the supercritical defocusing wave equation outside a ball

10:30 – 11:00 **Coffee Break**

11:00 – 11:40 **Herbert Koch** (Rheinische Friedrich-Wilhelms-Universität Bonn)
The renormalized nonlinear wave equation with additive white noise

11:45 – 12:25 **Naiara Arrizabalaga** (Universidad del País Vasco, Bilbao)
Asymptotic expansion of eigenvalues for the MIT bag model

12:30 – 14:00 **Lunch Break**

14:00 – 14:40 **Anne-Sophie de Suzzoni** (Université Paris 13)
Dispersion and the Dirac operator

14:45 – 15:25 **Federico Cacciafesta** (Università degli Studi di Padova)
Strichartz estimates for the Dirac equation on spherically symmetric manifolds

15:30 – 16:10 **Ruijun Wu** (Max-Planck-Institut, Leipzig)
Geometric analysis of the nonlinear sigma model with gravitino

19:00 **Conference Dinner**

Schedule: Tuesday May 29

Lecture Room: **V2-210/216**

09:00 – 09:40	**Nabile Boussaïd** (Université de Franche-Comté, Besançon) On stability of solitary waves of the nonlinear Dirac equation
09:45 – 10:25	**Roland Donninger** (Universität Wien) Linear stability of the Skyrmion
10:30 – 11:00	**Coffee Break**
11:00 – 11:40	**Cristian Gavrus** (University of California, Berkeley) Global well-posedness of the high dimensional Maxwell-Dirac equation for small critical data
11:45 – 12:25	**Joachim Krieger** (École polytechnique fédérale de Lausanne) On stability of blow up for co-rotational Wave Maps
12:30 – 14:00	**Lunch Break**
14:00 – 14:40	**Irfan Glogic** (Universität Wien) Existence and stability of blowup for wave maps into negatively curved targets
14:45 – 15:25	**Birgit Schörkhuber** (Karlsruher Institut für Technologie) Co-dimension one stable blowup in supercritical wave equations

Schedule: Wednesday May 30

Lecture Room: **V2-210/216**

09:00 – 09:40	**Eric Séré** (Université Paris-Dauphine) Taking into account the polarized Dirac sea: the nonlinear Euler-Heisenberg model
09:45 – 10:25	**William Borrelli** (Université Paris-Dauphine) Effective Dirac equations in honeycomb structures
10:30 – 11:00	**Coffee Break**
11:00 – 11:40	**Xian Liao** (Rheinische Friedrich-Wilhelms-Universität Bonn) Conserved energies for the one-dimensional Gross-Pitaevskii equation
11:45 – 12:25	**Daniel Tataru** (University of California, Berkeley) The energy critical Yang-Mills flow

Abstracts

Piero D'Ancona (Sapienza Universitá di Roma)

On the supercritical defocusing wave equation outside a ball

In this work I consider a defocusing semilinear wave equation, with a power nonlinearity, defined on the outside of the unit ball of $\mathbb{R}^n$, and with Dirichlet conditions at the boundary. The power is assumed to be sufficiently large, $p>O(n)$, and the space dimension is 3 or larger. Even in the radial case, the corresponding problem on $\mathbb{R}^n$ is completely open. Here I construct a family of large global solutions, whose data are small perturbations of radial initial data in suitable weighted Sobolev norms of higher order.

Herbert Koch (Rheinische Friedrich-Wilhelms-Universität Bonn)

The renormalized nonlinear wave equation with additive white noise

The talk is on joint work with M. Gubinelli and H. Oh. I will explain the renormalization and local wellposedness for the renormalized nonlinear wave equation with additive white noise. The proof consists of two parts: First we consider stochastic fields and renormalization, which reduces the problem to a particular deterministic problem. In a second step we study this deterministic problem using dispersive estimates.

Naiara Arrizabalaga (Universidad del País Vasco, Bilbao)

Asymptotic expansion of eigenvalues for the MIT bag model

In this talk we present some spectral properties of the MIT bag model. This MIT bag model is a particular case of the Dirac operator, $-i\alpha\cdot\nabla+m\beta$, defined on a smooth and bounded domain $\Omega\subset\mathbb{R}^3$. Specifically, $-i\beta(\alpha\cdot\mathbf{n})\psi=\psi$ must hold at the boundary of Ω, where $\mathbf{n}$ is the outward normal vector and $\psi\in H^1(\Omega,\mathbb{C}^4)$. We will see the relation between that model and the Dirac operator with electrostatic and Lorentz scalar shell potentials. We will also see under which conditions those potentials generate confinement with respect to the Dirac operator and the boundary of Ω. Finally, we study the self-adjointness of the operators and describe the limiting behavior of the eigenvalues of the MIT bag Dirac operator as the mass m tends to $\pm\infty$.

Anne-Sophie de Suzzoni (Université Paris 13)

Dispersion and the Dirac operator

In this talk, I will present some aspects of dispersion for the Dirac operator. I will start by partially reviewing what is known for the Dirac operator in a Minkowski space-time. Then, I will introduce the Dirac operator in a curved space-time, and present a result of dispersion for specific cases such as assymptotically flat or warped product geometries.
This is a joint work with F. Cacciafesta (Padova).

Federico Cacciafesta (Universitá degli Studi di Padova)

Strichartz estimates for the Dirac equation on spherically symmetric manifolds

In this talk we show how to exploit the "radial" structure of the Dirac operator to obtain dispersive estimates (local smoothing, Strichartz..) for the Dirac equation on spaces with spherical symmetry. Applications to the well-posedness for some nonlinear models will be also discussed. This is a joint (ongoing) work with A.S. de Suzzoni.

Ruijun Wu (Max-Planck-Institut, Leipzig)

Geometric analysis of the nonlinear sigma model with gravitino

This model comes from supersymmetric nonlinear sigma models in super string theory and is an extension of the Dirac-harmonic maps. The Dirac operator here is defined along a map from a Riemann surface into a Riemannian manifold. We combine the theories/tools for harmonic maps and for Dirac operators to get a understanding of this model. The gravitino couples the matter fields in a nonlinear way and we cope with this using some recent analysis methods.

Nabile Boussaïd (Université de Franche-Comté, Besançon)

On stability of solitary waves of the nonlinear Dirac equation

This is a joint work with Andrew Comech from Texas A&m. We construct bi-frequency solitary waves of the nonlinear Dirac equation with the scalar self-interaction, known as the Soler model (with an arbitrary nonlinearity and in arbitrary dimension) and the Dirac–Klein–Gordon with Yukawa self-interaction. We show the relation of $\pm 2i\omega$ eigenvalues of the linearization at a solitary wave, Bogoliubov $SU(1,1)$ symmetry, and the existence of bi-frequency solitary waves. We show that the spectral stability of these waves reduces to spectral stability of usual (one-frequency) solitary waves which we obtained in our previous work.

Roland Donninger (Universität Wien)

Linear stability of the Skyrmion

The Skyrme model is a relativistic field theory that arises by adding higher-order terms to the wave maps Lagrangian. This modification was proposed in order to prevent blow up and to allow for the existence of static solutions of finite energy in 3 dimensions. A particular static solution, the Skyrmion, is conjectured to be a global attractor for the nonlinear flow. We present a rigorous proof for the linear stability of the Skyrmion which is based on a mildly computer-assisted method of broad applicability. This is joint work with Matthew Creek, Wilhelm Schlag, and Stanley Snelson.

Cristian Gavrus (University of California, Berkeley)

Global well-posedness of the high dimensional Maxwell-Dirac equation for small critical data

We discuss the global well-posedness of the massless Maxwell-Dirac equation in Coulomb gauge on $\mathbb{R}^{1+d}$ $(d \geq 4)$ for data with small scale-critical Sobolev norm, as well as modified scattering of the solutions. Main components of our proof are A) uncovering null structure of Maxwell-Dirac in the Coulomb gauge, and B) proving solvability of the underlying covariant Dirac equation. A key step for achieving both is to exploit (and justify) a deep analogy between Maxwell-Dirac and Maxwell-Klein-Gordon (for which an analogous result was proved earlier by Krieger-Sterbenz-Tataru), which says that the most difficult part of Maxwell-Dirac takes essentially the same form as Maxwell-Klein-Gordon. We will also discuss the massive case. This is joint work with Sung-Jin Oh.

Joachim Krieger (École polytechnique fédérale de Lausanne)

On stability of blow up for co-rotational Wave Maps

I will discuss a recent result, joint with S. Miao, on stability of certain blow up solutions for critical co-rotational Wave Maps.

Irfan Glogic (Universität Wien)

Existence and stability of blowup for wave maps into negatively curved targets

Harmonic maps between Lorentzian and Riemannian manifolds go under the name of wave maps. We consider wave maps from $(1+d)$-dimensional Minkowski space into negatively curved Riemannian manifolds. Furthermore, we are concerned with the question of blowup. Due to their resemblance to defocussing nonlinear wave equations, this kind of wave maps were believed to not exhibit generic (stable) blowup. Cazenave, Shatah and Tahvildar-Zadeh proved in 1998 that self-similar blowup exists for $d=7$. We complement this result by constructing for each dimension $d\geq 8$ a negatively curved, d-dimensional target manifold that allows for the existence of a self-similar wave map. What is more, we prove that our solutions provide a stable blowup mechanism for the corresponding Cauchy problem. This, in addition to being the first example of stable blowup for wave maps with negatively curved targets, also shows that intuition from above was in fact false. This is joint work with Roland Donninger, University of Vienna.

Birgit Schörkhuber (Karlsruher Institut für Technologie)

Co-dimension one stable blowup in supercritical wave equations

We consider the focussing cubic NLW on $\mathbb{R}^7$. This model is energy supercritical and it is well-known that the stable blowup is described (at least locally) by a trivial self-similar solution referred to as ODE blowup. In this talk, we address blowup solutions with a non-trivial limiting profile. In particular, it will be shown that the equation has a non-trivial self-similar solution which is stable along a codimension one Lipschitz manifold of perturbations of the initial data. This is joint work with Irfan Glogic (Vienna).

Eric Séré (Université Paris-Dauphine)

Taking into account the polarized Dirac sea: the nonlinear Euler-Heisenberg model.

This is joint work with Philippe Gravejat and Mathieu Lewin (J. Math. Pures Appl., in press). The Euler-Heisenberg model provides a nonlinear system of equations for the electromagnetic field. The nonlinearity takes into account the interaction between the classical electromagnetic field and the Dirac sea. It depends on a small coupling parameter and one recovers the linear Maxwell equations when this parameter is set to zero. In most situations the linear (Maxwell) approximation is extremely accurate, but nonlinear effects cannot be neglected in very strong fields, as for instance on the surface of some neutron stars called «magnetars». We give the first rigorous derivation of the Euler-Heisenberg magnetic energy in the semi-classical limit of slowly varying, time-independent, magnetic fields. The question of (slowly) time-varying fields remains open.

William Borrelli (Université Paris-Dauphine)

Effective Dirac equations in honeycomb structures

Recently, new two-dimensional materials possessing Dirac fermions low-energy excitations have been discovered, the most famous being graphene. In those materials electrons at the Fermi level have zero apparent mass and can be described using the massless Dirac equation. More generally, Schrödinger operators with honeycomb potentials generically exhibit conical intersections (the so-called Dirac points) in their dispersion bands. This leads to the appearance of Dirac as the effective operator, describing the electron dynamics around Dirac points. The large, but finite, time-scale validity of the Dirac approximation has been proved for the linear case, and for cubic nonlinearities. The latter case corresponds to the *Gross-Pitaevski equation*, which is a fundamental model in the description of macroscopic quantum phenomena and in nonlinear optics. The cubic

Dirac equation in 2D is *critical* for the Sobolev embedding, and this makes the existence of stationary solutions a non-trivial problem. Describing finite size samples of graphene requires to choose suitable boundary conditions for the Dirac operator. Local well-posedness for a model of electron transport in graphene has been proved, and the existence of stationary solutions has also been adressed. In this talk I will give an overview of this results.

Xian Liao (Université Paris-Dauphine)

Conserved energies for the one-dimensional Gross-Pitaevskii equation

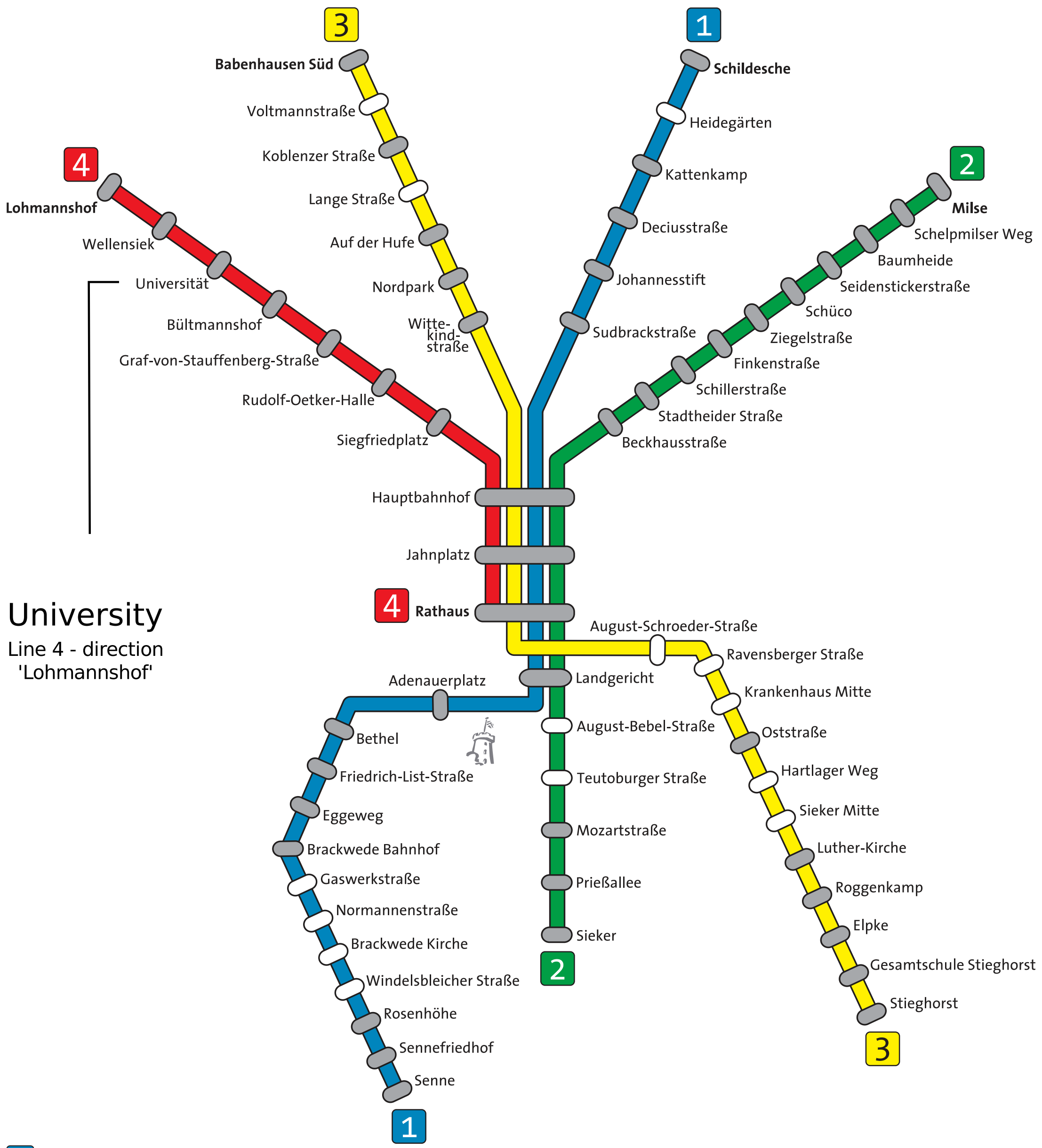
In this talk I will establish a family of conserved energies for the one dimensional Gross-Pitaevskii equation, which is simply the defocusing cubic nonlinear Schroedinger equation but with nonzero boundary condition at infinity. The formulation of the conserved energies relies on the detailed analysis of the invariant transmission coefficient, which is associated to the Lax operator of the GP equation. This is a joint work with Herbert Koch.

Daniel Tataru (University of California, Berkeley)

The energy critical Yang-Mills flow

The aim of the talk will be to provide an overview of the recent proof of the Threshold Conjecture for the energy critical Yang-Mills flow, in 4+1 dimensions. This is joint work with Sung-Jin Oh.

Tram map



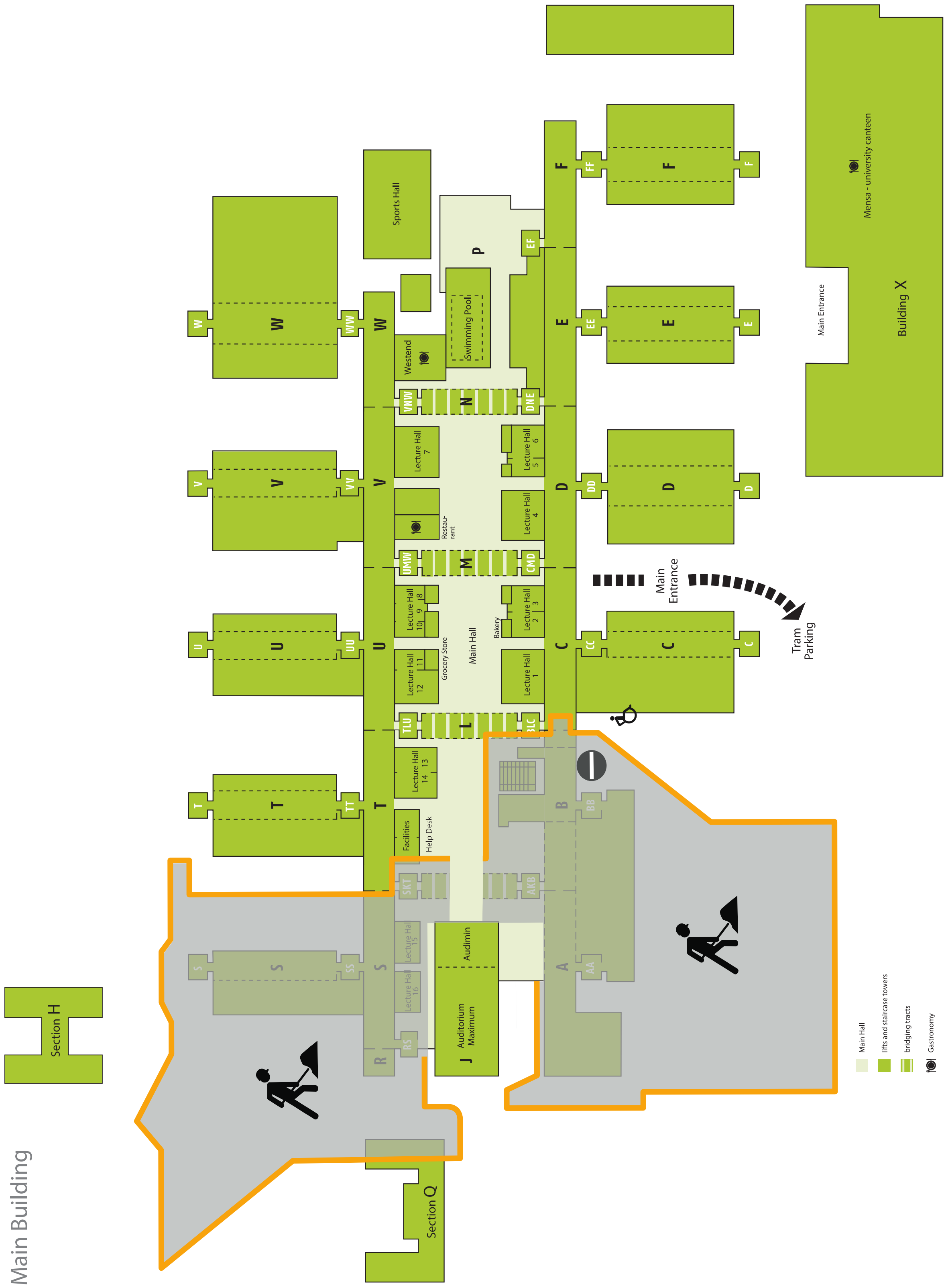
1 Senne – Jahnplatz – Schildesche

2 Sieker – Jahnplatz – Milse

3 Stieghorst – Jahnplatz – Babenhausen Süd

4 Rathaus – Jahnplatz – Universität – Lohmannshof

Campus map


Main Building
Section H
Section Q
Building X
Mensa - university canteen
Main Entrance
Main Entrance
Tram
Parking
Sports Hall
Swimming Pool
Westend
Main Hall
Grocery Store
Bakery
Restau-
rant
Facilities
Help Desk
Audimin
Auditorium
Maximum
Main Hall
lifts and staircase towers
bridging tracts
Gastronomy