CRC Retreat 2019: Abstracts

September 30 – October 1, 2019

On a class of infinite-dimensional singular stochastic control problems

Giorgio Ferrari

We study a class of infinite-dimensional singular stochastic control problems with applications in economic theory and finance. The control process linearly affects an abstract evolution equation on a suitable partially-ordered infinite-dimensional space X, it takes values in the positive cone of X, and it has right-continuous and nondecreasing paths. We first provide a rigorous formulation of the problem by properly defining the controlled dynamics and integrals with respect to the control process. We then exploit the concave structure of our problem and derive necessary and sufficient first-order conditions for optimality. The latter are finally exploited in a specification of the model where we find an explicit expression of the optimal control. The techniques used are those of semigroup theory, vector-valued integration, convex analysis, and general theory of stochastic processes. This talk is based on a joint work with Salvatore Federico, Michael Röckner, and Frank Riedel.

Numerics of stochastic games with asymmetric information

Tsiry Avisoa Randrianasolo

In this talk, we will present a convergent numerical approximation of the value function $V(t, x, p)$ of a stochastic game with asymmetric information. We will combine 3 numerical methods in t, in x, and in p and find the continuous approximation of V by piecewise linear interpolation. In particular a suitable choice of triangulation allows to construct a piecewise linear function that preserves the convexity in p of the solution. Eventually we will use a test problem to illustrate how the numerical method works. We use a particular algorithm to compute the convex data point and convex envelope but the whole numerical method is built so that general convex hull algorithms for high dimension (more than 4) such as the *Quickhull* algorithm works as well. This is a joint project with Ľubomír Baňas and Giorgio Ferrari.

Robust space-time a posteriori estimates for the non-smooth Cahn–Hilliard equation

Christian Vieth

We consider the Cahn–Hilliard equation with a non-smooth potential in the energy. The model consists of variational inequalities which require suitable numerical treatment. We propose a smooth regularization approach for the the non-smooth problem and construct a space-time adaptive numerical method for the approximation of the resulting nonlinear partial differential equation. We derive a posteriori error estimates to guarantee rigorous error control of the numerical approximation which is robust with respect to the model parameters. This is joint work with Lubomír Baňas.

Analysis on the Wasserstein space

Feng-Yu Wang

By constructing the diffusion processes generated by second-order differential operators on the Wasserstein space, non-linear Fokker-Planck equations are described by the corresponding linear ones. As applications, the exponential ergodicity as well as Schrödinger type PDEs on the Wasserstein space are investigated.

Solvability and ill-posedness of the isentropic Euler system

Martina Hofmanová

I will discuss several puzzling results related to solvability and ill-posedness of the isentropic Euler system. On the one hand, the method of convex integration can be used to construct infinitely many wild solutions as well as rather surprising approximation results. On the other hand, ideas from Markov selections and a new notion of dissipative solution allow to select a semiflow of physically reasonable solutions and to exclude oscillation defects in certain cases.

Innovation investment under financial constraints

Herbert Dawid

We study the effect of financial constraints on the optimal innovation investment of an incumbent firm facing uncertainty about the effect of its innovation effort and the evolution of demand as well as a risk of bankruptcy if its liquidity is negative. In the case of a monopoly it is shown that optimal investment of a firm depends in a U-shaped way on its liquidity and that a small degree of bankruptcy risk is sufficient to prevent the firm from going into debt in order to finance innovation investments. Furthermore, there is a non-monotone relationship between the profitability of the incumbent's existing market and the expected innovation time. We show that these results are in accordance with empirical evidence obtained from firm level data and discuss extensions of the setting to scenarios with competition.

A rough path approach to the stochastic Landau-Lifshitz-Gilbert equation

Emanuela Gussetti

The Landau-Lifshitz-Gilbert equation describes the behaviour of a ferromagnetic material on a bounded domain. I will present a result on existence and uniqueness of strong solutions to this problem posed on a one dimensional domain and perturbed by a linear multiplicative noise driven by a general rough path.

Ancestral lines in the diffusive mutation-selection model with pairwise interaction

Luigi Esercito

Dealing with selection is one of the main tasks in population genetics. The aim of this talk is to describe the Moran model with mutation and a particular type of frequency-dependent selection, introducing a backward process called the ancestral selection graph, and examining the relationship between the two.

Expectation values of characteristic polynomials in polynomial ensembles

Tim Würfel

Random matrices appear in a wide variety of scientific fields and applications, from statistical physics, signal and communication systems to multivariate statistics. We study random matrix ensembles with an external source which yield a biorthogonal structure in the sense of A. Borodin. In particular we are interested expectation values of characteristic polynomials which yield expressions for the correlation functions of the respective random matrix ensemble.

Upper envelopes of families of Feller semigroups and viscosity solutions to a class of nonlinear Cauchy problems

Max Nendel

We construct the smallest semigroup S that dominates a given family of linear Feller semigroups. The semigroup S is called the semigroup envelope or Nisio semigroup. We investigate strong continuity properties of the semigroup envelope and show that it is a viscosity solution to a nonlinear abstract Cauchy problem. We derive a condition for the existence of a Markov process under a nonlinear expectation for the case where the state space of the Feller processes is locally compact. The procedure is then applied to numerous examples, in particular nonlinear PDEs that arise from control problems for infinite dimensional Ornstein-Uhlenbeck processes and infinite dimensional Lévy processes.

Some mathematical aspects of machine learning

Zhiming Ma

In recent years the technique of machine learning has practically achieved great success. But its theoretic basis is still weak, the mathematical aspects of machine learning are far from satisfied. In this talk I shall briefly report some progress on machine learning. The progress is being made by my students and the colleagues working in MSRA, and its developement involves some mathematical thinking and mathematical methods.

Quantum graphs, aperiodic order, and locally constant cocycles

David Damanik

We discuss unbounded quantum graphs with spectra that are small in a suitable sense. For example, if the structure is generated by mechanisms from the theory of aperiodic order, it can be shown that the spectra have Lebesgue measure zero. In the proof of this statement one encounters the need to develop a trace map analysis for locally constant cocycles over suitable subshifts. This bears a resemblance to related work on random products of matrices with local correlations, which will be briefly discussed as well if time permits.

Ancestral lines in the diffusive mutation-selection model with pairwise interaction

Luigi Esercito

Dealing with selection is one of the main tasks in population genetics. The aim of this talk is to describe the Moran model with mutation and a particular type of frequency-dependent selection, introducing a backward process called the ancestral selection graph, and examining the relationship between the two.

Numerical approximation and the sharp-interface limit of the stochastic Cahn–Hilliard equation

Lubomír Baňas

The Cahn-Hilliard equation is a fourth oder parabolic partial differential equation (PDE) that is widely used as a phenomenological model to describe the evolution of interfaces in many practical problems, such as, the microstructure formation in materials, fluid flow, etc. I has been observed in the engineering literature that the stochastic version of the Cahn-Hilliard equation provides a better description of the experimentally observed evolution of complex microstructure. The equation belongs to a class of so-called phase-field models where the interface is replaced by a diffuse layer with small thickness proportional to an interfacial thickness parameter . It can be shown that for vanishing interfacial thickness the deterministic as well as the stochastic Cahn-Hilliard equation (with proper scaling of the noise) both converge to a sharp-interface limit which is given by the deterministic Hele-Shaw problem. We propose a time implicit numerical approximation of the stochastic Cahn-Hilliard equation which is robust with respect to the interfacial thickness parameter. We show that, with suitable scaling of the noise, the sharp-interface limit of the proposed numerical approximation converges to the deterministic Hele-Shaw problem. In addition we present numerical evidence that without the scaling of the noise the sharp-interface limit of the stochastic Cahn-Hilliard equation is a stochastic version of the Hele-Shaw problem. We propose a numerical approximation of the stochastic Hele-Shaw problem and present computational results which demonstrate the respective convergence of the stochastic Cahn-Hilliard equation to the deterministic or the stochastic version of the Hele-Shaw problem depending on scaling of the noise term.

Fourier transform of Rauzy fractals and point spectrum of 1D Pisot inflation tilings

Michael Baake

Primitive inflation tilings of the real line with finitely many tiles of natural length and a Pisot–Vijayaraghavan unit as inflation factor are considered. We present an approach to the pure point part of their diffraction spectrum on the basis of a Fourier matrix cocycle in internal space. This cocycle leads to a transfer matrix equation and thus to a closed expression of matrix Riesz product type for the Fourier transforms of the windows for the covering model sets. In general, these windows are complicated Rauzy fractals and thus difficult to handle. Equivalently, this approach permits a construction of the (always continuous) eigenfunctions for the translation dynamical system induced by the inflation rule. We review and further develop the underlying theory, and apply it to the family of Pisa substitutions, with special emphasis on the Tribonacci case.

Submodular mean field games: existence and approximation of solutions

Jodi Dianetti

Mean field games are limit models for non-cooperative symmetric N-player games with mean field interaction as the number of players N tends to infinity. In this talk, we introduce a class of mean field games with costs that are submodular with respect to a suitable order relation on the state and measure space. The submodularity assumption has a number of interesting consequences. Firstly, it allows us to prove existence of solutions via an application of Tarski's fixed point theorem, covering cases with discontinuous dependence on the measure variable. Secondly, it ensures that the set of solutions enjoys a lattice structure: in particular, there exist a minimal and a maximal solution. Finally, it guarantees that those two solutions can be obtained through a simple learning procedure based on the iterations of the best-response-map.

This talk is based on a joint work with Giorgio Ferrari, Markus Fischer and Max Nendel.

Numerical approximation of parabolic problems with fractional differentiability

Jörn Wichmann

We consider parabolic p-Laplace systems under fractional differentiability assumptions. At first we present a well known optimal convergence result for the implicite Euler finite element method. Then we adapt the method to the fractional setting and prove optimal convergence with dependents on the fractional differentiability index $\alpha_t, \alpha_x \in (0, 1]$.