

Gradient Flows, Large Deviation Theory, and Macroscopic Fluctuation Theory

June 17 – 21, 2024

Bielefeld University
ZIF - Center for Interdisciplinary Research

This workshop is part of the DFG-funded CRC 1283
*Taming uncertainty and profiting from randomness and low regularity
in analysis, stochastics and their applications*
at Bielefeld University

Organisers: Matthias Erbar and Benjamin Gess

https://www.sfb1283.uni-bielefeld.de/2024_MFT/

Schedule “Gradient flows, large deviations and MFT”						
	Mon (17.6)	Tue (18.6)	Wed (19.6.)		Thu (20.6)	Fri (21.6)
location	ZIF	ZIF	ZIF		ZIF	V2-210/216
9:15	Welcome					
9:30 - 10:30	Peletier	Peletier	Fischer		Perkowski	Gvalani
10:30 - 11:00	Coffee	Coffee	Coffee		Coffee	Coffee
11:00 - 12:00	Peletier	Fischer	Fischer		Heydecker	He
	Lunch				Djurdevac	Schlichting
12:00 - 14:00	Poster session	Lunch	Lunch		Lunch	Lunch
14:00 - 14:45		Zimmer	Tse		Konarovskyi	
14:45 - 15:30	Renger	Garrido	Stephan		Erhard	
15:30 - 16:00	Coffee	Coffee	Coffee		Coffee	
16:00 - 16:45	Leonard	Maes	Yoldas		Rameh	
16:45 - 17:30	Basile	Gabrielli	Menegaki		Rehmeier	
17:30 - 19:00	Reception					

Schedule: Monday June 17

Lecture Room: **ZiF**

09:15–09:30 **Welcome**

09:30–10:30 **Mark Peletier**
Why does entropy drive evolution equations?

10:30–11:00 **Coffee**

11:00–12:00 **Mark Peletier**
Why does entropy drive evolution equations?

12:00–13:00 **Lunch**

13:00–14:45 **Poster session**

14:45–15:30 **Michiel Renger**
Macroscopic Fluctuation Theory: MANERIC vs. GENERIC

15:30–16:00 **Coffee**

16:00–16:45 **Christian Léonard**
Time reversal applied to some large deviations from gradient flows

16:45–17:30 **Giada Basile**
Large deviations for binary collision models: energy non-conserving paths

17:30–19:00 **Reception**

Schedule: Tuesday June 18

Lecture Room: **ZiF**

- 09:30 – 10:30 **Mark Peletier**
Why does entropy drive evolution equations?
- 10:30 – 11:00 **Coffee**
- 11:00 – 12:00 **Julian Fischer**
Mathematical approaches to the rigorous justification of fluctuating hydrodynamics
- 12:00 – 14:00 **Lunch**
- 14:00 – 14:45 **Johannes Zimmer**
Fluctuating hydrodynamics for long-range interaction
- 14:45 – 15:30 **Pedro L. Garrido**
Nonequilibrium Potentials
- 15:30 – 16:00 **Coffee**
- 16:00 – 16:45 **Christian Maes**
It takes two to tango, Clausius entropy as Noether charge
- 16:45 – 17:30 **Davide Gabrielli**
Current fluctuations for the boundary driven zero-range process:
microscopic versus macroscopic approach and a theory of non-reversible
resistor-like networks

Schedule: Wednesday June 19

Lecture Room: **ZiF**

- 09:30 – 10:30 **Julian Fischer**
Mathematical approaches to the rigorous justification of fluctuating hydrodynamics
- 10:30 – 11:00 **Coffee**
- 11:00 – 12:00 **Julian Fischer**
Mathematical approaches to the rigorous justification of fluctuating hydrodynamics
- 12:00 – 14:00 **Lunch**
- 14:00 – 14:45 **Oliver Tse**
Title
- 14:45 – 15:30 **Artur Stephan**
On Multi-Scale Hamilton-Jacobi Equations for Chemical-Reaction Systems
- 15:30 – 16:00 **Coffee**
- 16:00 – 16:45 **Havva Yoldas**
A cross-diffusion system obtained via (convex) relaxation in the JKO scheme
- 16:45 – 17:30 **Angeliki Menegaki**
Quantitative framework for hydrodynamic limits

Schedule: Thursday June 20

Lecture Room: **ZiF**

09:45 – 10:30 **Nicolas Perkowski**
Title

10:30 – 11:00 **Coffee**

11:00 – 11:45 **Daniel Heydecker**
The Porous Medium Equation: Rescaled Zero-Range Process, Large Deviations and Gradient Flow

11:45 – 12:30 **Ana Djurdjevac**
An inertial PDE system for the Cucker-Smale model of flocking dynamics

12:30 – 14:00 **Lunch**

14:00 – 14:45 **Vitalii Konarovskyi**
Stochastic Modified Flows, Mean-Field Limits and Dynamics of Stochastic Gradient Descent

14:45 – 15:30 **Dirk Erhard**
The tube property for the Swiss cheese problem

15:30 – 16:00 **Coffee**

16:00 – 16:45 **Ons Rameh**
Mixing time of the Asymmetric Zero Range process on the segment

16:45 – 17:30 **Marco Rehmeier**
Nonlinear Fokker–Planck–Kolmogorov equations as gradient flows on the space of probability measures

Schedule: Friday June 21

Lecture Room: **UHG V2-210/216**

09:45 – 10:30 **Rishab Gvalani**
Title

10:30 – 11:00 **Coffee**

11:00 – 11:45 **Zihui He**
A variational approach to a fuzzy Boltzmann equation

11:45 – 12:30 **André Schlichting**
EDP convergence of exchange driven stochastic particle systems

12:30 – 14:00 **Lunch**

Abstracts

Lecture Series:

Julian Fischer (IST Austria)

Mathematical approaches to the rigorous justification of fluctuating hydrodynamics

The theory of fluctuating hydrodynamics attempts to describe density fluctuations in systems of interacting particles in the regime of large particle numbers by means of suitable SPDEs.

The Dean-Kawasaki equation - a strongly singular SPDE - is one of the most basic equations of fluctuating hydrodynamics; it has been proposed in the physics literature to describe the fluctuations of the density of N weakly interacting diffusing particles in the regime of large particle numbers N . The singular nature of the Dean-Kawasaki equation presents a substantial challenge for both its analysis and its rigorous mathematical justification: Besides being non-renormalizable by regularity structures or paracontrolled calculus, it has recently been shown to not even admit nontrivial martingale solutions.

In this lecture series, we discuss approaches to the rigorous justification of the Dean-Kawasaki equation as a quantitative model for density fluctuations. We show that structure-preserving discretizations of the Dean-Kawasaki equation may approximate the density fluctuations of N weakly interacting diffusing particles to arbitrary order in $1/N$ (in suitable weak metrics), the accuracy being only limited by the numerical scheme. We subsequently discuss how the situation differs in the case of the continuum (non-discretized) Dean-Kawasaki equation with regularized noise.

Mark Peletier (TU Eindhoven)

Why does entropy drive evolution equations?

‘Entropy’ has played a central role in the understanding of physical systems since the birth of thermodynamics, and it was known for a long time that many entropies decrease along dissipative evolutions such as diffusion, heat conduction, viscous flow, and many others.

The pioneering work of Jordan, Kinderlehrer and Otto gave such ‘entropies’ an even more important role: in addition to being Lyapunov functions, they often are the ‘driving force’ in a gradient flow. The introduction of GENERIC systems by Grmela and Oettinger further strengthened this point of view, by unifying gradient flows with Hamiltonian systems. This leads to an evolution driven by two functions, an ‘energy’ and an ‘entropy’. Well-known examples are the Boltzmann equation and the Navier-Stokes-Fourier equations.

What is less widely known, however, is the reason why entropies ‘drive’ evolution equations such as gradient flows and GENERIC systems. Entropy is a statistical-mechanical concept, often motivated by vague ideas such as ‘chaos’, ‘degeneracy’, or ‘lack of knowledge’; such descriptions do not explain why entropy should ‘drive’ evolution equations.

In addition, ‘entropy’ comes in many functional forms, and it’s not obvious why. In fact, sometimes different forms appear to be equivalent, but the axioms of GENERIC single out one of these equivalent forms - again without clear reason.

In this series of lectures I want to explore these questions: Why does ‘entropy’ drive so many evolution equations? Why are there many different forms? How can I understand the modelling, or physical, or chemical interpretation of a given entropy-like functional?

Talks:

Giada Basile (U Rome Sapienza)

Large deviations for binary collision models: energy non-conserving paths

I will present some large deviation results for binary collision stochastic models, such as the Kac's walk. I will exhibit some atypical paths that violate energy conservation and I will introduce a large deviation rate function which takes into account these trajectories. Finally, I will discuss the connection between the rate function and the gradient flow formulation of the homogeneous Boltzmann equation. Funded by the European Union - Next Generation EU.

Ana Djurdjevac (FU Berlin)

An inertial PDE system for the Cucker-Smale model of flocking dynamics

In particle systems, 'flocking' describes the phenomenon where particles' individual velocities eventually align over time. The Cucker-Smale model is a well-known mathematical framework for this behavior. Many continuous descriptions of the Cucker-Smale model use PDEs with both particle position and velocity as independent variables, thus providing a full description of the particles mean field dynamics. In this presentation, we explore the advantages of examining the Cucker-Smale model through a simplified inertial PDE system, consisting of two equations that depend solely on position. We will discuss the derivation, the approximations required for model reduction, analysis, simulations, and future research directions for this PDE system. This talk is based on a joint work in progress with Natasa Djurdjevac Conrad, Federico Cornalba, Sebastian Zimmer.

Dirk Erhard (U Bahia)

The tube property for the Swiss cheese problem

In 2001 Bolthausen, den Hollander and van den Berg obtained the asymptotics of the probability that the volume of a Wiener sausage at time t is smaller than expected by a fixed multiplicative constant. This asymptotics was given by a variational formula and they conjectured that the best strategy to achieve such a large deviation event is for the underlying Brownian motion to behave like a swiss cheese: stay most of the time inside a ball of subdiffusive size, visit most of the points but leave some random holes. They moreover conjectured that to do so the Brownian motion behaves like a Brownian motion in a drift field given by a function of the maximizer of the variational problem. In this talk I will talk about the corresponding problem for the random walk and will explain that conditioned to having a small range its properly defined empirical measure is indeed close to the maximizer of the above mentioned variational problem.

This is joint work with Julien Poisat.

Davide Gabrielli (U L'Aquila)

Current fluctuations for the boundary driven zero-range process: microscopic versus macroscopic approach and a theory of non-reversible resistor-like networks

We compute the joint large deviation rate functional in the limit of large time for the current flowing through the edges of a finite graph on which a boundary driven system of stochastic particles evolves with zero-range dynamics. This generalizes one-dimensional results previously obtained with different approaches; our alternative techniques illuminate various connections and complementary perspectives. In particular, we use a variational approach to derive the rate functional by contraction from a level 2.5 large deviation rate functional. We perform an exact minimization and finally obtain the rate functional as a variational problem involving a superposition of cost functions

for each edge. The contributions from different edges are not independent since they are related by the values of a potential function on the nodes of the graph. The rate functional on the graph is a microscopic version of the continuous rate functional predicted by the macroscopic fluctuation theory, and we show indeed a convergence in the scaling limit. If we split the graph into two connected regions by a cutset and are interested just in the current flowing through the cutset, we find that the result is the same as that of an effective system composed of only one effective edge. The characteristics of this effective edge are related to the “capacities” of the graph and can be obtained by a reduction using elementary transformations as in electric networks; specifically, we treat components in parallel, in series, and in N-star configurations (reduced to effective complete N-graphs). Our reduction procedure is directly related to the trace process. Joint work with R. J. Harris

Pedro L. Garrido (U Granada)

Nonequilibrium Potentials

The nonequilibrium potential, V , defines the stationary state of a system at the mesoscopic level when the system’s variables are described by fluctuating continuum fields. In this context, V is the unknown of a Hamilton-Jacobi equation that typically cannot be solved. We will show some strategies to determine V explicitly and/or its behavior near the deterministic stationary state by computing the field’s correlations.

Rishab Gvalani (MPI Leipzig)

Title

Abstract

Zihui He (U Bielefeld)

A variational approach to a fuzzy Boltzmann equation

We study a fuzzy Boltzmann equation where the particles interact via delocalized collisions. We discuss the existence and uniqueness of solutions and their convergence to the solutions of classical Boltzmann equations. We also provide a variational characterization casting the fuzzy Boltzmann equation into the framework of GENERIC (General Equations for Non-Equilibrium Reversible-Irreversible Coupling) systems.

This talk is based on joint work with Matthias Erbar.

Daniel Heydecker (MPI Leipzig)

The Porous Medium Equation: Rescaled Zero-Range Process, Large Deviations and Gradient Flow

We study a rescaling of the zero-range process with homogenous jump rates $g(k) = k^\alpha$ with arbitrary $\alpha \geq 1$. In a suitable scaling limit, we identify the dynamical large deviations from the porous medium equation, relying on pathwise regularity estimates. The large deviations rate function also allows us to express the porous medium equation as the gradient flow of the Boltzmann entropy with respect to a tailor-made Wasserstein-type distance under minimal regularity assumptions.

Vitalii Konarovskiy (U Hamburg)

Stochastic Modified Flows, Mean-Field Limits and Dynamics of Stochastic Gradient Descent

We will discuss new limiting dynamics for stochastic gradient descent in the small learning rate regime, called stochastic modified flows. These SDEs are driven by a cylindrical Brownian motion and improve the so-called stochastic modified equations by having regular diffusion coefficients and by matching the multipoint statistics. We will also introduce distribution-dependent stochastic modified flows, which we prove to describe the fluctuating limiting dynamics of stochastic gradient descent in the small learning rate – infinite width scaling regime. This is a joint work with Benjamin Gess and Sebastian Kassing.

Christian Léonard (Université Paris Nanterre)

Time reversal applied to some large deviations from gradient flows

The aim of this talk is to promote the use of time reversal when expressing large deviation rate functions of Brownian particle systems when the number of particles tends to infinity. Introducing stochastic derivatives, in particular the backward one, allows to express some relative entropies of diffusion processes as action functionals on the Otto-Wasserstein manifold of probability measures. As a first consequence, in the small time asymptotics, the rate function appears directly as an analogue of the objective function of the JKO scheme. Another immediate byproduct of this representation is that one can embed the limiting gradient flow evolution as a solution to some Newton's equation on the Otto-Wasserstein manifold. As an illustration, a quick overview of the early Universe reconstruction problem will be sketched.

Although this talk is based on several joint works, it is mainly motivated by an ongoing joint work with Johannes Zimmer.

Christian Maes (U Leuven)

It takes two to tango, Clausius entropy as Noether charge

Entropy as Noether charge for quasistatic gradient flow: Entropy increase is fundamentally related to time reversal of trajectories. By adding the 'extra dimension' associated to thermodynamic force, we extend that discrete symmetry to a continuous symmetry for the dynamical fluctuations around (nonlinear) gradient flow. The latter connects macroscopic equilibrium conditions upon introducing a quasistatic protocol of control parameters. The entropy state function becomes the Noether charge. As a result, and following ideas first expressed by Shin-ichi Sasa and co-workers, the first part of the Clausius heat theorem gets connected with the Noether theorem.

Joint work with Aaron Beyen

Angeliki Menegaki (Imperial College London)

Quantitative framework for hydrodynamic limits

We will present a new quantitative approach to the problem of proving hydrodynamic limits from microscopic stochastic particle systems, namely the zero-range, the simple exclusion and the Ginzburg-Landau process with Kawasaki dynamics, to macroscopic partial differential equations. Our method combines a modulated Wasserstein distance estimate comparing the law of the stochastic process to the local Gibbs measure, together with stability estimates à la Kruzhkov in weak distance and consistency estimates exploiting the regularity of the limit solution. It is simplified as it avoids the use of the block estimates. This is a joint work with Clément Mouhot (University of Cambridge) and Daniel Marahrens.

Nicolas Perkowski (FU Berlin)

Title

Abstract

Ons Rameh (U Paris Cité)

Mixing time of the Asymmetric Zero Range process on the segment

In this talk, we shall discuss the mixing time of the Asymmetric Zero Range process (AZRP) with non-decreasing jump rates on the segment. We will first study the hydrodynamic limit starting from the configuration where all particles are gathered on the left-most site. The macroscopic equilibrium time gives a lower bound on the mixing time. We will show the sharpness of this estimate when the system is asymmetric enough, which establishes the cut-off phenomenon. This extends earlier results of Labbe and Lacoïn in the context of the asymmetric simple exclusion process.

Marco Rehmeier (U Bielefeld)

Nonlinear Fokker–Planck–Kolmogorov equations as gradient flows on the space of probability measures

We propose a general method to identify nonlinear Fokker–Planck equations (FPEs) as gradient flows on the space of probability measures on R^d with a natural differential geometry. Our notion of gradient flow does not depend on any underlying metric structure such as the Wasserstein distance, but is derived from purely differential geometric principles. We explicitly identify the associated energy functions and show that these are Lyapunov functions for the FPE-solutions. These results cover classical and generalized porous media equations. Joint work with Michael Röckner (Bielefeld).

Michiel Renger (TU Munich)

Macroscopic Fluctuation Theory: MANERIC vs. GENERIC

Macroscopic Fluctuation Theory provides a rich framework to study non-equilibrium systems via their flux large deviations. The dynamics can then be decomposed into an entropic gradient flow part - symmetric under time reversal, and an antisymmetric part. The fully antisymmetric dynamics, obtained by completely shutting the entropic effects, often turns out to be a Hamiltonian system. Whenever this is the case, we call such systems “MANERIC”. We compare this structure to one other common framework to model non-equilibrium systems: “GENERIC”.

André Schlichting (U Münster)

EDP convergence of exchange driven stochastic particle systems

We consider the thermodynamic limit of mean-field stochastic particle systems on a complete graph. The evolution of occupation number at each vertex is driven by particle exchange with its rate depending on the population of the starting vertex and the destination vertex, including zero-range and misanthrope process. We show that under a weak detailed balance condition, the evolution equation of the law of the particle density has a variational description related to generalised gradient flows and large deviation rate functionals.

We show the variational convergence based on a suitable energy dissipation principle. The convergence of the system in this variational sense is established based on compactness of the density and flux and Γ -lower-semicontinuity of the energy-dissipation functional along solutions to the continuity equation. The model has a weak free energy, which is shown to converge in

the thermodynamic limit, after taking possible condensation phenomena into account.
 Joint work with Chun Yin Lam and Jasper Hoeksema

Artur Stephan (TU Wien)

On Multi-Scale Hamilton-Jacobi Equations for Chemical-Reaction Systems

Microscopically, chemical-reaction systems can be modeled by a time-dependent stochastic process, which, e.g., counts the number of species. The macroscopic behavior of a chemical-reaction system is characterized by a deterministic reaction-rate equation, which describes the dynamics of the concentration of species. The large-deviation rate functional characterizes the probability of stochastic fluctuations far from the deterministic limit. It is an action functional and the corresponding Hamiltonian provides a Hamilton-Jacobi equation (HJE) describing many interesting features of the chemical-reaction systems and is the starting point for our analysis.

Motivated by real-world applications, where chemical reactions appear naturally on different time-scales, we are interested in a fast-slow limit to reduce the complexity of the system. We show how a fast-slow limit can be rigorously derived on the level of the Hamilton-Jacobi equation. The methods we use include but are not limited to chemical reactions described by gradient flows. The talk is based on joint work with Yuan Gao (Purdue U).

Oliver Tse (TU Eindhoven)

Title

Abstract

Havva Yoldas (TU Delft)

A cross-diffusion system obtained via (convex) relaxation in the JKO scheme

We start from a cross-diffusion system that is a gradient flow for the Wasserstein distance of a certain non-lower-semi continuous functional. We consider the relaxation of the functional and prove existence of a solution in a suitable sense for the gradient flow of (the relaxed functional). This gradient flow has also a cross-diffusion structure, but the mixture between two different regimes, that are determined by the relaxation, makes the study non-trivial. This is a joint work (Calc. Var. PDE (2023) 62:23) with Romain Ducasse (Paris) and Filippo Santambrogio (Lyon).

Johannes Zimmer (TU Munich)

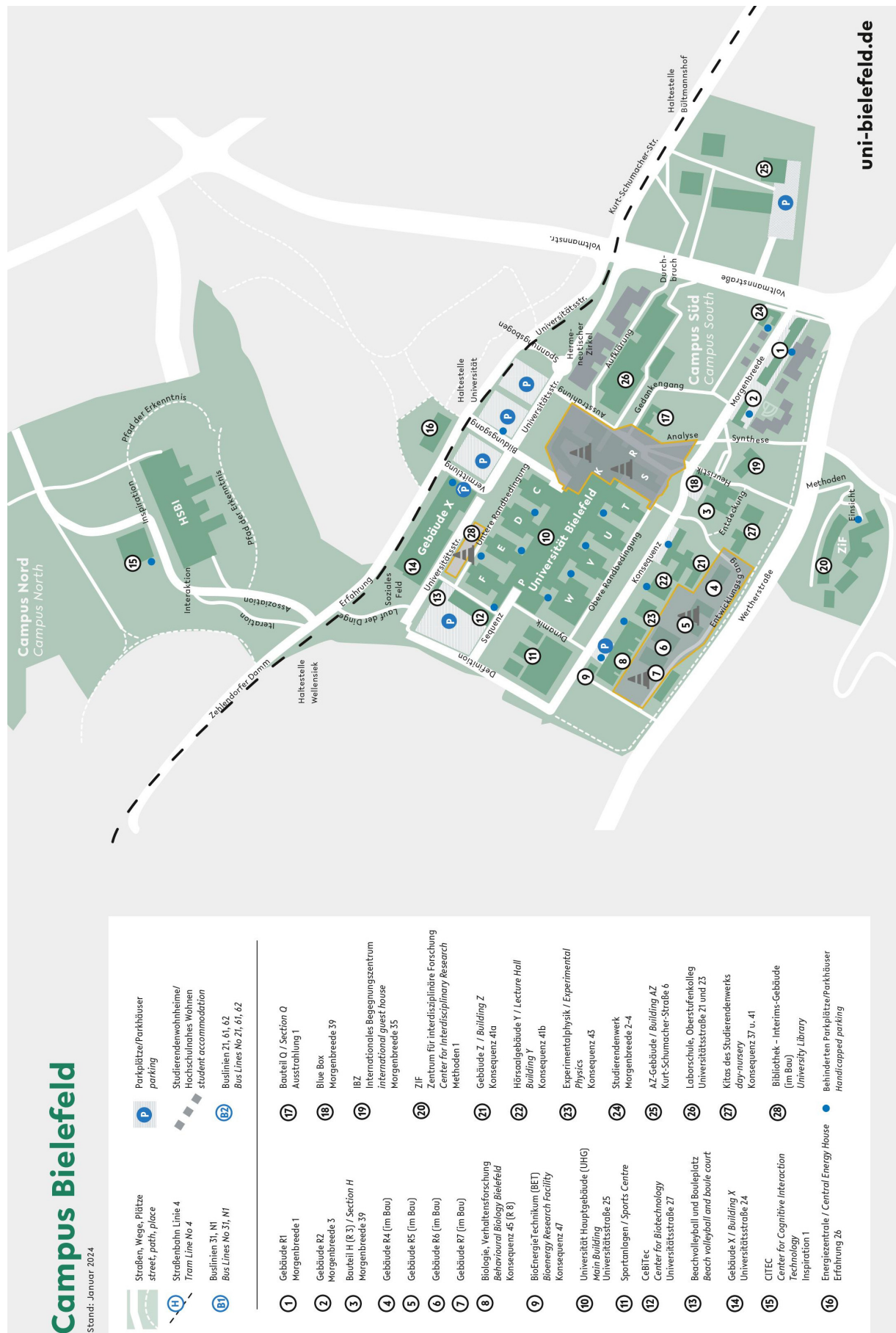
Fluctuating hydrodynamics for long-range interaction

We study a many-particle system in the limit of particle number going to infinity (hydrodynamic limit) as well as fluctuations around this limit, given by the evolution of many but finitely many particles. The description of the latter is often called fluctuation hydrodynamics or Dean-Kawasaki equation. We discuss the concepts, in particular for the so-called symmetric inclusion process in the case of long-range interaction, which leads to nonlocal limit equations.

This is joint work with Mario Ayala.

Registered participants

Family Name	Given Name	Affiliation
Abedi	Ehsan	Bielefeld University
Bechtold	Florian	Bielefeld University
Della Corte	Serena	Delft University of Technology
Erhard	Dirk	Universidade Federal da Bahia
Fischer	Julian	Institute of Science and Technology Austria
Fitoussi	Mathis	Université d'Evry Paris-Saclay
Gabrielli	Davide	University of L'Aquila
Garrido	Pedro L.	Universidad de Granada
He	Zihui	University Bielefeld
Hu	Yanyan	Delft University of Technology
Hu	Shanshan	Bielefeld University
Ingmanns	Jonas	ISTA (Institute of Science and Technology Austria)
Konarovskyi	Vitalii	Hamburg University
Léonard	Christian	Université Paris Nanterre
Maes	Christian	KU Leuven
Martini	Adrian	Oxford University
Niesdroy	Anne	Bielefeld University
Nikolaev	Paul	University of Mannheim
Palmieri	Matteo	Scuola Normale Superiore (SNS), Pisa
Perkowski	Nicolas	FU Berlin
Perkowski	Nicolas	FU Berlin
Prasad	Harsh	Universität Bielefeld
Quattrocchi	Filippo	Institute of Science and Technology Austria
Rameh	Ons	Universite Paris-Cite, ENS PSL
Schlichting	André	University of Münster
Schlichting	Andre	Münster
Shanshan	Hu	Bielefeld University
Wang	Wei	Montanauniversität Leoben
Yoldas	Havva	TU Delft
Zimmer	Johannes	TU München



Tram map



Some recommendations for restaurants

- (1) **Argentina-Steakhouse** (<https://argentina-steakhouse.de/>)
Argentinian beef at its best
- (2) **Brauhaus Joh. Albrecht** (<https://bielefeld.brauhaus-joh-albrecht.de/>)
Home made beer plus German style food
- (3) **Kometsu** (<http://www.kometsu.de/index.html>)
Authentic Japanese place for sushi
- (4) **KDW** (<http://www.kdw-restaurant.de/index.html>)
Fine Greek cuisine
- (5) **Numa** (<http://www.numa.de/>)
Asia meets East-Westphalia
- (6) **Wernings Weinstube** (<https://www.wernings-weinstube.de/>)
Some regional dishes plus a good selection of wines
- (7) **Wilde Kuh/ Wilde Kuh 2** (<https://www.facebook.com/WildeKuhBurger/>)
Excellent "build your own burger" place
- (8) **Three sixty** (<http://bielefeld.three-sixty.de/>)
Sports bar with burgers and other snacks
- (9) **Jivino** (<http://www.jivino-enoteca.de/>)
Spanish tapas
- (10) **Bernstein** (<https://www.the-bernstein.com/>)
Dinner plus cocktails in a fancy rooftop restaurant

Notes