



# Harmonic and Stochastic Analysis of Dispersive PDEs

June 3 – 7, 2024

# $\label{eq:constraint} \ensuremath{\text{Bielefeld University}} $$V2-210/216$ and ZIF - Center for Interdisciplinary Research$

This workshop is part of the DFG-funded CRC 1283 Taming uncertainty and profiting from randomness and low regularity in analysis, stochastics and their applications at Bielefeld University

Organisers: Sebastian Herr and Michael Röckner https://www.sfb1283.uni-bielefeld.de/2024\_HSADP/

Schedul	e "Harmonic and Sto	chastic Analysis of	Dispersive PDEs" (J	une 3, 2024 – June 7	, 2024)
Date:	Monday (3.6.24)	Tuesday (4.6.24)	Wednesday (5.6.24)	Thursday (6.6.24)	Friday (7.6.24)
Location:	V2-210/216	ZIF	V2-210/216	V2-210/216	V2-210/216
Time (CEST):					
09:00-09:30	Welcome address				
09:30-10:30	Koch	De Suzzoni	Tzvetkov	Weber	Barbu
10:30-11:00	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:00-12:00	Tolomeo	Ciotir	Burg	Deng	Okamoto
12:00-13:00	Ferrario	Forlano	Krieger	Pocovnicu	Lühmann
13:00-14:00	Lunch break	Lunch break / Conference picture	Lunch break	Lunch break	Lunch break
14:00-14:30	Poster session	Poster session	Poster session	Poster session	
14:30-15:30	De Bouard	Brzeźniak		Spitz	
15:30-16:00	Coffee break	Coffee break		Coffee break	
16:00-17:00	Robert	Zine		Zhang, D.	
17:00-18:00	Chapouto	Hausenblas		Thomann	
18:00-18:30			Easthall & Dear		
18:30-19:00					
From 19:00				Conference Dinner	

# Schedule: Monday June 3

#### Lecture Room: V2-210/216

- 09:00-09:30 Welcome address
- 09:30–10:30 Herbert Koch Almost sure scattering for NLS
- 10:30-11:00 **Coffee break** (V2-210/216)
- 11:00–12:00 **Leonardo Tolomeo** Statistical mechanics of the focusing nonlinear Schrödinger equation
- 12:00–13:00 **Benedetta Ferrario** Stationary solutions for nonlinear Schrödinger equation
- 13:00-14:00 Lunch break
- 14:00-14:30 Poster session
- 14:30–15:30 Anne De Bouard From stochastic additive Zakharov system to stochastic multiplicative nonlinear Schrödinger equation
- 15:30-16:00 Coffee break (V2-210/216)

#### 16:00–17:00 **Tristan Robert** Regularization by noise for some modulated dispersive PDEs

#### 17:00–18:00 Andreia Chapouto

Pathwise well-posedness of stochastic nonlinear dispersive equations with multiplicative noises

# Schedule: Tuesday June 4

#### Lecture Room: ${\bf ZiF}$

- 09:30-10:30 Anne-Sophie De Suzzoni Strichartz estimates for the Dirac equation on asymptotically flat manifolds
- 10:30-11:00 Coffee break (ZiF)

#### 11:00–12:00 **Ioana Ciotir** Stochastic porous media equation with Robin boundary conditions, gravity-driven infiltration and multiplicative noise

- 12:00–13:00 **Justin Forlano** Invariant measures for mKdV and KdV on the line
- 13:00–14:00 Lunch break / Conference picture
- 14:00-14:30 Poster session
- 14:30–15:30 Zdzisław Brzeźniak Wave maps in dimension 1+1 with an external forcing
- 15:30-16:00 Coffee break (ZiF)

#### 16:00-17:00 Younes Zine

Hyperbolic sine-Gordon model beyond the first threshold

17:00–18:00 Erika Hausenblas Stochastic Landau-Lifshitz-Gilbert equations (SLLGEs) driven by a rough path

# Schedule: Wednesday June 5

Lecture Room: V2-210/216

- 09:30–10:30 **Nikolay Tzvetkov** Low regularity well-posedness of the binormal flow
- 10:30-11:00 Coffee break (V2-210/216)
- 11:00–12:00 Nicolas Burq Gibbs' measure for the non linear Schrödinger on the sphere
- 12:00–13:00 **Joachim Krieger** Global controllability of a geometric wave equation
- 13:00-14:00 Lunch break
- 14:00-14:30 Poster session

#### 18:00–19:00 Football & Beer (Sports ground behind the University)

### Schedule: Thursday June 6

#### Lecture Room: V2-210/216

- 09:30–10:30 **Hendrik Weber** A priori bounds for the nonlinear Parabolic Anderson Model
- 10:30-11:00 Coffee break (V2-210/216)
- 11:00–12:00 Yu Deng Recent progress on mathematical wave turbulence

#### 12:00–13:00 **Oana Pocovnicu** Invariant Gibbs dynamics for fractional wave equations in negative Sobolev spaces

- 13:00-14:00 Lunch break
- 14:00-14:30 Poster session
- 14:30–15:30 Martin Spitz Well-posedness of the three dimensional stochastic Zakharov system
- 15:30-16:00 Coffee break (V2-210/216)

#### 16:00–17:00 **Deng Zhang** Multi-bubble blow-ups and multi-solitons to stochastic nonlinear Schrödinger equations

#### 17:00–18:00 Laurent Thomann Renormalization of a quadratic Schrödinger model with additive noise

From 19:00 Conference Dinner Wirtshaus 1802 (Kurt-Schumacher-Straße 17a, 33615 Bielefeld)

# Schedule: Friday June 7

Lecture Room: V2-210/216

- 09:30–10:30 Viorel Barbu Optimal control in real time for nonlinear Schrödinger equation
- 10:30-11:00 Coffee break (V2-210/216)
- 11:00-12:00Mamuro OkamotoOn the stochastic quantization of the  $\Phi_3^3$ -model
- 12:00–13:00 **Jonas Lührmann** Asymptotic stability of solitary waves for the 1D cubic NLS under even perturbations
- 13:00-14:00 Lunch break

# Abstracts

#### Viorel Barbu (Iasi)

#### Optimal control in real time for nonlinear Schrödinger equation

One discusses the existence and construction of feedback optimal controllers for the optimal control problem governed by nonlinear Schrödinger equations.

#### Zdzislaw Brzeźniak (York)

#### Wave maps in dimension 1+1 with an external forcing

I will talk about the local and global well-posedness theory in  $L^1$ , inspired by the approach of Keel and Tao from 1998 paper "Local and global well-posedness of wave maps on  $\mathbb{R}^{1+1}$  for rough data", for the forced wave map equation in the "external" formalism. In this context, the target manifold is treated as a submanifold of a Euclidean space. As a byproduct, we can reprove Y. Zhou's uniqueness result from 1999 paper "Uniqueness of weak solutions of 1+1 dimensional wave maps", leading to the uniqueness of weak solutions with locally finite energy. Additionally, we achieve the scattering of such solutions through a conformal compactification argument. This talk is based on a joint paper with Jacek Jendrej (Paris) and Nimit Rana (York) of the same title, arXiv:2404.09195.

#### Nicolas Burq (Paris)

#### Gibbs' measure for the non linear Schrödinger on the sphere

In this talk I will present recent results allowing to prove global existence for the non linear Schrödinger equations on the sphere for random initial data This result is a natural development on both Bourgain's result on Gibb's measures on the two dimensional torus and our work (with P. Gérard and N. Tzvetkov) on the non linear Schrödinger on spheres. I will highlight the difficulties with respect to these previous works and give some ideas about the strategy of proof. This is joint work with N. Camps, C. Sun and N. Tzvetkov.

#### Andreia Chapouto (Edinburgh)

Pathwise well-posedness of stochastic nonlinear dispersive equations with multiplicative noises Over the last decades, the well-posedness issue of stochastic dispersive PDEs with multiplicative noises has been extensively studied. However, this comes mostly from the viewpoint of Ito solution theory, and pathwise well-posedness remains completely open. In this talk, I will present the first pathwise well-posedness results for stochastic nonlinear wave equations (SNLW) and stochastic nonlinear Schrödinger equations (SNLS) with multiplicative white-in-time/coloured-in-space noise. In proving pathwise well-posedness, we combine the operator-value controlled rough paths adapted to dispersive flows, together with random tensor estimates, and the Fourier restriction norm method adapted to controlled rough paths.

#### Ioana Ciotir (Saint-Étienne-du-Rouvray)

Stochastic porous media equation with Robin boundary conditions, gravity-driven infiltration and multiplicative noise

We aim at studying a novel mathematical model associated to a physical phenomenon of infiltration in an homogeneous porous medium. The particularities of our system are connected to the presence of a gravitational acceleration term proportional to the level of saturation, and of a Brownian multiplicative perturbation. Furthermore, the boundary conditions intervene in a Robin manner with the distinction of the behavior along the inflow and outflow respectively. We provide qualitative results of well-posedness, the investigation being conducted through a functional approach.

#### Anne De Bouard (Palaiseau)

# From stochastic additive Zakharov system to stochastic multiplicative nonlinear Schrödinger equation

We study the limit of a Zakharov system with an additive spatially correlated noise to a multiplicative stochastic nonlinear Schrödinger equation, as the velocity of the ion density, described by the wave equation, tends to infinity. Note that in this limit the evolution of the energy becomes singular and the scaling is a diffusion-approximation regime, requiring the use a predictor-corrector method. When considering only linear drift, we obtain a convergence rate by using an expansion of the solution the Kolmogorov equation.

These are joint works with G. Barrué, A. Debussche and R. Nader (ENS Rennes).

#### Yu Deng (Los Angeles)

#### Recent progress on mathematical wave turbulence

The theory of wave turbulence, which started in the 1920s as the wave analog of Boltzmann's kinetic theory, has been an active field of physics in the last century, with substantial applications in science. In this talk I will review some recent works, joint with Zaher Hani, that establish the rigorous mathematical foundation of this subject. In particular, we present the justification of the wave kinetic equation up to arbitrarily large kinetic time, which is the first long time result ever obtained in any nonlinear kinetic limit.

#### Anne-Sophie De Suzzoni (Palaiseau)

#### Strichartz estimates for the Dirac equation on asymptotically flat manifolds

In this talk, we discuss Strichartz estimates for the Dirac equation on asymptotically flat manifolds. We will present the Dirac equation in curved setting, and some of its symmetries. To obtain Strichartz estimates, we see the Dirac equation as a perturbation of the KleinGordon or wave equation and combine weak dispersive estimates with Strichartz and smoothing estimates for the wave and Klein–Gordon flows exploiting previous results in the same geometrical setting.

#### Benedetta Ferrario (Pavia)

#### Stationary solutions for nonlinear Schrödinger equation

We consider a nonlinear Schrödinger equation with a random force in a bounded domain. We prove the existence of martingale solutions and of stationary martingale solutions when there is a damping term of order zero. Then, following the technique by Kuksin, we let the intensity of the damping and of the noise vanish; in the limit we obtain stationary solutions of the deterministic nonlinear Schrödinger equation. Finally we prove some properties of the limit process and compare our results with those of Kuksin and collaborators.

#### Justin Forlano (Edinburgh)

#### Invariant measures for mKdV and KdV on the line

I will discuss a recent result on the invariance of the Gibbs measure on the line for the real-valued defocusing modified Korteweg de-Vries equation (mKdV). Along the way, we give a new proof of the invariance of the Gibbs measure for the periodic mKdV. Finally, by applying the Miura transform, we discover a new invariant probability measure for KdV on the line, with the same local regularity as the white noise but which is non-Gaussian. This talk is based on joint work with R. Killip and M. Visan (UCLA).

#### Erika Hausenblas (Leoben)

#### Stochastic Landau-Lifshitz-Gilbert equations (SLLGEs) driven by a rough path

The stochastic Landau-Lifshitz-Gilbert equations (SLLGEs) describe the behaviour of the magnetisation under the influence of the randomly fluctuating effective field.

In a joint work by Mukherjee and Fahim, we adapted Lyons' rough paths theory to study Landau-Lifshitz-Gilbert equations (LLGEs). Here, we considered the LLG equation driven by geometric rough paths in one dimension, with non-zero exchange energy only. By proposing a suitable transformation, we convert the LLGEs to a highly nonlinear time-dependent partial differential equation without rough paths term. Our point of interest is the regular approximation of the geometric rough path. We investigate the limit equation, the form of the correction term, and its rate of convergence in controlled rough path spaces. The key ingredients for the construction of the solution and its corresponding convergence results are the Doss-Sussmann transformation, maximal regularity property, and the geometric rough path theory.

#### Herbert Koch (Bonn)

#### Almost sure scattering for NLS

We consider cubic NLS in dimensions 2,3 and 4 and we prove that almost surely solutions scatter for random data at low and even negative regularity. Moreover we establish some smoothing properties of the associated scattering operator and convergence rates to the scattering data. This is joint work with Nicolas Burq, Nikolay Tzbetkov and Nicola Visciglia.

#### Joachim Krieger (Lausanne)

#### Global controllability of a geometric wave equation

I will explain recent results, obtained jointly with J.-M. Coron and S. Xiang, on global controllability of the 1+1 dimensional wave maps equation, and also discuss numerous natural challenges that remain. This in particular involves the issue of potentially rough forcings.

#### Jonas Lührmann (College Station)

Asymptotic stability of solitary waves for the 1D cubic NLS under even perturbations

I will present a perturbative proof of the asymptotic stability of solitary waves for the 1D cubic NLS under even perturbations. This is joint work with Yongming Li (Texas A&M University).

#### Mamuro Okamoto (Osaka)

#### On the stochastic quantization of the $\Phi_3^3$ -model

We study the construction of the  $\Phi_3^3$ -measure and complete the program on the construction of the focusing Gibbs measures, initiated by Lebowitz, Rose, and Speer (1988). We also study the dynamical problem for the canonical stochastic quantization of the  $\Phi_3^3$ -measure. This talk is based on a joint work with Tadahiro Oh (The University of Edinburgh) and Leonardo Tolomeo (The University of Edinburgh).

#### Oana Pocovnicu (Edinburgh)

#### Invariant Gibbs dynamics for fractional wave equations in negative Sobolev spaces

In this talk, we consider a fractional nonlinear wave equation with a general power-type nonlinearity (FNLW) on the two-dimensional torus. Our main goal is to construct invariant global-in-time Gibbs dynamics for FNLW. We first construct the Gibbs measure associated with this equation. By introducing a suitable renormalisation, we then prove almost sure local well-posedness with respect to Gibbsian initial data. Finally, we extend solutions globally in time by applying Bourgain's invariant measure argument.

We also consider the case of initial data consisting of the randomisation of a given pair of functions of negative regularities. We show that, in this case, probabilistic well-posedness fails unless we impose that the given pair has additional Fourier-Lebesgue regularity.

#### Martin Spitz (Bielefeld)

#### Well-posedness of the three dimensional stochastic Zakharov system

The Zakharov system is a model in plasma physics describing rapid oscillations of the electric field in a conducting plasma. It consists of a Schrödinger and a wave equation with quadratic coupling. In this talk we show that the stochastic Zakharov system is well-posed in the energy space up to the maximal existence time. The proof intertwines probabilistic techniques such as refined rescaling transforms with dispersive techniques such as the normal form method, Strichartz and local smoothing estimates.

We also present a regularization by noise result which states that finite time blowup before any given time can be prevented with high probability by adding sufficiently large non-conservative noise. The talk is based on joint work with Sebastian Herr, Michael Röckner, and Deng Zhang.

#### Laurent Thomann (Nancy)

#### Renormalization of a quadratic Schrödinger model with additive noise

The study is devoted to the interpretation and wellposedness of a quadratic stochastic NLS model with an additive space-time fractional noise. We will focus on the case where the noise is rough, and we exhibit an explicit Bourgain-Wick renormalization procedure allowing to restore the convergence of some approximated solutions. This a joint work with Aurélien Deya (Université de Lorraine) and Reika Fukuizumi (Waseda University).

#### Leonardo Tolomeo (Edinburgh)

Statistical mechanics of the focusing nonlinear Schrödinger equation

In this talk, we discuss a number of results related to (non-)construction of the Gibbs measures for focusing nonlinear Schrödinger equations.

The program was initiated by Lebowitz-Rose-Speer (1988), who built the focusing  $\Phi^p$  measure in dimension 1 by introducing a suitable mass cutoff. A number of open questions were raised in this work, namely the existence of certain phase transitions and if the construction can be repeated in higher dimension.

In this talk, we show the solution to most of those questions. After surveying various results, we will discuss the general strategy for tackling these problems, and the main techniques developed to find the answer.

This talk is based on joint works with T. Oh (Edinburgh), M. Okamoto (Osaka), H. Weber (Münster) and J. Forlano (Edinburgh).

#### Tristan Robert (Nancy)

#### Regularization by noise for some modulated dispersive PDEs

It is known since pioneering works by Veretennikov and Krylov-Röckner that for ODEs driven by a rough vector field, uniqueness of the solution can be recovered by adjunction of an additive noise in the equation. Improvement on the behavior of an ODE or PDE by adding a noise term is therefore referred to as a regularization by noise phenomenon, and is widely believed to hold for a large class of ODEs/PDEs and perturbative noises. In this talk, I will consider nonlinear dispersive PDEs where a deterministic noise is added as a distributional time coefficient in front of the dispersion. Despite the roughness of the noise term, we will see that any semilinear dispersive PDE with this noise term is well-posed at least in the same range of regularity as its noiseless counterpart, as soon as well-posedness relies on linear space-time estimates. Perhaps more surprisingly, provided that the noise is irregular enough, we will observe several regularization by noise phenomena: large data global well-posedness for focusing mass-critical equations, well-posedness at super-critical regularity for strongly non-resonant equations through improved multilinear estimates, and improvement on the Cauchy theory for Kadomtsev-Petviashvili equations through short-time multilinear estimates on longer time scales.

#### Nikolay Tzvetkov (Lyon)

#### Low regularity well-posedness of the binormal flow

We focus on a class of solutions of the binormal flow, modelling the evolution of vortex filaments, that generates several corner singularities in finite time. After proving a deterministic result, we will also see how improvements can be obtained by a suitable randomization of the curvature and torsion of the vortex filament. To do so, we will prove a scattering result for a quasi invariant measure associated with a suitable 1D cubic nonlinear Schroedinger model that we consider of independent interest. This is a joint work with V. Banica, R. Luca and L. Vega.

#### Hendrik Weber (Münster)

#### A priori bounds for the nonlinear Parabolic Anderson Model

We show a priori bounds that exclude finite time explosion for the solution of the nonlinear (or generalised) Parabolic Anderson model (gPAM)  $(\partial_t - \Delta)u = \sigma(u)\xi$  in the framework of Hairer's regularity structures. gPAM is the archetype of an equation that can be treated using regularity structures and it was one of the first equations for which a local solution theory was developed using this theory. While global existence in the linear case  $\sigma(u) = u$  follows from Hairer's and Gubinelli-Imkeller-Perkowski's original works, the case of non-linear  $\sigma$  is significantly more challenging and has remained open so far. Our results apply to noise terms  $\xi$  of (ir)regularity  $> -1 - \kappa$  for some  $\kappa > 0$ , including in particular the case of 2-dimensional spacial white noise. As a corollary we obtain global existence and the existence of an invariant measure for the dynamic Sine-Gordon model on the two-dimensional torus in the regime  $\beta^2 \in (4\pi, (1+\kappa)4\pi)$ .

This is joint work with Guilherme Feltes (Münster) and Ajay Chandra (Imperial College).

#### Deng Zhang (Shanghai)

#### Multi-bubble blow-ups and multi-solitons to stochastic nonlinear Schrödinger equations

In this talk we will review some recent results on multi-bubble blow-ups and multi-solitons to the focusing nonlinear Schrödinger equations in both the stochastic and deterministic case. We will show the construction and conditional uniqueness of multi-bubble Bourgain-Wang type blow-up solutions and non-pure multi-solitons, which provide new examples for the mass quantization conjecture and the soliton resolution conjecture. In the low asymptotic regime, the refined uniqueness is also derived. Furthermore, we will show the direct construction of stochastic multi-solitons in the mass critical and subcritical cases, for which the classical pseudo-conformal symmetry is absent.

#### Younes Zine (Lausanne)

#### Hyperbolic sine-Gordon model beyond the first threshold

Over the past two decades, significant progress has been made in understanding singular random dispersive PDEs with polynomial nonlinearities. However, non-polynomial nonlinearities remain poorly understood. This talk presents recent advancements in this direction, focusing on the well-posedness of the two-dimensional damped wave equation with a sine nonlinearity, driven by additive space-time white noise.

I will introduce the physical Fourier restriction norm method, a novel framework that tackles the challenges of non-polynomial settings. This method leverages recent developments in the Fourier restriction theory for the cone to establish key deterministic estimates. Additionally, I will discuss the proof of nonlinear smoothing for the imaginary Gaussian multiplicative chaos, the main probabilistic component of our argument, utilizing combinatorial and geometric techniques. This is joint work with Tadahiro Oh (Edinburgh, UK).

# Registered participants

Family Name	Given Name	Affiliation
Alonso Ruiz	Patricia	Texas A&M
Barbu	Viorel	Romanian Academy of Sciences
Brzeźniak	Zdzislaw	University of York
Burq	Nicolas	Université Paris-Saclay
Chapouto	Andreia	University of Edinburgh
Ciotir	Ioana	INSA Rouen
Coe	James	University of Edinburgh
De Bouard	Anne	Ecole Polytechnique Palaiseau
Deng	Yu	University of Southern California
De Suzzoni	Anne-Sophie	Ecole Polytechnique Palaiseau
Ferrario	Benedetta	University of Pavia
Forlano	Justin	University of Edinburgh
Geiß	Sarah	Bielefeld University
Hausenblas	Erika	Montanuniversität Leoben
Hientzsch	Lars Eric	Bielefeld University
Höfer	Fabian	University of Münster
Koch	Herbert	Universität Bonn
Krieger	Joachim	EPFL Lausanne
Kumar	Ankit	Indian Institute of Technology Roorkee, India
Li	Yongming	Texas A&M University
Liang	Rui	University of Birmingham
Liu	Shao	University of Bonn
Lührmann	Jonas	Texas A&M University
Martínez Martini	María Eugenia	Université Claude Bernard Lyon 1
Maulén	Christopher	Universität Bielefeld
Niesdroy	Anne	Bielefeld University
Nikov	Niko	University of Edinburgh
Okamoto	Mamuro	Osaka University
Patterson	James	University of Birmingham
Pocovnicu	Oana	Heriot-Watt University
Pompili	Lorenzo	University of Bonn
Prasad	Harsh	Bielefeld University
Robert	Tristan	Université de Lorraine
Schippa	Robert	UC Berkeley
Seong	Kihoon	Cornell University
Spitz	Martin	Universität Bielefeld
Täufer	Matthias	FernUniversität in Hagen
Tolomeo	Leonardo	University of Edinburgh
Robert	Tristan	Université de Lorraine
Tzvetkov	Nikolay	ENS Lyon
Weber	Hendrik	Universität Münster
Zhang	Deng	Shanghai Jiao Tong University
Zine	Younes	EPFL



# Tram map



#### Some recommendations for restaurants

- (1) Argentina-Steakhouse (https://argentina-steakhouse.de/) Argentinian beef at its best
- (2) Brauhaus Joh. Albrecht (https://bielefeld.brauhaus-joh-albrecht.de/) Home made beer plus German style food
- (3) Kometsu (http://www.kometsu.de/index.html) Authentic Japanese place for sushi
- (4) **KDW** (http://www.kdw-restaurant.de/index.html) Fine Greek cuisine
- (5) Numa (http://www.numa.de/) Asia meets East-Westphalia
- (6) Wernings Weinstube (https://www.wernings-weinstube.de/) Some regional dishes plus a good selection of wines
- (7) Wilde Kuh/ Wilde Kuh 2 (https://www.facebook.com/WildeKuhBurger/) Excellent "build your own burger" place
- (8) Three sixty (http://bielefeld.three-sixty.de/) Sports bar with burgers and other snacks
- (9) Jivino (http://www.jivino-enoteca.de/)
  Spanish tapas
- (10) Bernstein (https://www.the-bernstein.com/) Dinner plus cocktails in a fancy rooftop restaurant

Notes