CRC Retreat 2022

Program

Start	End	Projects	Title	Speakers
Thursday, September 1				
08:00	10:00	Bus: Bielefeld - Hofgeismar		
10:00	10:45	Arrival and Welcome Coffee		
10:45	10:50		Opening	
10:50	11:20	B1-B3-B8	Numerics for self-organized criticality	L. Banas
11:25	11:55	B8	Conservative SPDEs as fluctuating mean field limits of stochastic gradient descent	V. Konarovskyi
12:00	13:30	Lunch		
13:30	14:00	C4-C5	Submodular mean field games: results and perspectives	J. Dianetti
14:05	14:35	C4	Optimal execution with multiplicative price impact and incomplete information on the return	F. Dammann
14:40	15:10	C1	Random walk theory meets population genetics: The Wright-Fisher model with efficiency	H. Dopmeyer
15:15	16:00	Coffee		
16:00	16:45	Mercator Fellow	Synchronization in a Kuramoto mean field game	H. Mete Soner (online)
16:50	17:35	C7 (new PI)	Asymptotic parametrization of Wasserstein balls and perturbed Markovian transition semigroups	M. Nendel
18:00	19:00	Dinner		
19:15	20:00	Guided tour / walk: Brunnenpark Hofgeismar		
20:30		Pub quiz		
Friday, September 2				
07:30	09:00	Breakfast		
09:00	09:45	New project	AR(1)-sequences with Rademacher innovations	V. Wachtel
09:50	10:20	C6	Boundary behavior of 2D-Coulomb gases	L. Molag
10:25	10:50	Coffee		
10:50	11:20	A1-B1-B8	Dispersive effects in stochastic PDE	A. Niesdroy
11:25	11:55	A3-A7	Higher differentiability for nonlocal problems	S. Nowak
12:00	13:30	Lunch		
13:00	*14:00*	*Meeting of principal investigators*		
14:00	14:45	A7 (new Co-PI)	Behind the regularity: qualitative and quantitative analysis of (non)local models admitting the energy gaps	A. Balci
14:50	15:20	A3	Volume growth and heat kernel bounds on Riemannian manifolds with ends	P. Sürig
15:25	16:00	Coffee		
16:00	18:00	Bus: Hofgeismar - Bielefeld		

CRC Retreat 2022: Abstracts

September 1 – September 2, 2022

Numerics for self-organized criticality

Lubomír Baňas

The concept of self organized criticality (SOC) describes a property of complex systems that evolve towards a sub-critical state which is then maintained due to the occurrence of critical events (so-called avalanches) which occur with a power-law frequency. The SOC theory has applications in a range of areas such as, e.g., neurology, econophysics, geology, etc. We consider a degenerate stochastic PDE which is obtained as an asymptotic limit of discrete SOC models. We propose explicit and implicit discretization schemes for the approximation of the considered SPDE(s) and present several numerical studies which demonstrate some interesting properties of the continuous models.

Conservative SPDEs as fluctuating mean field limits of stochastic gradient descent

Vitalii Konarovskyi

In my talk, the convergence of stochastic interacting particle systems in the mean-field limit to solutions to conservative stochastic partial differential equations will be shown. We will discuss the optimal rate of convergence and derive a quantitative central limit theorem for such SPDEs. The results apply in particular to the convergence in the mean-field scaling of stochastic gradient descent dynamics in overparametrized, shallow neural networks to solutions to SPDEs. Moreover, we will see that the inclusion of fluctuations in the limiting SPDE improves the rate of convergence, and retains information about the fluctuations of stochastic gradient descent in the continuum limit. The talk is based on joint work with Benjamin Gess and Rishabh S. Gvalani.

Submodular mean field games: results and perspectives

Jodi Dianetti

We study mean field games with scalar Itô-type dynamics and costs that are submodular with respect to a suitable order relation on the state and measure space. The submodularity assumption has a number of interesting consequences. Firstly, it allows us to prove existence of solutions via an application of Tarski's fixed point theorem, covering cases with discontinuous dependence on the measure variable. Secondly, it ensures that the set of solutions enjoys a lattice structure: in particular, there exist a minimal and a maximal solution. Thirdly, it guarantees that those two solutions can be obtained through a simple learning procedure based on the iterations of the best-response-map. Our approach also allows to treat submodular mean field games with common noise, as well as mean field games with singular controls, optimal stopping and reflecting boundary conditions.

This talks is based on some joint works together with Giorgio Ferrari, Markus Fischer and Max Nendel.

Optimal execution with multiplicative price impact and incomplete information on the return

Felix Dammann

In this project we study a singular stochastic control problem in which we model the problem of an investor who posseses a finite amount of assets and aims at selling them on the market. We assume that the investor has only incomplete information on the drift of the stock price, but is able to monitor its evolution on the market in order to form a belief regarding the future price trend.

Our mathematical modelling leads to a singular stochastic control problem featuring a finite-fuel constraint and partial observation. We provide the complete analysis by exploiting the connection to a related two-dimensional optimal stopping problem. The solution is then given in terms of a belief-dependent free boundary b, that triggers the optimal liquidation rule.

This talk is based on joint work with Giorgio Ferrari.

Random walk theory meets population genetics: The Wright-Fisher model with efficiency

Hannah Dopmeyer

Availability of a resource defines the size of a population. But how do different consumption strategies of that resource influence the composition and evolution of the population? To answer this question a suitable model, the Wright-Fisher Model with Efficiency, is constructed and studied using the methods from population genetics combined with random walk theory.

In this discrete model the amount of resource K is fixed and individuals need either 1 (inefficient) or 1-kappa (efficient) units of resource to reproduce. The next generation is formed by randomly (uniformly with replacement) picking an individual of the previous generation as parent. The offspring is generated using the corresponding amount of resource and inherits the consumption strategy of its parent. This procedure is repeated until the amount of resource is fully consumed. The last individual is either accepted (Ending 1, overshoot the resource) or rejected (Ending 2, undershoot the resource). The proportion of inefficient individuals in generation n with K units of resource is denoted by X_n^K.

In this talk the Wright-Fisher model and its refinement, the Wright-Fisher model with Efficiency are introduced. The setting is then translated into the context of a random walk. Using classical results from random walk theory, key properties of the process will be presented. Finally a sketch of the proof about the convergence of the process X on the evolutionary time-scale towards the solution of a stochastic differential equation will be given.

This project is joint work with Fernando Cordero and Ellen Baake.

Synchronization in a Kuramoto mean field game

Halil Mete Soner (online talk)

Originally motivated by systems of chemical and biological oscillators, the classical Kuramoto model has found an amazing range of applications from neuroscience to Josephson junctions in superconductors, and has become a key mathematical model to describe self organization in complex systems. These autonomous oscillators are coupled through a nonlinear interaction term which plays a central role in the long term behavior of the system. While the system is unsynchronized when this term is not sufficiently strong, fascinatingly, they exhibit an abrupt transition to a full synchronization above a critical value of the interaction parameter. We explore this system in the mean field formalism. We treat the system of oscillators as an infinite particle system, but instead of positing the dynamics of the particles, we let the individual particles determine endogenously their behaviors by minimizing a cost functional and eventually, settling in a Nash equilibrium. The mean field game also exhibits a bifurcation from unsynhcronization to self-organization. This approach has found interesting applications including circadian rhythms and jet-lag recovery. This is joint work with Rene Carmona and Quentin Cornier.

Asymptotic parametrization of Wasserstein balls and perturbed Markovian transition semigroups

Max Nendel

In mathematical finance and actuarial science, the assessment of risk is closely related to matching the distribution of the underlying risk factors in an accurate way; a major issue in this process is the so-called model uncertainty or epistemic uncertainty. The latter refer to the impossibility of perfectly capturing the randomness of future states in a single stochastic framework. We first explore a static setting that takes into account model uncertainty in the distribution of a risk factor by allowing for perturbations around a baseline model, measured via Wasserstein distances. We try to understand to which extent this nonparametric form of probabilistic imprecision can be parametrized. In a second step, we deal with a class of time-homogeneous continuous-time Markov processes with transition probabilities bearing a nonparametric uncertainty. The uncertainty is again modeled by considering perturbations of the transition probabilities within a proximity in Wasserstein distance. As a limit over progressively finer time periods, on which the level of uncertainty scales proportionally, we obtain a convex semigroup, which solves a Hamilton-Jacobi-Bellman equation in a viscosity sense. We show that, in standard situations, the nonlinear transition operators arising from Wasserstein uncertainty coincide with the value function of an optimal control problem and provide sensitivity bounds for the convex semigroup relative to the reference model. The talk is based on joint works with Sven Fuhrmann, Michael Kupper, and Alessandro Sgarabottolo

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AR(1)-sequences with Rademacher innovations

Vitali Wachtel

In this talk I am going to discuss persistence probabilities for autoregressive processes. I shall primerly focus on the case when innovations have a Rademacher distribution. In this particular case one has rather interesting connection to dynamical systems in discrete time. This allows one to reduce the original problem on persistence probabilities to a first-passage problem for finite Markov chains.

Boundary behavior of 2D-Coulomb gases

Leslie Molag

Static 2D Coulomb gases describe a finite temperature system of a fixed number of particles, that repel each other via a logarithmic interaction, and are confined to the origin by some potential. It is expected that all such gases exhibit a peaked behavior at the level of correlations, on the boundary of some 2D region around the origin. For a particular choice of the temperature and potential, the "elliptic Ginibre ensemble", we prove the presence of this peaked behavior, and we show in detail what it looks like.

Dispersive effects in stochastic PDE

Anne Niesdroy

Both dispersive effects as well as regularization by noise can result in gain of regularity in stochastic partial differential equations. We aim to analyze the interaction of these effects and hope to get some improvements in regularity by combining both.

In this talk we focus on the setting of stochastic kinetic transport equations.

The topic is part of a joint subproject of A1-B1-B8.

Higher differentiability for nonlocal problems

Simon Nowak

We present some fine higher regularity results for nonlocal equations with irregular coefficients. A striking phenomenon highlighted by our results is that in contrast to the setting of local second-order elliptic equations, in our nonlocal setting we are able to gain differentiability under less restrictive assumptions on the coefficients.

Behind the regularity: qualitative and quantitative analysis of (non)local models admitting the energy gaps

Anna Balci

We study several local and (non)local models in the situation of the presence of the Lavrentiev phenomenon. This effect leads to the lack of higher regularity, special kind of non-uniqueness of the solution and non-density of smooth functions in the corresponding energy spaces. We design the examples on energy gaps using fractal geometry of the barrier sets. Besides this we investigate the non-conforming finite element scheme with convergence to the global minimizer. The numerical experiments give additional analytical insights.

This talk is based on several joint works with Lars Diening, Moritz Kaßmann, Christoph Ortner, Johannes Storn and Mikhail Surnachev.

Volume growth and heat kernel bounds on Riemannian manifolds with ends

Philipp Sürig

We investigate on-diagonal heat kernel bounds on Riemannian manifolds having at least one end with a polynomial volume growth.
