CRC Retreat 2024
Programme

SFB 1283

Tuesday, 27 August				
Start	**End**	**Projects**	**Title**	**Speakers**
08:00	10:00	Bus: Bielefeld - Hofgeismar		
10:00	10:45	Arrival and Welcome Coffee		
10:45	10:50		Opening	
10:50	11:20	A6-C6	Random point processes in the plane and applications to birds of prey	G. Akemann
11:25	11:55	A9	Self-similar phenomena in nonlinear PDEs: low regularity theory	I. Glogic
12:00	13:30	Lunch		
13:30	14:00	A1-B3	Numerical approximation of biharmonic wave maps into spheres equation	L. Banas
14:05	14:35	B8	Understanding the Geometry of the Loss Landscape to Accelerate Stochastic Optimization Algorithms	S. Kassing
14:40	15:10	C1	Killing versus catastrophes in birth-death processes and an application to population genetics	E. Di Gaspero
15:15	16:00	Coffee		
16:00	*18:00*	*Collaborative research*		
18:00	19:00	Dinner		
19:15	Bowling			

Wednesday, 28 August				
07:30	09:00	Breakfast		
09:00	09:30	A6-C1	The multiple coupon collection process and Markov embedding	E. Baake
09:35	10:05	B11	Statistical analysis of empirical graph Laplacians	M. Wahl
10:05	10:50	Coffee		
10:50	11:20	A7	Fractional Sobolev spaces with degenerate weights	J. Rolfes
11:25	11:55	B5	Sudakov-type concentration results and local laws for weighted random matrices	H. Sambale
12:00	13:30	Lunch		
13:00	*14:30*	*Meeting of principal investigators/Collaborative research*		
14:40	15:10	B1-C3-C4	Climate Change Models and Singular Control of SPDEs in Hilbert Spaces	F. Riedel
15:15	16:00	Coffee		
16:00	18:00	Bus: Hofgeismar - Bielefeld		

Check-in: Tuesday, around 13:00/14:00
Check-out: Wednesday, 09:00

CRC Retreat 2024 – Abstracts

27 – 28 August 2024

Random point processes in the plane and applications to birds of prey

Gernot Akemann

Random point processes in the plane are popular models in ecology. In this talk I will put charged particles in the two-dimensional Coulomb gas at general inverse temperature beta in a such a perspective. Away from the integrable point beta=2, corresponding to the complex eigenvalues of random matrices with complex normal entries, and the Poisson point process of independent points corresponding to beta=0, very little is known about the local statistics of this point process. We therefore resort to numerical simulations to determine the nearest and next-to-nearest spacing distribution between points in the Coulomb gas to model data from biology. An alternative, approximate description is based on a 2x2 random matrix ensemble valid for small values of beta. Annual ensembles of nests of three different birds of prey in the area of the Teutoburger Wald close to Bielefeld are modelled by such a simple random point process, by fitting an effective beta to the data. In such a way the "repulsion" between the nests of these very territorial birds can be quantified. We compare the inter and intra-species repulsion, as well as their change over time.

This is joint work with Adam Mielke, Patricia Paessler and the group of Oliver Krueger in Biology.

Self-similar phenomena in nonlinear PDEs: low regularity theory

Irfan Glogić

The notion of *self-similarity* is an often-used concept to capture universal properties of physical and natural processes. It typically describes late stages of both *finite-time blowup* and *global-in-time behavior*, therefore representing one of the most important shared dynamical features of mathematical models of natural phenomena.

Motivated by the observations above, our aim is to study the *existence* of self-similar structures as well as their *stability* (which is a key requirement for their physical observability) across a range of *supercritical* nonlinear PDEs.

We will illustrate our point of view on a specific equation: the Keller-Segel model for bacterial chemotaxis, which is a parabolic-elliptic system that is observed to exhibit a variety of self-similar phenomena. Stemming from the desire to gain physical insight into the modeled phenomena, the questions we plan to answer concern

- deterministic stability analysis *at the critical regularity*, and
- probabilistic stability and non-uniqueness of solutions *below the critical regularity.*

To successfully answer these questions, we will, in particular, need to develop a range of Strichartz estimates (both deterministic and probabilistic) in similarity variables, and devise (possibly computer-assisted) techniques to treat the spectral problems arising in the stability analysis.

Numerical approximation of biharmonic wave maps into spheres equation

L'ubomír Baňas

We propose a practical structure preserving non-conforming finite element approximation of biharmonic wave maps into spheres equation. The scheme satisfies a discrete energy law and enforces the non-convex sphere constraint at the mesh-points via a discrete Lagrange multiplier. This approach restricts the spatial approximation to the (non-conforming) linear finite elements which introduces several complications in the analysis of the numerical scheme. Using discrete compactness arguments adapted to the non-conforming setting, we show that the numerical approximation converges to the weak solution of the continuous problem in spatial dimension $d = 1$. The convergence analysis in dimensions $d > 1$ is complicated by the lack of a discrete chain rule as well as the low regularity of the numerical approximation in the non-conforming setting. Hence, we show convergence of the numerical approximation in higher-dimensions by introducing additional stabilization terms in the numerical approximation. We present numerical experiments to demonstrate the performance of the proposed numerical approximation and to illustrate the regularizing effect of the bi-Laplacian which prevents the formation of discrete blow-ups.

The talk is based on a joint work between projects B3 and A1.

Understanding the Geometry of the Loss Landscape to Accelerate Stochastic Optimization Algorithms

Sebastian Kassing

In machine learning, loss functions are typically non-convex and contain multiple minima. Though this poses a substantial hurdle in the theoretical analysis, stochastic optimization methods, such as Stochastic Gradient Descent and Stochastic Heavy Ball, perform surprisingly well in practice. A popular property that ensures fast convergence for non-convex loss functions is the Polyak-Lojasiewicz inequality. Interestingly, this simple gradient inequality has very strong implications on the geometry of the loss landscape and the set of minimizers. In this talk, I plan to explain the geometric structure hidden behind the PL-inequality and present works on its implications in stochastic optimization. I hope to encourage a discussion in search of further applications of this connection between Riemannian geometry, Stochastic Optimization and Machine Learning.

Killing versus catastrophes in birth-death processes and an application to population genetics

Enrico Di Gaspero

We establish connections between the absorption probabilities of a class of birth-death processes with killing, and the stationary tail distributions of a related class of birth-death processes with catastrophes. Major ingredients of the proofs are an excursion decomposition of sample paths, a generalised detailed-balance condition, and representations of our processes in terms of superpositions of simpler processes. An overarching role is played by Siegmund duality, which allows us to invert the relationship between the processes. We apply our results to a pair of ancestral processes in population genetics, namely the killed ancestral selection graph and the pruned lookdown ancestral selection graph, in a finite population setting and its diffusion limit.

The multiple coupon collection process and Markov embedding

Ellen Baake

The embedding problem of Markov transition matrices into Markov semigroups is a classic problem that regained a lot of impetus and activities in the last few years. We consider it here for the following generalisation of the classical coupon collection process: from a set $S = \{1, 2, \ldots, m\}$ of distinct types of objects, a subset $K \subseteq S$ is drawn with probability p_K in every time step, $\sum_{K \subseteq S} p_K = 1$, and united with the set of types sampled so far. We obtain explicit conditions for the resulting discrete-time Markov chain to be representable as the semigroup of a continuous-time process.

This is joint work with Michael Baake.

Statistical analysis of empirical graph Laplacians

Martin Wahl

Laplacian Eigenmaps and Diffusion Maps are nonlinear dimensionality reduction methods that use the eigenvalues and eigenvectors of (un)normalized graph Laplacians. Both methods are applied when the data is sampled from a low-dimensional manifold, embedded in a high-dimensional Euclidean space. In this talk, I will show how these empirical graph Laplacians can be understood as spectral approximations of the Laplace-Beltrami operator on the underlying Riemannian manifold. I will discuss graph Laplacians as well as higher-order generalizations of these (so-called Hodge Laplacians).

Fractional Sobolev spaces with degenerate weights

Julian Rolfes

We give a definition for fractional weighted Sobolev spaces with degenerate weights. We provide embeddings and Poincare inequalities for these spaces and show robust convergence when the parameter of fractional differentiability goes to 1.

Sudakov-type concentration results and local laws for weighted random matrices

Holger Sambale

Generalizing Sudakov's classical results for weighted sums of independent (or mildly correlated) random variables with weights from the Euclidean unit sphere, we develop a model of weighted random matrices which leads to similar results in terms of the respective Stieltjes transforms. We also show closeness to the Stieltjes transform of the semicircle distribution, hence establishing a local semicircular law.

Climate Change Models and Singular Control of SPDEs in Hilbert Spaces

Frank Riedel

Policy makers frequently rely on highly simplified (zero-dimensional) models of climate change and CO2 emissions to evaluate potential economic impacts. We contend that these overly simplistic models can lead to inaccurate conclusions. This paper argues that realistic models require an understanding of the control of physical systems described by stochastic partial differential equations (SPDEs) in conjunction with economic modeling. We develop the optimal control theory to address these complex models.