UNIVERSITÄT BIELEFELD

Internal Workshop of the Projects C3, C4, C5 - CRC1283: Taming Uncertainty in Economics and Finance

Bielefeld, June 17 – June 18, 2020

IMW

SFB 1283

Information

Organizers

Frank Riedel	Bielefeld University	frank.riedel@uni-bielefeld.de
Giorgio Ferrari	Bielefeld University	giorgio.ferrari@uni-bielefeld.de

Venue

The workshop will be held online on Zoom.

Program

Wednesday, June 17

09:15 - 10:00	Introductory Words
	Session 1 — Chairman: Giorgio Ferrari
09:30 - 10:00	**Hanwu Li**, *A Knightian Irreversible Investment Problem*
10:00 - 10:30	**Julian Hölzermann**, *Pricing Interest Rate Derivatives under Volatility Uncertainty*
10:30 - 11:00	- Coffee Break -
	Session 2 — Chairman: Frank Riedel
11:00 - 11:45	**Fausto Gozzi**, *Understanding the Time-Space Evolution of Economic Activities: Recent Mathematical Models*
11:45 - 12:15	**Max Nendel**, *On nonlinear semigroups and model uncertainty in finance*
12:15 - 14:00	- Lunch Break -
	Session 3 — Chairman: Max Nendel
14:00 - 14:45	**Mihail Zervos**, *Risk-sharing with Two-Sided Commitment: a Continuous Time Version*
14:45 - 15:15	**Jodi Dianetti**, *Multidimensional Singular Control and Related Skorokhod Problem over the infinite Time-horizon*
15:15 - 15:45	**Patrick Schuhmann**, *An Optimal Production Problem under Regime Switching*
15:45 - 16:15	- Coffee Break -
	Session 4 — Chairman: Hanwu Li
16:15 - 17:00	**Jacco Thijssen**, *Optimal Timing of Interventions during an Epidemic*
17:00 - 17:45	Concluding remarks

Thursday, June 18

10:00 - 12:00	Internal discussion of the research groups C3-C4-C5

Abstracts

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1. **Jodi Dianetti, Bielefeld University**

   Title: Multidimensional Singular Control and Related Skorokhod Problem over the infinite Time-horizon

   Abstract: In this talk we consider a bounded variation control problem; that is, the problemof linearly controlling a diffusion through a bounded variation process, in order to minimizea discounted cost functional over the infinite time horizon. The setting is multidimesional,and the dynamics of the state and the costs are convex. Under some structural conditionson the data, we characterize the unique optimal policy as a solution to a related Skorokhodproblem. In particular, we prove that the optimal control acts only when the underlying diffu-sion touches the free boundary related to the Hamilton-Jacobi-Bellman variational inquelity,and that the direction of this action is prescribed by the derivative of the value function.Our approach is based on the study of some monotonicity property of the derivative of thevalue function through its interpretation as the value of an optimal stopping game. Thismonotonicity allows to construct nearly optimal policies as solutions to Skorokhod problemswhich reflect the underlying diffusion near the original free boundary. The limit of thisapproximation scheme is then studied, providing the desired characterization.

   This talk is based on a joint work with Giorgio Ferrari.

2. **Fausto Gozzi, LUISS G. Carli, Rome**

   Title: Understanding the Time-Space Evolution of Economic Activities:

   Recent Mathematical Models

   Abstract: The goal of this talk is to present a review of recent models on the time evolution of most important economic variables (e.g. labour and capital) across different locations, taking into account space heterogeneity. In particular we focus on two recent works (partly in progress): - one looking at the macro level where there is one planner which, in a spatial Ramsey setting, maximizes utility across space with heterogeneous productivity. - one looking at the micro level where the agents move across space maximizing their own utility which also depend on the position of the other agents.

3. **Julian Hölzermann, Bielefeld University**

Title: Pricing Interest Rate Derivatives under Volatility Uncertainty

Abstract: We study the pricing of contracts in fixed income markets in the presence of volatility uncertainty. We consider an arbitrage-free bond market under volatility uncertainty. The uncertainty about the volatility is modeled by a G-Brownian motion, which drives the forward rate dynamics. The absence of arbitrage is ensured by a drift condition. In such a setting we obtain a sublinear pricing measure for additional contracts. Similar to the forward measure approach, we de fine a forward sublinear expectation to simplify the valuation of cashflows. Under the forward sublinear expectation, we obtain a robust version of the expectations hypothesis and a valuation method for bond options. With these tools, we derive robust pricing rules for the most common interest rate derivatives: fixed coupon bonds, floating rate notes, interest rate swaps, swaptions, caps, and floors. For fi xed coupon bonds, floating rate notes, and interest rate swaps, we obtain a single price, which is the same as in traditional models. For swaptions, caps, and floors, we obtain a range of prices, which is bounded by the prices from traditional models with the highest and lowest possible volatility. Due to these pricing formulas, the model naturally exhibits unspanned stochastic volatility.

4. **Hanwu Li, Bielefeld University**

Title: A Knightian Irreversible Investment Problem

Abstract: In this talk, we study an irreversible investment problem under Knightian uncertainty. In a general framework, in which Knightian uncertainty is modeled through a set of multiple priors, we prove existence and uniqueness of the optimal investment plan, and derive necessary and sufficient conditions for optimality. This allows us to construct the optimal policy in terms of the solution to a stochastic backward equation under the worst-case scenario. In a time-homogeneous setting – where risk is driven by a geometric Brownian motion and Knightian uncertainty is realized through a so-called "κ-ignorance" – we are able to provide the explicit form of the optimal irreversible investment plan.

This is based on joint work with Giorgio Ferrari and Frank Riedel.

5. **Max Nendel, Bielefeld University**

Title: On nonlinear semigroups and model uncertainty in finance

Abstract: In mathematical finance, model uncertainty or ambiguity is an almost omnipresentphenomenon, which, for example, appears when certain aspects of an underlying asset can-not be determined precisely or in presence of insufficient data in order to perform reliable sta-tistical estimation methods for the parameters of a stochastic process. The latter typicallyleads to so-called parameter uncertainty in the generator of a stochastic process. Under thistype of uncertainty, worst case considerations together with dynamic consistency require-ments lead to a stochastic optimal control problem, where, roughly speaking, "nature" tries tocontrol the system into the worst possible scenario. In this talk, we illustrate how a class ofoptimal control problems arising in this context can be tackled using a semigroup-theoreticapproach proposed by M. Nisio. We conclude by considering a second approach, where theuncertainty is captured in terms of a Wasserstein distance around a reference model

6. **Patrick Schuhmann, Bielefeld University**

Title: An Optimal Production Problem under RegimeSwitching

Abstract: In this talk we discuss a stochastic inventory control problem with regimeswitching. The cumulative demand of a good is considered as a Brownian Mo-tion with drift and volatility coefficients modulated by a continuous time Markovchain representing the regime of the economy. The manager of the firm choosesa non-negative production strategy to maintain the inventory level as close aspossible to a given target value. The firm is penalized for derivations fromthe target value and the objective of the manager is to minimize the total costassociated to the production strategy. We consider two different scenarios:In the first one, we model the control problem as a singular stochastic controlproblem with regime switching. In particular, the production strategy is con-sidered to be non-negative and unbounded from above. Since this assumptionmight not be realistic in many applications, we also model the control problem asa classical stochastic control problem with bounded velocity and regime switch-ing. In particular, the production strategy is considered to be non-negative andbounded from above by an exogenously given boundK(possibly depending onthe Markov chain).We use a Guess-and-Verify approach to find a solution for the value functionin the two different scenarios. In particular, we show that the optimal solutionin the first case is to hold the inventory always above a certain thresholdbdepending on the state of the economy. In the second case, we show that theoptimal production rate is of bang-bang type, i. e. there exists a state-dependentthresholdbsuch that the production rate is zero if the inventory is above this andmaximal if the inventory is below. Furthermore, we show that these boundariescan be characterized by a nonlinear system of equations.We finish our analysis by

providing a numerical solution for the boundariesand we study the dependency of them with respect to certain model parameters.

Based on joint work with Abel Cadenillas.

7. **Jacco Thijssen, University of York**

Title: Optimal Timing of Interventions during an Epidemic

Abstract: The recent outbreak of the SARS-CoV-2 virus, causing infectious disease COVID-19, has shown that government (non-pharmaceutical) intervention can have a significant impact on the rate at which a virus spreads. It has been deemed imperative to reduce and delay the peak ("flattening the curve"). Flattening the curve is especially important when considering the impact of an outbreak on the health care system. Delaying the peak gives hospitals and (local) governments more time to prepare for later outbreaks and the (anticipated) influx of patients, but, more importantly, when the peak number of people requiring care simultaneously is reduced, there is a lower risk of the healthcare system being overwhelmed. It remains unclear, though, under what circumstances non-pharmaceutical intervention is optimal and perhaps even more importantly, once measures are in place, when to stop non-pharmaceutical interventions. When measures are lifted too early a second outbreak could appear. Intervention lasting too long could lead to unnecessary long term economic consequences. This paper uses a continuous-time Markov chain model to study the value and optimal exercise decision of two (sequential) options: the option to intervene and, after intervention has started, the option to end it. We find that early intervention is optimal and that measures should stay in place until the probability of absorption is very high. Also, if the initial number of infectives is too high, then it is optimal not to have (non-pharmaceutical) intervention at all.

8. **Mihail Zervos, London School of Economics**

Title: Risk-sharing with Two-Sided Commitment: a Continuous Time Version

Abstract: We consider two agents receiving exogenous endowments in a non-storable good. On their own, each of the agents derives utility by consuming their individual endowment, which they receive continuously over an infinite time horizon. The two agents may decide to pull their endowments and agree on a consumption allocation with a view to risk-sharing. We assume that both agents have limited commitment, which gives rise to the constraints that the utilities they derive from their share of the allocation should be greater than or equal to their autarky utilities at all times. We address

this problem using a duality approach. The dual formulation, which involves Lagrange multipliers to enforce the participation constraints, gives rise to a singular stochastic control problem. As an application, we consider a symmetric environment where endowment shares are driven by a mean-reverting diffusion process, allowing for aggregate uncertainty. The relevant co-state acts as a time-varying Pareto weight that determines the consumption allocation. We analyse the HJB equation associated with this problem and solve for the free-boundaries that delineate regions of the state space in which participation constraints become binding. In some configurations, perfect risk-sharing is sustainable.

List of Participants

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