

Realizing the Potential – Theory in Bielefeld

September 28th, 2019

Bielefeld University
Lecture Room: V2-210/216

This workshop is funded by:

- Faculty of Mathematics
- Collaborative Research Center *CRC 1283*
- International Research Training Group *IRTG 2235*

Convenor: Moritz Kassmann

https://www.sfb1283.uni-bielefeld.de/2019_RTP/

Schedule: Saturday September 28

Lecture Room: **V2-210/216**

- 09:20–09:30 **Opening**
- 09:30–10:05 **W. Hansen (Bielefeld)**
Extremal positive solutions for $\Delta u + u\mu = 0$
- 10:10–10:45 **K. Bogdan (Wroclaw)**
Shot-down jump processes
- 10:45–11:15 **Coffee break**
- 11:15–11:50 **T. Kumagai (Kyoto)**
Stability of heat kernel estimates and parabolic Harnack inequalities for symmetric Dirichlet forms
- 11:55–12:30 **P. Kim (Seoul)**
Dirichlet heat estimates of symmetric stable processes on horn-shaped regions
- 12:30–14:00 **Lunch break**
- 14:00–14:55 **G. Grubb (Copenhagen)**
Regularity of solutions to fractional-order boundary problems
- 15:00–15:35 **T. Kuusi (Helsinki)**
Higher-order linearization and regularity in nonlinear homogenization
- 15:40–16:15 **J. Serra (Zürich)**
Analysis of singularities in the classical obstacle problem and a conjecture of Schaeffer
- 16:15–16:45 **Coffee break**
- 16:45–17:20 **T. Kulczycki (Wroclaw)**
Semigroup properties of solutions of SDEs driven by Lévy processes with independent coordinates
- 17:25–18:00 **L. Beznea (Bucharest)**
Potential theory for infinite dimensional processes
- 19:00– **Conference dinner**

Wolfhard Hansen

Extremal positive solutions for $\Delta u + u\mu = 0$

Given a measure μ on $R^d \setminus \{0\}$, $d \geq 2$, which locally generates continuous real potentials, let $\mathcal{H}_0^{\Delta+\mu}(U)$ be the set of all continuous real solutions $u \geq 0$ to the equation $\Delta u + u\mu = 0$ on the punctured unit ball U satisfying $\lim_{|x| \rightarrow 1} u(x) = 0$. Annoyingly, it is still an open question if, for $d \geq 3$, the convex cone $\mathcal{H}_0^{\Delta+\mu}(U)$ always consists of multiples of *one* function (*Picard principle*). In this talk various partial results, obtained by joint work with Ivan Netuka (Charles University, Prague), shall be presented.

Krzysztof Bogdan

Shot-down jump processes

The shot-down process is a strong Markov process which is annihilated (shot down) when jumping “over” or to the complement of a given domain of the Euclidean space. Due to specific features of the shot-down time, such processes offer a novel mathematical framework for nonlocal interactions. In this work we construct the shot-down process for the fractional Laplacian in bounded smooth domains. We define its transition density and characterize the Dirichlet form. We show that the shot-down Green function is comparable to that of the fractional Laplacian with Dirichlet conditions on the same domain. However, we also prove that for non-convex domains the shot-down transition density is incomparable with the usual Dirichlet heat kernel of the fractional Laplacian and the Harnack inequality may fail for harmonic functions of the shot-down process. This is a joint work with Kajetan Jastrzębski, Moritz Kassmann, Michał Kijaczko and Paweł Popławski.

Takashi Kumagai

Stability of heat kernel estimates and parabolic Harnack inequalities for symmetric Dirichlet forms

We consider symmetric Dirichlet forms that consist of strongly local (diffusion) part and non-local (jump) part on a metric measure space. Under general volume doubling condition and some mild assumptions on scaling functions, we establish stability of two-sided heat kernel estimates in terms of Poincaré inequalities, jumping kernels and generalized capacity inequalities. We also discuss characterizations of the associated parabolic Harnack inequalities. Our results apply to symmetric diffusions with jumps even when the underlying spaces have walk dimensions larger than 2. This is a joint work with Z.Q. Chen (Seattle) and J. Wang (Fuzhou).

Panki Kim

Dirichlet heat estimates of symmetric stable processes on horn-shaped regions

In this talk, we consider symmetric stable processes on (unbounded) horn-shaped regions which are non-uniformly $C^{1,1}$ near infinity. By making full use of probabilistic approaches, we establish two-sided Dirichlet heat estimates (global in time) of such processes. The estimates are very sensitive with respect to the reference function of each horn-shaped region. Our results also cover the case that the associated Dirichlet semigroup is not intrinsically ultracontractive.

Gerd Grubb

Regularity of solutions to fractional-order boundary problems

Fractional powers of the Laplacian, $(-\Delta)^a$ with $0 < a < 1$, have been the focus of much research in recent years, because they have an important role as generators of Lévy processes of interest in financial theory, and enter also in mathematical physics and differential geometry. Various methods have been used, mostly from probability and potential theory. Our special interest is pseudodifferential methods (which combine the integral operator point of view with the Fourier transform). We consider this and other related operators of order $2a$, restricted to act on a bounded smooth subset Ω of R^n . Here one can define a homogeneous Dirichlet problem, whose precise domain has been a subject of research. We shall explain a fairly elementary characterization of the domain (corresponding to data in Sobolev or Hölder spaces), which has been worked out recently. Next, this is used in a study of the regularity of solutions to time-dependent ("heat") problems, and resolvent problems, associated with the operator. It is found that, contrary to problems without a time-parameter or a spectral parameter, the boundary regularity of solutions does not increase to infinity when the regularity of the data grows to infinity — unless extra boundary conditions are imposed.

Tuomo Kuusi

Higher-order linearization and regularity in nonlinear homogenization

The analysis of higher-order linearized equations lets us develop an incisive large-scale higher regularity theory for solutions of nonlinear elliptic equations in the context of homogenization. We proceed in analogy to the role of the Schauder theory in resolving Hilbert's 19th problem on the regularity of solutions to nonlinear equations with smooth coefficients.

Joaquim Serra

Analysis of singularities in the classical obstacle problem and a conjecture of Schaeffer

Caffarelli obtained in the 1970's a fundamental breakthrough: he gave a robust sufficient condition that implies the local smoothness of the free boundary. Complementarily, in the last years we worked towards obtaining a more complete understanding of singularities. This has lead us to proving, in dimensions three and four, a conjecture of Schaeffer which asserts that for generic boundary data there are no singularities (although one can construct examples of boundary data and obstacles for which the singular set is as large as the regular set!).

Tadeusz Kulczycki

Semigroup properties of solutions of SDEs driven by Lévy processes with independent coordinates

We study the stochastic differential equation $dX_t = A(X_t) dZ_t$, $X_0 = x$, where $Z_t = (Z_t^{(1)}, \dots, Z_t^{(d)})^T$ and $Z_t^{(1)}, \dots, Z_t^{(d)}$ are independent one-dimensional Lévy processes with characteristic exponents $\psi_1, \dots, \psi_d$. We assume that each ψ_i satisfies a weak lower scaling condition $WLSC(\alpha, 0, \underline{C})$, a weak upper scaling condition $WUSC(\beta, 1, \overline{C})$ (where $0 < \alpha \leq \beta < 2$) and some additional regularity properties. We consider two mutually exclusive assumptions: either (i) all $\psi_1, \dots, \psi_d$ are the same and α, β are arbitrary, or (ii) not all $\psi_1, \dots, \psi_d$ are the same and $\alpha > (2/3)\beta$. We also assume that the determinant of $A(x) = (a_{ij}(x))$ is bounded away from zero, and $a_{ij}(x)$ are bounded and Lipschitz continuous. In both cases (i) and (ii) we prove that for any fixed $\gamma \in (0, \alpha) \cap (0, 1]$ the semigroup P_t of the process X_t satisfies $|P_t f(x) - P_t f(y)| \leq ct^{-\gamma/\alpha} |x - y|^\gamma \|f\|_\infty$ for arbitrary bounded Borel function f . We also show the existence of a transition density of the process X_t .

Lucian Beznea

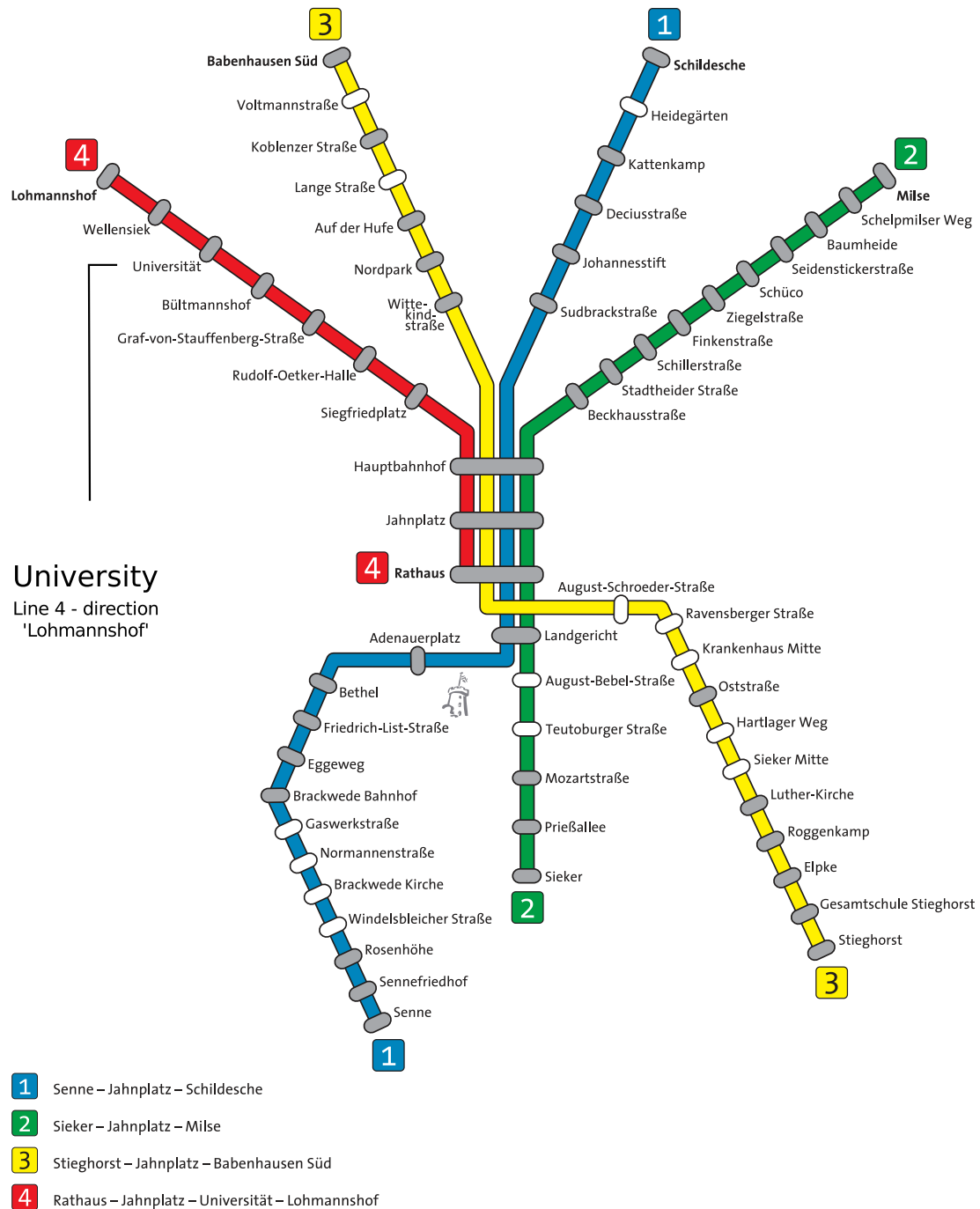
Potential theory for infinite dimensional processes

We survey results obtained during a long time collaboration between the potential theory and stochastic analysis groups from Bucharest and Bielefeld. In particular, we present recent applications of several cone of potentials tools like the reduced function, Choquet capacities, resolvent families of kernels, excessive functions, and their probabilistic interpretations for Markov processes on general state spaces, allowing applications to infinite dimensional situations.

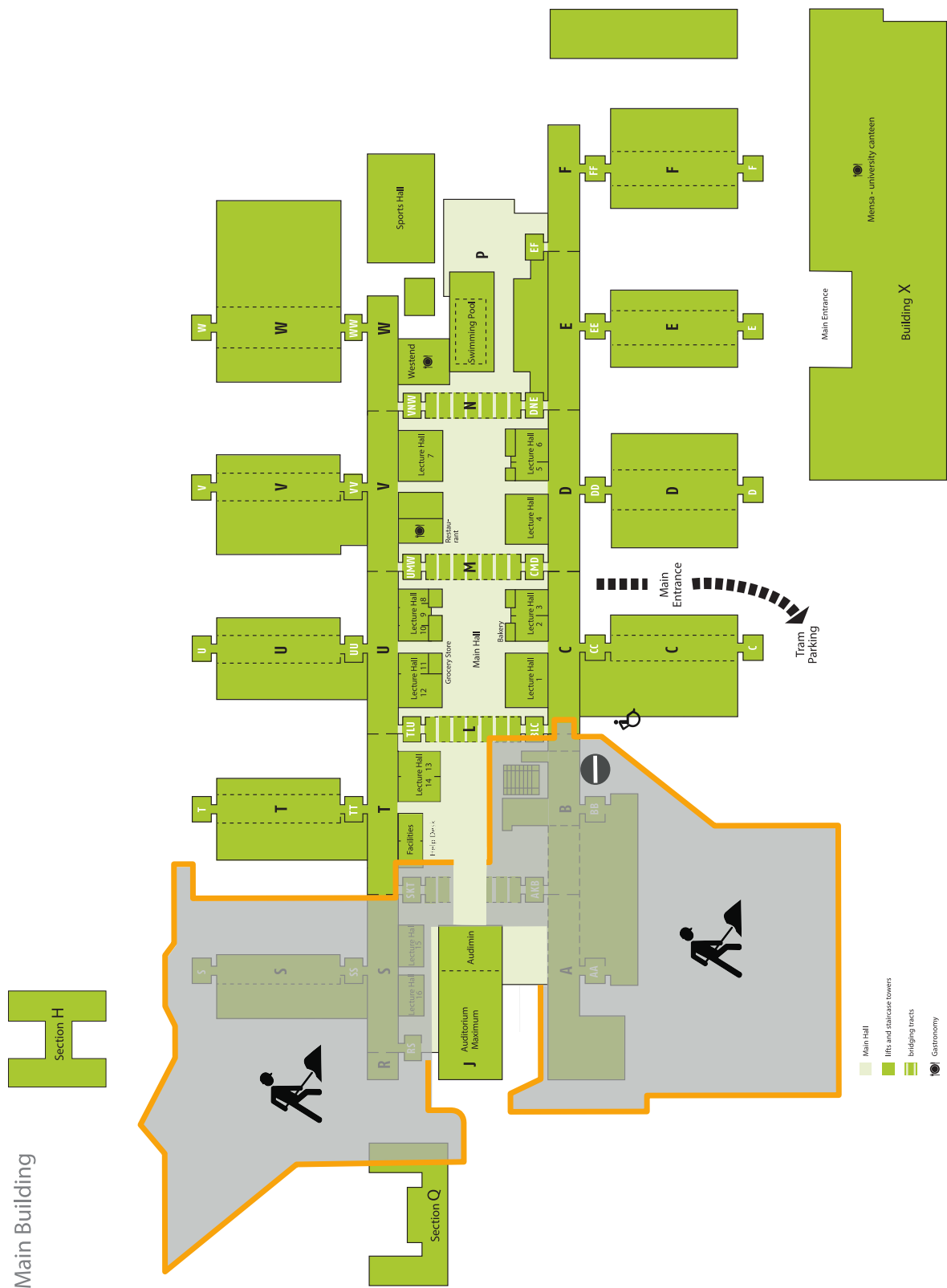
Some recommendations for restaurants

- (1) **Argentina-Steakhouse** (<https://argentina-steakhouse.de/>)
Argentinian beef at its best
- (2) **Brauhaus** (<https://bielefeld.brauhaus-joh-albrecht.de/>)
Home made beer plus German style food
- (3) **Kometsu** (<http://www.kometsu.de/index.html>)
Authentic Japanese place for sushi
- (4) **KDW** (<http://www.kdw-restaurant.de/index.html>)
Fine Greek cuisine
- (5) **Numa** (<http://www.numa.de/>)
Asia meets East-Westphalia
- (6) **Wernings Weinstube** (<https://www.wernings-weinstube.de/>)
Some regional dishes plus a good selection of wines
- (7) **Sparrenburg** (<https://www.restaurant-sparrenburg.de/>)
German style food at the castle above Bielefeld
- (8) **Wilde Kuh/ Wilde Kuh 2** (<https://www.facebook.com/WildeKuhBurger/>)
Excellent "build your own burger" place
- (9) **Three sixty** (<http://bielefeld.three-sixty.de/>)
Sports bar with burgers and other snacks
- (10) **Jivino** (<http://www.jivino-enoteca.de/>)
Spanish tapas
- (11) **Bernstein** (<https://www.the-bernstein.com/>)
Dinner plus cocktails in a fancy rooftop restaurant

Tram map



Campus map



Line 4

Rathaus

Linie

4

Jahnplatz - Hauptbahnhof - Rudolf-Oetker-Halle - Universität - Lohmannshof

Linie

10

Jahnplatz - Hauptbahnhof - Rudolf-Oetker-Halle - Universität - Lohmannshof

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Ⓢ : fährt nur an Schultagen, Änderungen möglich

Diesen Fahrplan erhalten Sie in unseren Kundenzentren und unter www.moBiel.de.

Bitte beachten Sie die Sonderfahrpläne am 24.12. und 31.12.

Das Angebot ab 5.00 Uhr samstags, sonntags und an Feiertagen finden Sie in den NachtBus-Fahrplänen.

Nächste Verkaufagentur:

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Infotelefon moBiel, rund um die Uhr (05 21) 51-45 45

gültig ab: 17.06.2018

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Universität																moBiel			
Linie 4 Rudolf-Oetker-Halle - Hauptbahnhof - Jahnplatz - Rathaus																			
Linie 10 Rudolf-Oetker-Halle - Hauptbahnhof - Jahnplatz - Rathaus - Krankenhaus Mitte - Sieker Mitte - Stieghorst																			
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A : fährt als StadtBahn-Linie 2 nach Sieker B : fährt freitags und vor Feiertagen als StadtBahn-Linie 4, montags bis donnerstags und an Sonn- und Feiertagen als StadtBahn-Linie 2 nach Sieker C : fährt freitags und vor Feiertagen als StadtBahn-Linie 4, montags bis donnerstags und an Sonn- und Feiertagen als StadtBahn-Linie 10 D : fährt freitags und vor Feiertagen als StadtBahn-Linie 2 nach Sieker, montags bis donnerstags und sonn- und feiertags als StadtBahn-Linie 10 bis Sieker Mitte E : StadtBahn-Linie 10 F : fährt nur bis Sieker Mitte ☉ : fährt nur an Schultagen, Änderungen möglich Diesen Fahrplan erhalten Sie in unseren Kundenzentren und unter www.moBiel.de. Bitte beachten Sie die Sonderfahrpläne am 24.12. und 31.12. Das Angebot ab 5.00 Uhr samstags, sonntags und an Feiertagen finden Sie in den NachtBus-Fahrplänen. gültig ab: 17.06.2018 Infotelefon moBiel, rund um die Uhr (05 21) 51-45 45 www.moBiel.de																			