Delaying Product Introduction: A Dynamic Analysis with Endogenous Time Horizon*

Serhat Gezer†

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Abstract

We consider a capital accumulating incumbent firm which produces an established product and has the option to introduce an improved substitute product to the market by incurring adoption costs. We find that depending on the initial capacities on the established market and the value of adoption costs, three scenarios are possible, namely introducing immediately, later or abstaining from product introduction. In case of delayed product introduction, the incumbent reduces capacities for the established product before the new product is introduced. We find that the higher the adoption costs, the higher is the gain by delaying the product introduction compared to immediate introduction. From a welfare perspective, the product introduction is welfare enhancing but the option of delay decreases the welfare gain. The model is calibrated using data on hard disk and solid state drives.

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†Department of Business Administration and Economics and Center for Mathematical Economics, Bielefeld University, Universitätsstraße 25, 33615 Bielefeld, Germany, E-mail address: sgezer@uni-bielefeld.de

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1 Introduction

For many firms, especially for those operating in the high-tech sector, whenever a new technology is available, they have to decide whether to adjust the product range by incorporating the new technology and if yes, when to do so.

Wang and Hui (2012) provide examples of firms hesitating to incorporate new available technologies and choosing to stay with the old technology for a while. Examples include the technology of DVD that has been developed well before vendors started promoting DVDs. Another example is the MP3 standard.

In an empirical investigation, Chandy and Tellis (2000) have found that a large fraction of product innovations has been achieved by incumbents. Indeed, we face such a situation described above often in real-world markets and in many industries, submarkets evolve and coexist with the established product. An example is the TV Industry where CRT televisions and flatscreens were sold simultaneously for a long time (cf. Dawid et al. (2015)). Another example is the storage device industry where solid-state drives (SSD) have been introduced to the market in addition to hard disk drives (HDD). We use recent data from this industry on worldwide sales to calibrate our model.[1]

We consider an incumbent firm which has the option to introduce a horizontally and vertically differentiated substitute product which has a higher quality than the established one. For realizing this option, it incurs one-time adoption costs. Thus, the firm has to determine if the product introduction is profitable and if yes, when the optimal time of product introduction is. After introduction, we assume that the firm sells both products.

[1]Data on HDD and SSD sales stem from quarterly reports of Western Digital and *Storage-Newsletter*. The latter is provided by TrendFocus which is a market research and consulting firm which is specialized in the data storage industry.

The firm faces the following trade-off: On the one hand, by launching the new product it cannibalizes demand for the established product. On the other hand, it benefits from the new product with higher quality by exploiting higher willingness to pay of the consumers.

We find that if the firm is strong on the established market, i.e. its capacities are at a high level, then the firm decides to wait and to introduce the improved product later. By delaying, the firm benefits from discounting adoption costs (e.g. coming from adjustment costs, advertisement activities or fees paid to developers for using their technologies) while it decreases the capacity of the established product before the new product is introduced. Due to this reduction of capacity, prices for both products increase which reduces cannibalization. Amongst others, this enables the incumbent to build-up capacities for the new product faster when it is introduced, compared to immediate introduction.

There is a large literature on capital accumulating firms which has been extended by Dawid et al. (2015) who analyze the optimal R&D effort for product innovation and the optimal capital accumulation of established and new products, where the breakthrough probability of developing a new product depends on both, the knowledge stock and the current R&D effort. Hence, in that paper innovation time is stochastic and it is assumed that the new product is introduced immediately once it is available. We focus on the optimal timing of product introduction and optimal investment in capacities and differ from Dawid et al. (2015) in not considering R&D efforts to develop a new product and not linking successful development to market introduction but considering the time of market introduction as a choice variable.

The classical literature on optimal timing of technology adoption (see, e.g., Kamien and Schwartz (1972) for a single firm and Reinganum (1981) and Fudenberg and Tirole (1985) for a duopoly) assumes that quality increases due to technological progress and the only decision variable is the time of technology adoption (see Hoppe (2002) for a survey on theoretical models and empicial evidence). Farzin et al. (1998) and Doraszelski (2004) extend this stream of literature by modeling the quality improvement as a stochastic process. In contrast, in our model, the quality of the new product is fixed and the firm cannot gain additional quality by delaying.

Our analysis focuses on the dependence on initial characteristics whose importance has been addressed a lot, e.g., in Hinloopen et al. (2013) where initial marginal costs determine if a technology is developed further or not. Here, initial levels of capacity determine whether and when a new product is going to be introduced.

Real options models (see, e.g., Dixit and Pindyck (1994)) have focussed on optimal timing in continuous time where demand is stochastic, e.g., evolving according to a Brownian motion. A simultaneous analysis of optimal timing and optimal investment in capacities in the real options literature has been provided by Huisman and Kort (2015) where the price of the good is stochastic. We differ from that stream of literature by considering a deterministic environment and continuous adjustments of capacities.

The problem of an incumbent delaying product introduction has been addressed in Wang and Hui (2012). They apply a discrete three-period time framework where capacity adjustments are not taken into account.

Hendricks and Singhal (1997) estimate empirically the impact of being late to the market, i.e. not fulfilling promise of preannouncements. While reasons for not meeting an announced introduction date include problems in development and the need to redesign products, managerial reasons are given in Adaku et al. (2018). In contrast to those works, intentional delay is considered in the production management literature where existing inventory is identified to cause delays of new product introductions in different industries (see Avlonitis (1983), Billington et al. (1998), Koca et al. (2010), Li et al. (2010) and Katana et al. (2017)).

The model is calibrated in order to replicate the recent dynamics in the HDD and SSD industry. While worldwide shipments of HDDs decrease, SSDs' shipment increases strongly. SSDs are considered to be superior to HDDs in many aspects accompanied with the disadvantage of comparably high price. Even though those are not monopoly markets, they are quite concentrated and dominated by a small number of firms such that we believe that this data is to some extent suitable for our parametrization.

From a technical perspective, we employ Pontryagin's Maximum Principle for free end time (see, e.g., Grass et al. (2008)) to obtain analytical results concerning the optimal investments and the optimal time of market introduction. Moreover, in this optimal control problem, due to the non-concave structure of the value function, the Arrow-Mangasarian sufficiency conditions are not met which might lead to the presence of multiple optimal investment paths. In particular, we characterize situations in which the firm is indifferent between introducing and not introducing the new product to the market. In such models, qualitative properties of solutions depend very much on parameters (cf. Hinloopen et al. (2013)). Therefore, we use a bifurcation analysis to assess industry dynamics for different values of adoption costs where the state space is divided in parts which correspond to immediate, delayed or no product introduction, respectively.

The analysis in this paper is carried out for a monopoly setting. Even though the real-world examples we have raised stem from competitive environments, we believe that it is important to consider the monopoly as it is interesting in its own right. Indeed, timing of product introduction is not only influenced by competing firms but also by competing substitute products even if there is only a single firm. As the established and new product are substitutes, there is 'internal' competition between those two products. In order to disentangle rivalry between products and between firms, it is reasonable to analyze the monopoly case before proceeding to the competition case.

The paper is organized as follows. We introduce the model in Sect. 2. Sect. 3 is devoted to the technical analysis. In Sect. 4, we provide an economic interpretation, conduct a bifurcation analysis and present optimal timing curves. Sect. 5 analyzes welfare effects of delaying product introduction. Model assumptions are discussed in Sect. 6 and Sect. 7 concludes.

2 Model

We consider an incumbent firm which has initial capacity K_1^{ini} to produce an established product. A new substitute product with higher quality has been developed and is ready for market introduction. Product introduction comes with lump-sum adoption costs F. An important assumption is that the incumbent cannot invest in capacities of the new product before introducing it, i.e. there are no capacities at the time of introduction for the new product.

We follow the literature on optimal capital accumulation by relying on a standard linear model (see, e.g., Dockner et al. (2000)). Thus, the firm faces a linear inverse demand function which is given by

$$p_1(t) = 1 - K_1(t). \tag{1}$$

After product introduction, the inverse linear demand system[2] is given by

$$p_1(t) = 1 - K_1(t) - \eta K_2(t), \tag{2}$$

and

$$p_2(t) = 1 + \theta - \eta K_1(t) - K_2(t), \tag{3}$$

where $0 < \eta < 1$ measures the degree of horizontal and $\theta > 0$ the degree of vertical differentiation of the substitutes.

The firm wants to determine the optimal time of product introduction T and the optimal investment strategies before and after product introduction. There is no inventory, i.e. capacities equal sales[3]. The capacity dynamics are

$$\dot{K}_i(t) = I_i(t) - \delta K_i(t), \quad i = 1, 2, \tag{4}$$

$$K_1(0) = K_1^{ini}, \quad K_2(t) = K_2^{ini} = 0 \; \forall \; t \leq T, \tag{5}$$

where $\delta > 0$ measures the depreciation rate. As has been done in Dawid et al. (2015), we allow the firm to intentionally scrap capacities, i.e. $I_i \in \mathbb{R}$ while capacities have to remain non-negative:

$$K_i(t) \geq 0 \quad \forall \; t \geq 0, i = 1, 2. \tag{6}$$

[2]This demand system is motivated by the fact that the two products are substitutes and competing with each other. According to the seminal result of Kreps and Scheinkman (1983), setting prices optimally subject to ex-ante capacity commitments reduces to a Cournot setting which we adopt here.

[3]This assumption has been used in large parts of the literature on dynamic capacity investment, see, e.g., Goyal and Netessine (2007). See Section 6 for a discussion of this assumption.

Adjusting capacities is costly, in particular it comes with quadratic costs

$$C\big(I_i(t)\big) = \frac{\gamma}{2} I_i^2(t), \qquad i = 1, 2. \tag{7}$$

Normalizing production costs to zero, the objective function of the firm is given by the following expression:

$$\begin{aligned}\max_{T, I_1(t), I_2(t)} J = &\int_0^T e^{-rt}\big(p_1(t)K_1(t) - C(I_1)\big)dt \\ &+ \int_T^\infty e^{-rt}\big(p_1(t)K_1(t) + p_2(t)K_2(t) - C(I_1) - C(I_2)\big)dt - e^{-rT}F.\end{aligned} \tag{8}$$

We refer to this problem as $\mathcal{P}(K_1^{ini})$.

3 Analysis

In case that the firm wants to introduce the improved product at some finite time T, there will be a structural change of the model. Therefore, we denote by mode 1 (m_1) the optimal control problem up to time T and by mode 2 (m_2) the problem after T, where T might be infinite. Denote by $V^{m_1}(K_1)$ and $V^{m_2}(K_1, K_2)$ the corresponding value functions of the infinite horizon control problems where the mode is fixed and hence does not change[4]. The optimal control problem at hand where the mode m might change is denoted by $V(K_1, K_2, t, m)$ and we refer to this problem as the *optimal control problem with introduction option*.

The subproblem in m_2 is linear-quadratic with infinite time horizon which can be solved easily, as has been done in Dawid et al. (2015). The optimal strategy and the value function are stationary for this problem, i.e.

$$V(K_1, K_2, t, m_2) = V^{m_2}(K_1, K_2) - F. \tag{9}$$

There is a unique globally asymptotically stable steady state under the optimal strategy and the value function is given by[5]

$$V^{m_2}(K_1, K_2) = aK_1^2 + bK_1 + dK_1K_2 + eK_2^2 + fK_2 + g. \tag{10}$$

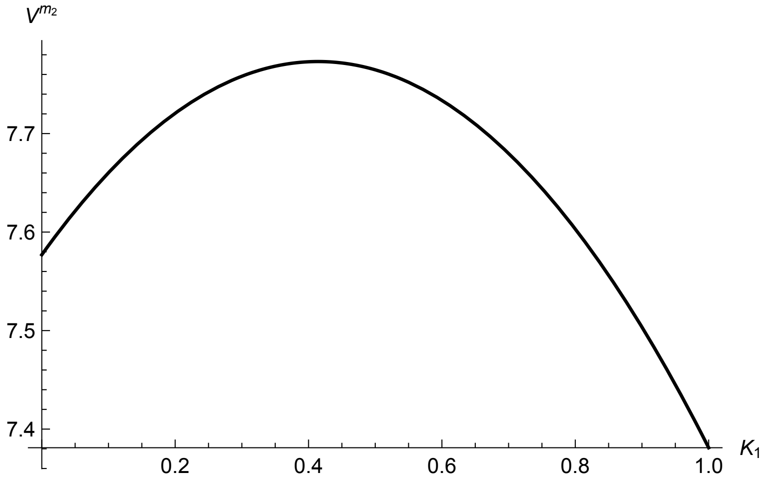


Figure 1: Value function of m_2 at T, i.e. for $K_2 = 0$. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.154472, \gamma = 5.6$.

The typical shape of the value function of m_2 is depicted in Figure 1.[6] In m_2, due to the higher willingness to pay for the new product, steady state profits will be higher compared to m_1. However, as we assume that capacities at hand cannot be transferred to the production of the new product, capacities for the new product has to be build up which temporarily reduces profits when switching to m_2.

By regarding the value function of the subproblem as the salvage value of the optimal control problem with introduction option, we can rewrite (8) by

$$\max_{T, I_1(t)} J = \int_0^T e^{-rt}\big(p_1(t)K_1(t) - C(I_1(t))\big)dt + e^{-rT}S\big(K_1(T)\big), \tag{11}$$

where $S\big(K_1(T)\big) = V^{m_2}(K_1(T), 0) - F$.[7] This problem can be solved analytically by Pontryagin's Maximum Principle for variable terminal time. The Hamiltonian is

$$H(K_1, I_1, \lambda, t) = (1 - K_1)K_1 - \frac{\gamma}{2}I_1^2 + \lambda(I_1 - \delta K_1), \tag{12}$$

[4] We suppress the argument t wherever it is possible and no confusion may arise.

[5] Coefficients are derived numerically from a system of nonlinear equations which is given in Dawid et al. (2015).

[6] The marginal value of K_1 is decreasing and even becomes negative due to the assumption that there is no inventory and hence capacities equal quantities.

[7] $K_2(T) = 0$ since there are no capacities for the new product at T, yet.

where λ is the co-state variable and the optimal investment is given by

$$I_1 = \frac{\lambda}{\gamma}. \tag{13}$$

The co-state equation reads

$$\dot{\lambda} = (r + \delta)\lambda - (1 - 2K_1), \tag{14}$$

and the transversality condition is given by[8]

$$\lambda(T) = S_{K_1} = V^{m_2}_{K_1}(K_1, 0), \tag{15}$$

where $S_{K_1} = \frac{\partial S(K_1(t))}{\partial K_1(t)}$. For nonzero finite T^*, let $\big(K_1^*(\cdot), I_1^*(\cdot)\big)$ be an optimal solution to (11) on the optimal time interval $[0, T^*]$ for m_1. Pontryagin's Maximum Principle for variable end time implies an additional constraint for the terminal time, which is given by

$$H(K_1^*(T^*), I_1^*(T^*), \lambda(T^*), T^*) = rS\big(K_1^*(T^*)\big) - S_T\big(K_1^*(T^*)\big). \tag{16}$$

Note that the salvage value does not depend explicitly on T^* and hence,

$$S_T\big(K_1^*(T^*)\big) = 0. \tag{17}$$

Equation (16) is obtained by considering the right hand side of (11) as a function of terminal time and maximizing the function with respect to terminal time (see Grass et al. (2008)). Intuitively, equation (16) requires that staying marginally in m_1 and introducing afterwards is as good as introducing the new product immediately.

In Lemma 2 in Appendix A.2, we state that there are two solutions for equation (16). We denote the two solutions of (16) by K_1^{lb} and K_1^{ub}, respectively for lower and upper bound of an interval (where $K_1^{lb} \leq K_1^{ub}$) which we will analyze further below. For $F = 0$, both solutions coincide[9], i.e. $K_1^{lb} = K_1^{ub}$ (see Appendix A.2), which we denote by $K_1^{F=0}$.

[8] The canonical system, isoclines, the steady state for staying in m_1 (which is denoted by K_1^{ss,m_1}) and its stability properties are given in Appendix A.1.

[9] Technically, in case of no adoption costs, H and rS are tangential at $K_1^{F=0}$:

$$\frac{\partial}{\partial K_1} H(K_1^{F=0}, I_1^*(T^*), \lambda(T^*), T^*) = \frac{\partial}{\partial K_1} rV^{m_2}(K_1^{F=0}, 0). \tag{18}$$

Note that equation (16) is a necessary condition and hence, delaying not only marginally but for a longer time might yield higher value. Thus, we have to figure out whether K_1^{lb} and K_1^{ub} are indeed optimal.

To answer this question, we state the following lemma where we focus on the dependence of K_1^{lb} on F.

Lemma 1. *K_1^{lb} is decreasing in F.*

Proof. See Appendix A.3. □

Moreover, K_1^{ub} is increasing in F. Thus, for increasing F, the interval $[K_1^{lb}, K_1^{ub}]$ expands around $K_1^{F=0}$.

In the proof of Lemma 2 in Appendix A.2, we find that

$$\frac{\partial V^{m_2}}{\partial K_2}(K_1^{F=0}, 0) = dK_1^{F=0} + f = 0. \tag{19}$$

It can easily be seen that $\frac{\partial V^{m_2}}{\partial K_2}(K_1^{F=0}, 0)$ is monotone in K_1 and thus,

$$\frac{\partial V^{m_2}}{\partial K_2}(K_1^{F=0}, 0) < 0 \quad \text{for} \quad K_1 > K_1^{F=0}, \tag{20}$$

and

$$\frac{\partial V^{m_2}}{\partial K_2}(K_1^{F=0}, 0) > 0 \quad \text{for} \quad K_1 < K_1^{F=0}. \tag{21}$$

If the firm were to introduce the new product for $K_1 > K_1^{F=0}$, this would yield a negative investment for K_2 in m_2 which would violate the non-negativity constraint of K_2 as we have assumed that no capacities for the new product are installed when introducing the new product. Hence, investments in K_2 would be restricted to be 0 as long as K_1 stays above $K_1^{F=0}$. Hence, for $K_1 > K_1^{F=0}$, introducing immediately cannot be optimal. As $K_1^{ub} \geq K_1^{F=0}$, as above, investments in m_2 would be restricted such that introducing is not optimal. We show in Appendix A.4 in Lemma 4 that in case it is optimal to introduce the new product, then K_1^{lb} is the optimal capacity to introduce the new product at. Hence, for higher capacities, it is optimal not to introduce and for lower capacities, it is optimal to introduce right away.

As the optimal introduction time depends on the size of capacity, we consider it as a correspondence depending on K_1^{ini} and denote it by

$$T^*(K_1^{ini}).^{10} \tag{22}$$

It is a correspondence since there are situations with multiple optimal values as we will discuss in the following main Proposition 1.

Before stating the main Proposition 1 we need to prove several results, which is done in Appendix A.5 and A.6. In particular, there exists a threshold value for adoption costs $\tilde{F}$, above which the firm finds it optimal to abstain from introducing the new product if the current capacity exceeds a certain threshold $\tilde{K}_1$. The latter is decreasing in F.

Proposition 1.

i) For $0 \leq F < \tilde{F}$,

$$T^*(K_1) = 0 \qquad \textit{for all } K_1 \leq K_1^{lb}, \tag{23}$$

$$0 < T^*(K_1) < \infty \qquad \textit{for all } K_1^{lb} < K_1. \tag{24}$$

ii) For $F = \tilde{F}$,

$$T^*(K_1) = 0 \qquad \textit{for all } K_1 \leq K_1^{lb}, \tag{25}$$

$$0 < T^*(K_1) < \infty \qquad \textit{for all } K_1^{lb} < K_1 < \tilde{K}_1, \tag{26}$$

$$T^*(K_1) = \infty \qquad \textit{for all } \tilde{K}_1 \leq K_1. \tag{27}$$

iii) For $\tilde{F} < F < \bar{F}$,

$$T^*(K_1) = 0 \qquad \textit{for all } K_1 \leq K_1^{lb}, \tag{28}$$

$$0 < T^*(K_1) < \infty \qquad \textit{for all } K_1^{lb} < K_1 \leq \tilde{K}_1, \tag{29}$$

$$T^*(K_1) = \infty \qquad \textit{for all } \tilde{K}_1 \leq K_1. \tag{30}$$

[10] An alternative would have been to define a function which gives the remaining time in m_1 not depending on the initial but current capacity (cf. Long et al. (2017)).

iv) For $\bar{F} \leq F$,

$$T^*(K_1) = 0 \qquad \textit{for all } K_1 \leq \tilde{K}_1, \tag{31}$$

$$T^*(K_1) = \infty \qquad \textit{for all } \tilde{K}_1 \leq K_1. \tag{32}$$

Proof. See Appendix A.5 and A.6. □

Proposition 1 states that immediate introduction is optimal if capacity for the established product is lower than a certain threshold (either K_1^{lb} or $\tilde{K}_1$ depending on the level of adoption costs) whereas for capacities above, it is either optimal to wait and to decrease capacities on the established market before product introduction or not to introduce at all.

Note that in Proposition 1 *iii)* and *iv)*, there are two different solutions at $K_1 = \tilde{K}_1$ that are both optimal. Hence, for $F > \tilde{F}$ at $\tilde{K}_1$, the firm is indifferent between introducing the new product (possibly after some delay) or not introducing at all[11]. At $\tilde{F}$, $\tilde{K}_1$ (the threshold separating finite and infinite solutions for T) is K_1^{ss,m_1} which is the long run capacity value if the firm stays with its established product (see Appendix A.1. That is, for $K_1 \geq K_1^{ss,m_1}$ the firm prefers not to innovate and stays in m_1, whereas for $K_1 < K_1^{ss,m_1}$ the firm decreases[12] capacities to K_1^{lb} where the new product is introduced eventually.

For $K_1^{F=0} < K_1$, as mentioned above, the non-negativity constraint for K_2 would have been active in m_2. Capacities of the established product would be lowered until $K_1^{F=0}$ but there, lowering further until K_1^{lb} would be optimal and hence, the product introduction would take place at K_1^{lb}.

By Proposition 3 in Appendix A.6, $\tilde{K}_1$ decreases in F and hence the range of capacities where the firm stays with only one product enlarges.

Note that for $F < \bar{F}$, the value function of m_2 and the value function of the problem with introduction option paste smoothly at K_1^{lb}, i.e.[13]

$$\frac{\partial V(K_1^{lb}, 0, m_1)}{\partial K_1} = \frac{\partial V^{m_2}(K_1^{lb})}{\partial K_1} . \tag{33}$$

[11]There is no other value of capacity where both solutions are optimal.

[12]In Appendix A.5 in Lemma 6, we show that at $\tilde{F}$, $K_1^{lb} \leq K^{ss,m_1}$ holds.

[13]Note that the value function is time-invariant and hence the time argument can be omitted, i.e. $V(K_1^{lb}, K_1^{lb}, t, m) = V(K_1^{lb}, K_1^{lb}, m)$.

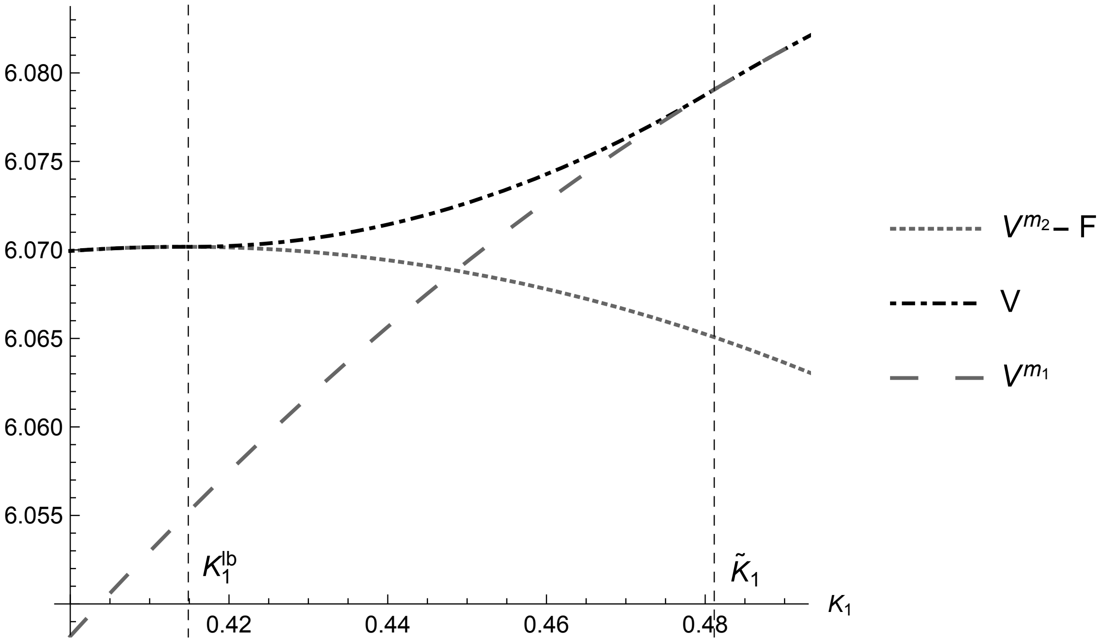


Figure 2: Value function for $\tilde{F} \approx 1.7029$. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.154472, \gamma = 5.6$.

Furthermore, at $\tilde{F}$, the value function of the problem with introduction option and the value function of m_1 paste smoothly at $\tilde{K}_1$ (see Figure 2) whereas for $F > \tilde{F}$ the value function has a kink at $\tilde{K}_1$.

In total, as long as F is intermediate (i.e. $\tilde{F} < F < \bar{F}$), we can split the state space in three parts:

i) 'Immediate introduction': $K_1 \leq K_1^{lb}$: Firm innovates immediately, $T^* = 0$.

ii) 'Delayed product introduction': $K_1^{lb} < K_1 \leq \tilde{K}_1$: Firm delays introduction and introduces the product later at $0 < T^* < \infty$.

iii) 'No introduction': $K_1 \geq \tilde{K}_1$: Firm delays introduction infinitely, i.e. there is no product introduction.

For increasing F the indifference point $\tilde{K}_1$ shifts to the left and eventually the waiting region vanishes where $\tilde{K}_1$ and K_1^{lb} coincide and only two possibilities remain: Either the firm innovates immediately (for low capacities) or never (for high capacities). Hence, for $F \geq \bar{F}$, the value function is given by the upper curve of the value functions V^{m_1} and V^{m_2}.

For $K_1^{lb} < K_1^{ini}$, the higher K_1^{ini} the longer it takes to arrive at K_1^{lb} where the firm wants to launch the new product, i.e. the stronger the firm is on the established market, the more it delays the introduction of the new product. On the other hand, due to Lemma 1, the higher the adoption costs, the lower is the switching capacity, i.e. the firm wants to reduce capacities more in advance before switching to m_2. Thus, higher capacities and higher adoption costs, both lead to a longer delay.

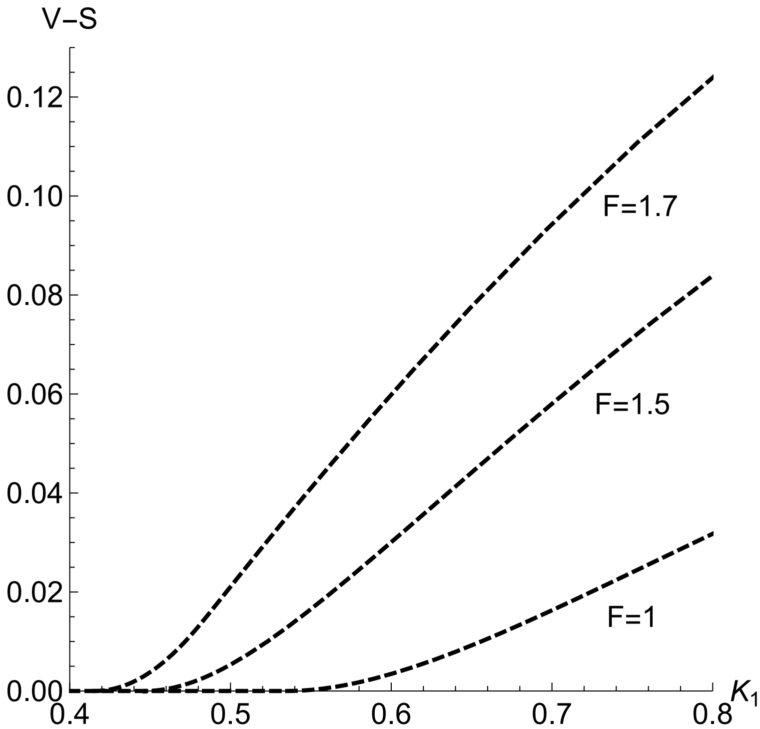


Figure 3: Gain by delay. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.154472, \gamma = 5.6$.

In Figure 3, we illustrate how the difference between the value function of the problem with introduction option (V) and the problem corresponding to immediate introduction (S) evolves as F increases. By Proposition 1, for $K_1 > K_1^{lb}$, $V > S$. As F increases and discounting adoption costs become more important, the difference of the value function with introduction option and the scrap value function gets larger. In other words, the higher the adoption costs, the more valuable the option to delay the product introduction.

Furthermore, as the products are vertically differentiated, the value of the problem of m_2 is higher than that of m_1 for no adoption costs. Thus, the value of the problem with introduction option is higher than the value of the infinite problem of m_1. Obviously, for large enough F, product introduction will not be sufficiently attractive anymore and the incumbent will stay with its established product, i.e. $V = V^{m_1}$.

4 Dynamics

After having analyzed analytically the possible cases for different adoption costs and different established capacities, in this section, we describe optimal capacity investments and provide economic intuition about the optimal timing decision. A bifurcation analysis is presented in Section 4.2. Optimal timing curves and their dependence on parameters of horizontal and vertical differentiation are given in Section 4.3.

In order to derive dynamics, we consider the following default parameter setting motivated by sales data from the storage device industry

$$r = 0.04,\ \delta = 0.1,\ \eta = 0.9,\ \theta = 0.154472,\ \gamma = 5.6. \tag{34}$$

Quarterly worldwide sales data of HDD's and SSD's from 2015 to 2017 are used to calibrate the linear inverse demand system by selecting the parameters η and θ such that the price of SSD is approximately three[14] times as much as HDD's price which is consistent with market observations[15]. Inserting data pairs into the linear inverse demand system yields prices which fluctuate extensively. However, both prices exhibit a decreasing trend which fits to price observations of the market for those products where the price of SSDs decrease faster than the price of HDDs. Assuming standard values for parameters r and δ, the remaining parameter γ

[14] Aggregating annual capacities, on average, the price ratio is 3.1739.

[15] As prices differ along size, capacity, model etc., prices of bestselling HDDs and SSDs for private users with capacity of 1TB are considered. For simplicity, production costs are assumed to be zero. However, the high differences in prices arise to some extent due to different production costs. See Igami (2017) for a structural analysis in the hard disk drive industry.

could not be estimated[16] such that the parameter has been selected in order to get a crowding out of HDDs eventually.

4.1 Economic Interpretation

The intuition for the 'Immediate Introduction' and 'No Introduction' scenario is straight forward. The benefit from the new product is either so high that the firm does not want to wait or the benefit is too low such that the firm stays with the established product. Thus, we focus on the interpretation of the interesting case of delay. The firm exploits profits in m_1 before moving to m_2. In economic terms, the following mechanisms can be identified.

First, the delay in time leads to stronger discounting of the scrap value $V^{m_2} - F$. The firm saves adoption costs as F is paid as a lump-sum, but gets V^{m_2} later as well. The latter is smoothed by the concave structure of the value function of m_2 as the firm reduces capacities of the established product and hence V^{m_2} increases[17].

Second, as mentioned above,

$$\frac{\partial V^{m_2}}{\partial K_2}(K_1^{F=0}, 0) = 0, \tag{35}$$

holds, which has an interesting economic intuition. In contrast to m_1, in m_2, the firm is able to invest in K_2. For $F = 0$ at $K_1^{F=0}$, there is no reason for waiting. But for higher $F > 0$, waiting yields discounting of adoption costs while there is no disadvantage of not being able to invest in the new product's capacity since at $K_1^{F=0}$, (35) still holds. Thus, by postponing the product introduction, the incumbent can decrease the capacity of K_1 before switching such that $\frac{\partial V^{m_2}}{\partial K_2}(K_1^{lb}, 0) > 0$, i.e. when switching, the marginal value of the new product's capacity is positive and hence there is an immediate gain from investment in K_2. Hence, the investment pattern in m_2 is affected, where due to the reduced capacity of the established product, the firm has stronger incentives to build-up capacities for the new product and

[16]The evolution of quantities in this model differs from observed data. Here, the firm would decrease SSD quantities in order to increase the price of HDDs. The reason seems to be the competitive nature of the dataset.

[17]This holds as long as the switching capacity K_1^{lb} is greater than the maximal argument of V^{m_2} which is true for the considered parameter setting.

the disinvestment in the established product is weaker[18] than it would be without delay. In m_2, temporarily profits drop and are lower than in m_1 as there is a strong investment in capacities of the new product but sales increase only gradually for the new product. By delaying, the firm can postpone this drop in profits and enjoy 'high' profits in m_1. However, the drop in profits is stronger compared to immediate introduction.

4.2 Bifurcation Analysis

From Figure 2, it is clear that the value function is not concave in K_1 and hence does not satisfy the Arrow-Mangasarian sufficiency conditions. Thus, as mentioned earlier, in this section we examine the qualitative properties of the steady states of the control problem with introduction option with respect to the parameter F. If the firm starts in the immediate introduction area, it introduces the new

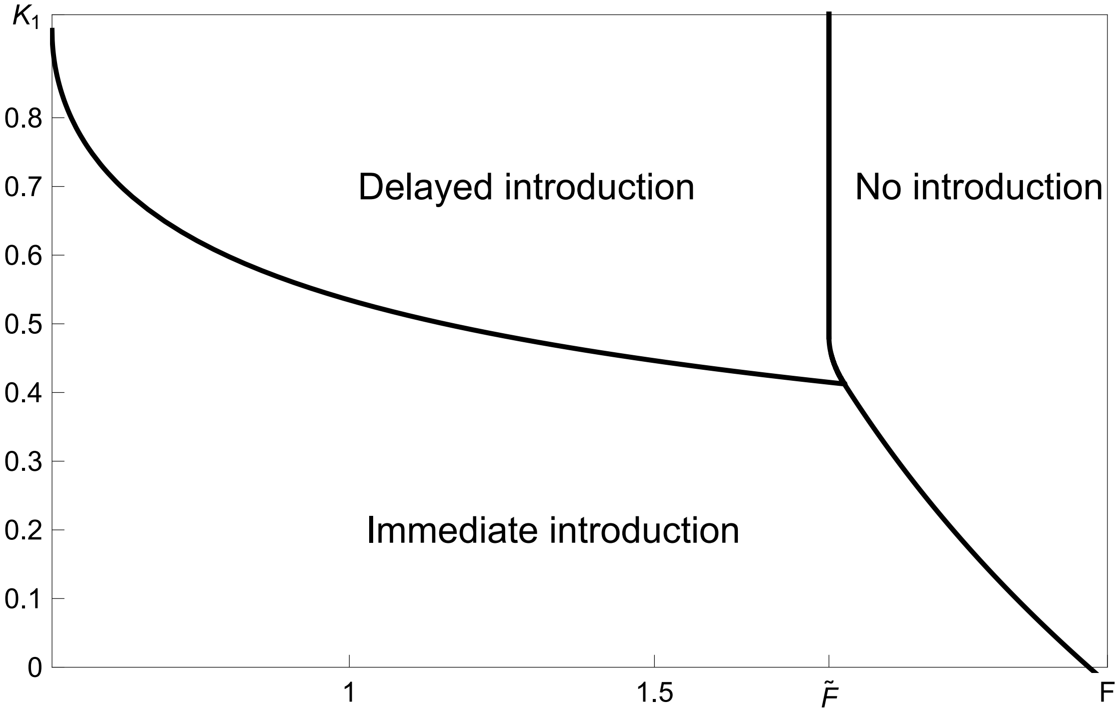


Figure 4: Regions.

product immediately and hence is no more in m_1 but in m_2. In the no introduction

[18]This is due to the increased marginal value of the established capacity.

area, the new product is never introduced. In the delayed introduction area, K_1 will be decreased until it hits the line seperating this area from the immediate introduction area.

Denote by $\bar{\bar{F}}$ the value of adoption costs where thereafter finite solutions for T disappear for the first time[19], i.e.

$$T^*(0) = \infty \ . \tag{36}$$

Hence for $F > \bar{\bar{F}}$, only the no introduction area remains.

As we are interested in characterizing dynamics in m_1 and in m_2 together, we draw a superimposed bifurcation diagram of both modes (cf. Hinloopen et al. (2017)) in Figure 5. For $F < \tilde{F}$, we have a unique stable steady state. No matter

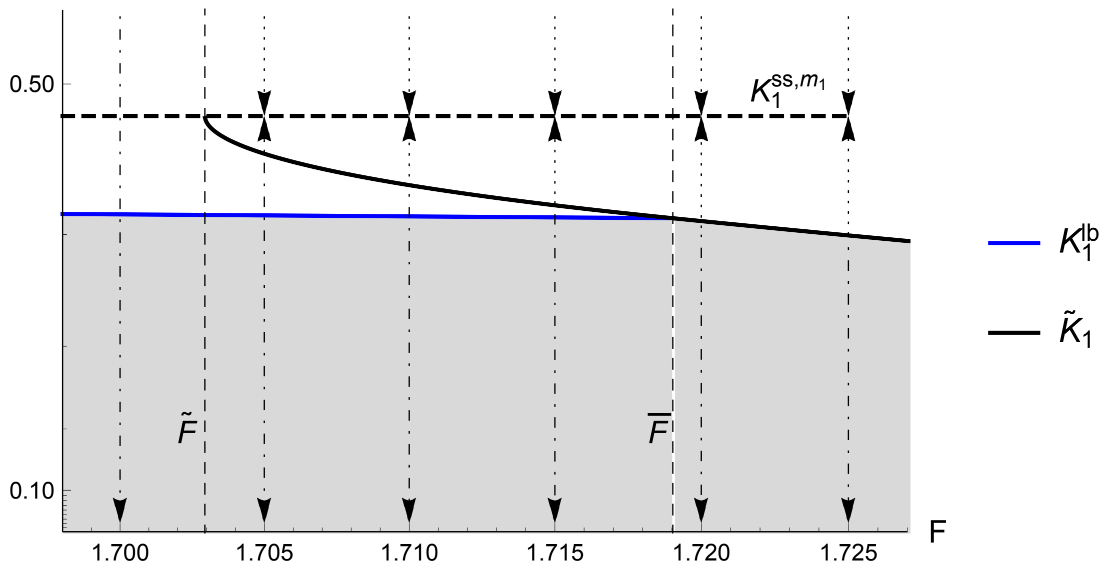


Figure 5: Superimposed diagram.

if the firm delays product introduction or not, it will eventually arrive at the steady state level of K_1 in m_2 which is 0 here. As analyzed before, at $\tilde{F}$ there arises a second steady state where for initial capacities $\tilde{K}_1 \leq K_1$, the firm stays in m_1 and eventually arrives at K_1^{ss,m_1}.

At $\tilde{F}$, we have an indifference-attractor bifurcation which is a heteroclinic bifurcation (see Wagener (2003) and Kiseleva and Wagener (2010) for a characteri-

[19]As $\tilde{K}_1$ is decreasing in F, at $\bar{\bar{F}}$, $K_1 = 0$ is the only remaining value for capacity such that the firm is indifferent between immediate and no product introduction.

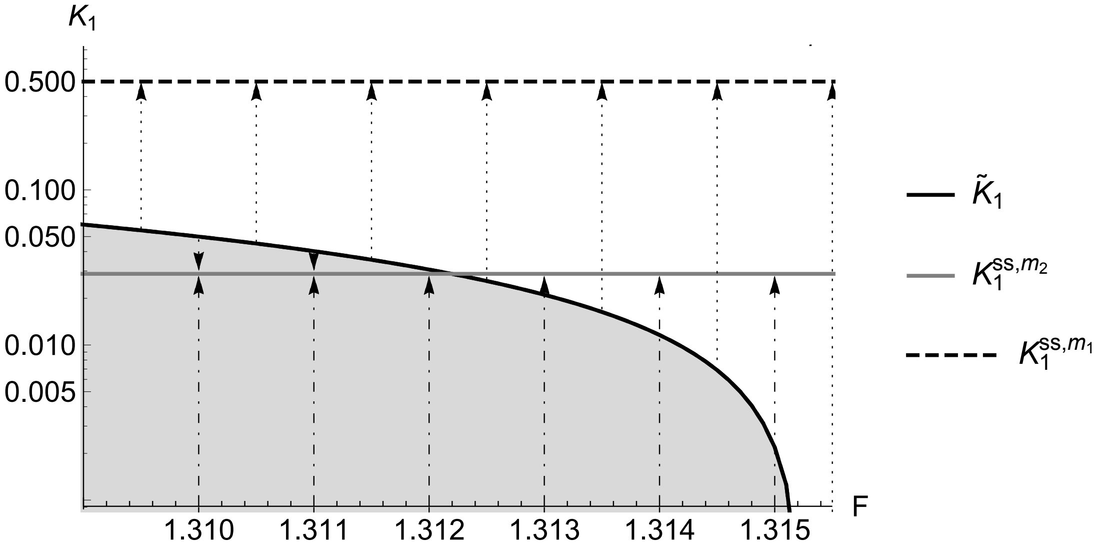


Figure 6: Dynamics around $\bar{\bar{F}}$ for an alternative parametrization: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.1, \gamma = 0.15$.

zation). Initially, the optimal solution is characterized by only one stable steady state. At $\tilde{F}$, a second equilibrium (K_1^{ss,m_1}) arises 'out of the blue', where a repelling curve separates the two basins of attraction. For very high F, only the second equilibrium remains. The black solid curve is the Skiba curve[20] (which is repelling except at $\tilde{F}$ and $\bar{\bar{F}}$ where it is semi-stable). Here, for capacities on the Skiba curve, optimal paths are moving in opposite directions. However, for a different parameter setting with a positive steady state of K_1 in m_2, for capacities on the Skiba curve below K_1^{ss,m_2} both optimal paths would move in same direction (see Figure 6).

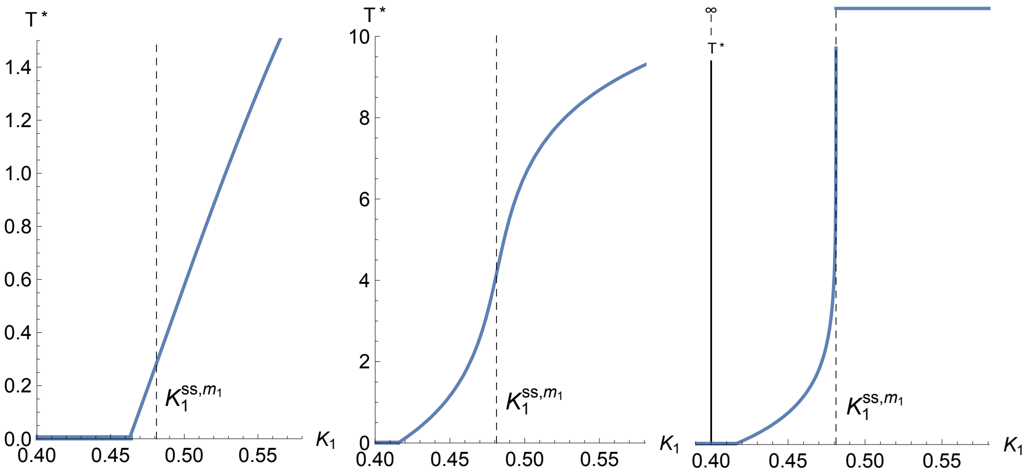
Note that this is a superimposed diagram and not a bifurcation diagram in the classical sense and the latter is possible since there the firm either jumps immediately to m_2 or never which means that we actually consider two disjoint optimal control problems where the mode can be interpreted as a further state variable.

[20]Note that at $\tilde{F}$, the firm is actually not indifferent and hence $\tilde{K_1}$ is not a Skiba point.

4.3 Characterization of Optimal Timing Curves

As discussed in Section 3, for $F > \tilde{F}$, $\tilde{K_1}$ separates finite and infinite solutions for the optimal introduction time. Thus, T^* jumps at $\tilde{K_1}$ to infinity. Hence, for $\tilde{K_1} \leq K_1$, the value function of the problem with introduction option is equal to the value function of the problem without introduction option.

We now investigate in detail what happens when F approaches $\tilde{F}$. The graphs of the optimal introduction time are depicted in Figure 7. For low adoption costs,



$F = 1.4$ $F = 1.7$ $F = \tilde{F} \approx 1.7029$

Figure 7: Optimal time of switching for increasing F.

the correspondence is concave for $K_1 \geq K_1^{lb}$. As analyzed in Section 3, it is finite for low adoption costs whereas it becomes infinite at $\tilde{F}$ for $K_1 \geq \tilde{K_1} = K_1^{ss,m1}$. For F approaching $\tilde{F}$, $T^*(K_1)$ becomes convex-concave and very steep at K_1^{ss,m_1}, i.e K_1^{ss,m_1} becomes an inflection point (see Figure 7) which means that the firm decreases higher capacities and 'stays around' K_1^{ss,m_1} for a while until it starts decreasing again down to K_1^{lb}. Note that for $F < \tilde{F}$, $T^*(K_1)$ is finite everywhere, whereas at $\tilde{F}$, $T^*(K_1)$ is infinite for $K_1 \geq K_1^{ss,m1}$.

Figure 8 depicts optimal curves in the (K_1, I_1) space for the interesting case of intermediate adoption costs (i.e. $\tilde{F} < F < \bar{F}$) where $\tilde{K_1}$ separates the two basins of attraction. For $K_1^{lb} < K_1 < \tilde{K_1}$, the firm decreases capacities down to K_1^{lb} and introduces the new product. In m_2, it continues decreasing capacities of K_1 down to K_1^{ss,m_2} while it builds up capacities for the new product up to K_2^{ss,m_2}.

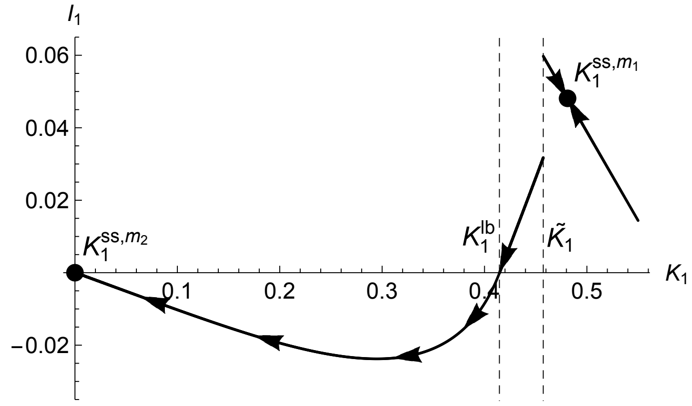


Figure 8: Capacity-investment dynamics for $F = 1.705$.

4.3.1 Effect of Horizontal and Vertical Differentiation

For decreasing degree of horizontal differentiation η, the products become more differentiated and thus the firm is expected to benefit from this. As both markets get more independent we expect that the firm is willing to introduce the new product earlier. Numerical experiments are in line with this intuition (see Figure 9). Analogously, for decreasing θ we get similar results in the opposite direction.

5 Welfare Implications

For analyzing welfare implications, note that the inverse demand functions stem from the following utility function of the consumers where M is the initial endowment:

$$CS(t) = u(K_1, K_2) = K_1 + (1+\theta)K_2 - \frac{1}{2}(K_1^2 + K_2^2) - \eta K_1 K_2 + (M - p_1 K_1 - p_2 K_2). \tag{37}$$

Welfare depends on the interpretation of adoption costs. If it is paid to the developer of the technology, then it is considered as a transfer and it is always profitable to introduce the new product immediately (given that investment in K_2 is positive in m_2, i.e. $K_1 < K_1^{F=0}$, cf. section 3). But if it is considered as 'real' costs, then it has to be taken into account. In that case, the social planner maximizes the

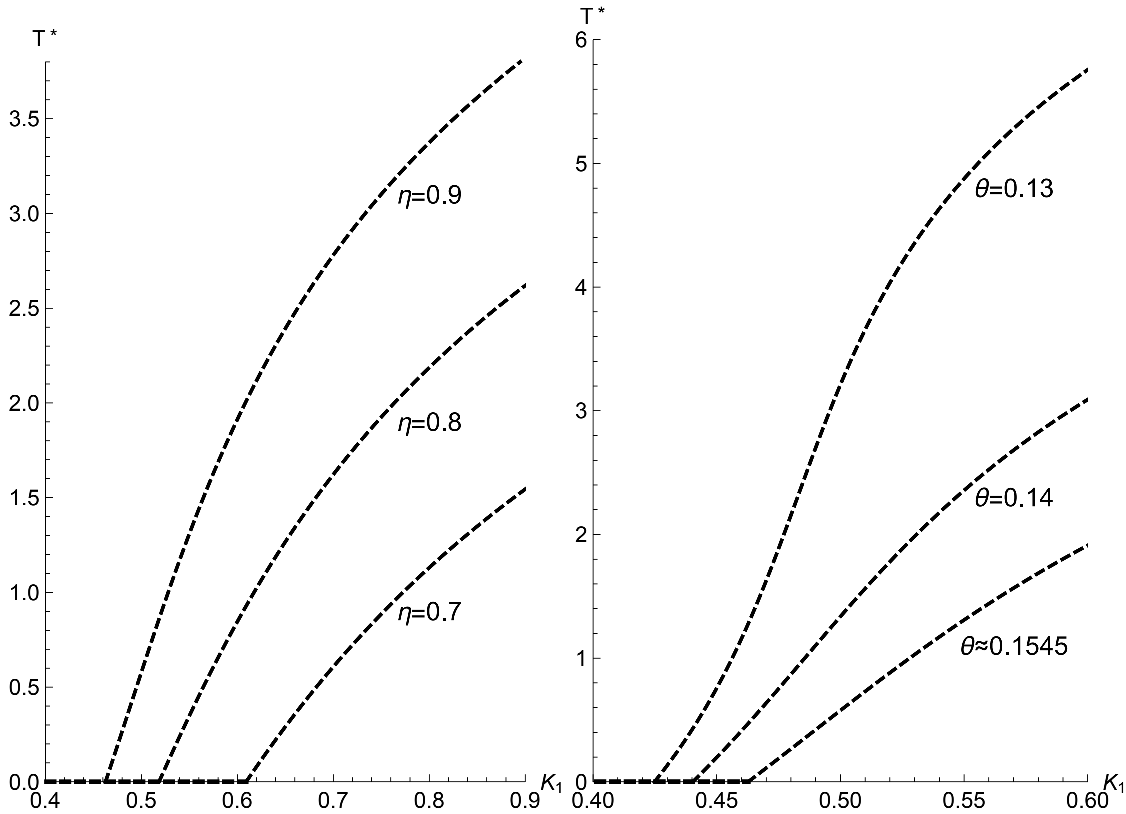


Figure 9: Optimal time of switching for different parameterizations of η and θ for the default parameter setting and $F = 1.4$.

difference of consumer surplus and costs of investment and adoption:

$$\max_{T,I_1(t),I_2(t)} J = \int_0^T e^{-rt}\big(u(K_1,0) - \frac{\gamma}{2}I_1^2\big)dt + \int_T^\infty e^{-rt}\big(u(K_1,K_2) - \frac{\gamma}{2}(I_1^2 + I_2^2)\big)dt - e^{-rT}F. \tag{38}$$

We expect that product introduction is favorable from a social point of view as in m_2, there is a new product of higher quality which affects the consumer only positively.

For the case of 'real' costs and a given initial capacity K_1^{ini}, the welfare difference of the situation of a profit maximizing firm and the social planner is given by

$$\Delta W(F; K_1^{ini}) = W(F; K_1^{ini}) - W^{sp}(F; K_1^{ini}), \tag{39}$$

where $W(F; K_1^{ini})$ and $W^{sp}(F; K_1^{ini})$ are the welfare functions of the profit maximizing firm and of a social planner, respectively.

We find that for the considered parameterization, from the perspective of a social planner, it is optimal to introduce immediately for a wider range of F and capacities. For instance, for $K_1^{ini} = K_1^{ss,m_1}$, delay occurs only for $F > 5.4277$ which is substantially higher than for the case of a profit maximizing firm ($F = 1.2910$).

The welfare loss for $K_1^{ini} = K_1^{ss,m_1}$ is depicted in Figure 10. The welfare loss

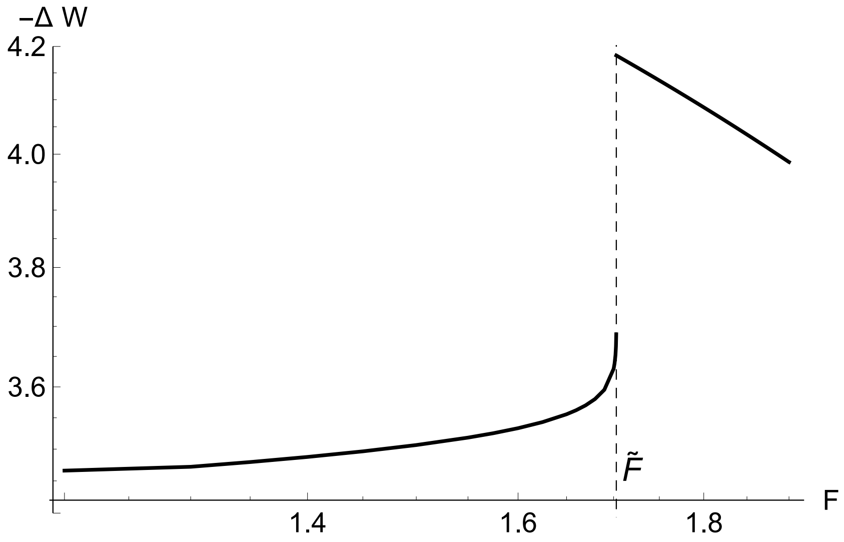


Figure 10: Welfare loss for $K_1 = K_1^{ss,m_1}$.

is initially constant as in both situations, immediate introduction is optimal (as long as $K_1^{ss,m_1} < K_1^{lb}$) but at some critical F (where $K_1^{lb} < K_1^{ss,m_1}$), the firm starts delaying the product introduction which increases the welfare loss. However, for $F \geq \tilde{F}$, the welfare loss decreases as the firm stays in m_1 where F does not have an effect whereas the welfare for the social planner decreases as costs of switching to m_2 increase.

We see that government intervention by subsidizing costs which come along with introducing new products would be welfare enhancing if it leads to faster introduction or introduction at all. However, for low adoption costs, subsidies would not have an impact.

6 Discussion of Results and Assumptions

From an economic perspective, delay was expected in order to discount adoption costs and to smooth cannibalization. Our analysis shows that the decrease of established capacities is accompanied by a larger marginal value for the new product in m_2, i.e. investing in the capacities of the new product is stronger than it would be with immediate introduction.

In our analysis, we abstract from competition. However, a monopoly could turn into a competing environment if entry is possible. Thus, if there is a threat of possible entrants, we expect that this would accelerate product introductions.

Another issue is that we do not consider the phase of development of the new product. For the interpretation that the new product is developed by the incumbent himself, it is clear that the firm is not going to engage in R&D activities if the product is not introduced eventually. In the case where the product is introduced with some delay, we expect that R&D efforts would be less in the development phase which would have a similar impact on the introduction time.

For the interpretation of external developers generating a new technology where adoption costs mainly consist of buying the patent for the new technology, an alternative option to adoption costs which has to be paid once when the product is introduced, would be to consider fees per unit which has to be paid to the owner of the patent. There, as long as the fee per unit is constant and less than θ, introduction would occur immediately since fees are paid continuously, so adoption costs are 'spread over time'.

We made the assumption that capacities are fully used, i.e. production equals sales. We believe that this assumption is of minor consequence to our results since in our model, there are no capacities for the new product in T and investment in capacities is accompanied with quadratic costs such that capacities are not build up as a 'lump-sum' but slowly while the capacity of the established product is reduced slowly. Moreover, in the case of delay, the incumbent starts reducing capacities even in m_1. A rigorous analysis of the full usage of capacity assumption yields that it is optimal to exploit full capacity if the following conditions hold:

$$2K_1 + \eta K_2 \leq 1, \tag{40}$$

$$\eta K_1 + 2K_2 \leq 1 + \theta. \tag{41}$$

Numerical experiments suggest that conditions (40) and (41) seem to be satisfied for reasonable values of K_1 ($\leq K_1^{ss,m_1}$)[21].

Furthermore, e.g., for decreasing demand, it is argued that in practice firms reduce prices in order to maintain production rather than reducing production due to contracts with employees and suppliers, even though such contracts are not modeled here (cf. Goyal and Netessine (2007)). However, counterexamples exist as well where firms have excess capacity, e.g., for deterring entry (see Chicu (2012)).

This analysis focuses on the effect of adoption costs. However, for some products, not adoption costs but differences in production costs may be the main reason for firms to abstain from product introduction, in particular if the old and new product's production costs differ a lot. Apple had developed a mouse in 1979 whose production costs were so high such that Apple abstained from further development of this mouse and hence from introducing it (cf. Hinloopen et al. (2013)).

7 Conclusion

Using a fully dynamic framework we identify different scenarios in which the firm's behavior depends crucially on the capacity of the established product and on the level of adoption costs. There is an interesting case where it is not optimal for the firm to introduce the new product immediately but to delay product introduction.

[21]In the case of no horizontal and vertical differentiation, i.e. $\eta = 1$ and $\theta = 0$, conditions (40) and (41) are satisfied if

$$K_1 \geq \frac{1}{3} \wedge K_2 \leq \frac{1}{3} \ , \tag{42}$$

or

$$K_1 \leq \frac{1}{3} \wedge K_2 \geq \frac{1}{3} \ . \tag{43}$$

For our default parameter setting with $F = 1.705$, (42) and (43) are satisfied. In the case of horizontal and vertical differentiation, (40) and (41) are weakened. For higher θ, the incumbent wants to build up capacities for the new product faster, but to decrease capacities of the established product faster as well. For lower η, as products are more differentiated and competition between the established and the new product is weakened, investment in the new product's and disinvestment of the established product's capacities are slower. Thus, in both cases, we expect that (40) and (41) are not affected much.

By delay in time, adoption costs are discounted while the firm prepares for product introduction by reducing capacities on the established market which increases the marginal value of the established and new products' capacities and hence reduces cannibalization. Moreover, the incumbent postpones investment in new capacity and hence benefits longer from high profits before product introduction. Noteworthy is the occurrence of Skiba points where the firm is indifferent in approaching different steady states which affects the number of products produced by the firm. We assume that firms cannot invest in capacities beforehand. Allowing for investment before introduction might have an effect on the time of introduction, in particular we expect that this would accelerate product introduction while we think that qualitative results would remain unaffected. Furthermore, we abstained from competition which would be the natural next step.

A Appendix

A.1

The canonical system is given by

$$\begin{aligned} \dot{K}_1 &= \frac{\lambda}{\gamma} - \delta K_1, \\ \dot{\lambda} &= (r+\delta)\lambda - (1-2K_1), \end{aligned} \tag{44}$$

and the isoclines are

$$\begin{aligned} \dot{K}_1 = 0 \quad &\Leftrightarrow \quad \lambda = \delta\gamma K_1, \\ \dot{\lambda} = 0 \quad &\Leftrightarrow \quad \lambda = \frac{1-2K_1}{r+\delta}. \end{aligned} \tag{45}$$

If the firm does not introduce the new product, i.e. for staying in m_1 infinitely, there is a unique steady state

$$K_1^{ss,m_1} = \frac{1}{\delta\gamma(r+\delta)+2}, \quad \lambda^{ss,m_1} = \frac{\delta\gamma}{\delta\gamma(r+\delta)+2} \ . \tag{46}$$

The steady state is a saddle point as the Jacobian is

$$\begin{pmatrix} -\delta & \frac{1}{\gamma} \\ 2 & r+\delta \end{pmatrix} \tag{47}$$

with

$$\det J = -\delta(r+\delta) - \frac{2}{\gamma} < 0. \tag{48}$$

The eigenvalues are given by

$$\mu_{1,2} = \frac{r}{2} \pm \sqrt{\left(\frac{r}{2}\right)^2 + \delta(r+\delta)}, \tag{49}$$

so eigenvalues have different sign and the steady state is indeed a saddle point.

A.2

Lemma 2. *Condition (16) holds for*

$$\left(K_1^*\right)_{1,2} = -\frac{d}{f} \pm \sqrt{\frac{2\gamma r F}{f^2}}. \tag{50}$$

Proof.

Consider the terminal condition[22] (16):

$$H(K_1^*, I_1^*, \lambda(T^*), T^*) = rS\big(K_1^*\big) \tag{51}$$

⇔

$$(1-K_1^*)K_1^* - \frac{\gamma}{2} I_1^{*2} + \lambda(T^*)(I_1^* - \delta K_1^*) = r(V^{m_2}(K_1^*) - F) \tag{52}$$

⇔

$$(1-K_1^*)K_1^* - \frac{\gamma}{2} I_1^{*2} + \frac{\partial V^{m_2}}{\partial K_1}(I_1^* - \delta K_1^*) = r(V^{m_2}(K_1^*) - F). \tag{53}$$

The HJB-equation in m_2 at T^* is given by[23]

$$(1-K_1^*)K_1^* - \frac{\gamma}{2}(I_1^{*2} + I_2^{*2}) + \frac{\partial V^{m_2}}{\partial K_1}(I_1^* - \delta K_1^*) + \frac{\partial V^{m_2}}{\partial K_2} I_2^* = rV^{m_2}(K_1^*). \tag{54}$$

For $I_2^* = \frac{V_{K_2}^{m_2}}{\gamma}$, we have:

$$(1-K_1^*)K_1^* - \frac{\gamma}{2} I_1^{*2} + \frac{\partial V^{m_2}}{\partial K_1}(I_1^* - \delta K_1^*) + \frac{1}{2\gamma}\Big(\frac{\partial V^{m_2}}{\partial K_2}\Big)^2 = rV^{m_2}(K_1^*). \tag{55}$$

[22]For convenience, we henceforth omit the dependence of state and control variables on T^*.

[23]Note that F is paid for switching to m_2 and does not occur in m_2 anymore.

Using (55) and (53) yields

$$rF = \frac{1}{2\gamma}\Big(\frac{\partial V^{m_2}}{\partial K_2}\Big)^2, \tag{56}$$

which under consideration of $K_2 = 0$ yields the two solutions

$$K_1^{lb} := -\frac{d}{f} - \sqrt{\frac{2\gamma r F}{f^2}}, \tag{57}$$

and

$$K_1^{ub} := -\frac{d}{f} + \sqrt{\frac{2\gamma r F}{f^2}}. \tag{58}$$

□

A.3

Proof of Lemma 1. Taking the derivative of K_1^{lb} with respect to F yields

$$\frac{\partial K_1^{lb}}{\partial F} = -\frac{2\gamma r}{2f^2\sqrt{\frac{2\gamma r F}{f^2}}} = -\sqrt{\frac{\gamma r}{2Ff^2}} < 0\ . \tag{59}$$

□

A.4

Lemma 3. *For $K_1 < K_1^{lb}$,*

$$H < rS, \tag{60}$$

holds and for $K_1^{lb} < K_1 < K_1^{ub}$,

$$H > rS, \tag{61}$$

holds.

Proof. By Lemma 2 in Appendix A.2, we know that for $F = 0$ the terminal condition of the Maximum Principle holds for $K_1^{F=0}$ and $H < rS$ for other values of capacity[24]. For $F > 0$, F occurs negatively on the right hand side of the terminal

[24]Cf. Appendix A.2. For $F = 0$, the square root in (50) vanishes and both solutions coincide. Moreover, note that for $F = 0$, the only extra term in (55) in comparison to (53) is $\frac{1}{2\gamma}\left(\frac{\partial V^{m_2}}{\partial K_2}\right)^2$ which is non-negative. Hence for all K_1, H is less or equal than rS (it is equal for $K_1^{lb}(= K_1^{ub})$ as $\frac{1}{2\gamma}\left(\frac{\partial V^{m_2}}{\partial K_2}\right)^2 = 0$).

condition and only there. Thus, there arises an interval whose bounds are given by (57) and (58) wherein $H > rS$. For K_1^{ini} outside the interval, the opposite holds. □

Lemma 4. *If $T^*(K_1)$ is finite for all K_1, then for all $K_1 \leq K_1^{lb}$, it is optimal to innovate immediately. For all $K_1^{lb} < K_1$, it is optimal to reduce capacities and to innovate when the capacity reaches K_1^{lb}, i.e. $T^*(K_1) > 0$.*

Proof of Lemma 4. Whenever $H > rS$, delaying the introduction of the new product marginally and introducing it afterwards is better than introducing it immediately. Due to Lemma 3, $H > rS$ holds in the interval (K_1^{lb}, K_1^{ub}). Moreover, note that introducing is not optimal for $K_1 > K_1^{F=0}$. For $K_1^{lb} < K_1 \leq K_1^{F=0}$, along the path to K_1^{lb}, delaying marginally and introducing dominates the option of introducing immediately. Hence, introducing at K_1^{lb} is indeed optimal. For $K_1 < K_1^{lb}$, again due to Lemma 3, $H < rS$ holds. Investing in K_1 such that K_1 increases and hits the switching candidate K_1^{lb} and introducing then is worse than having introduced earlier since along the path, at every value of capacity, introducing immediately would have been better compared to delaying marginally and introducing afterwards. Hence, for $K_1 \leq K_1^{lb}$, introducing immediately is optimal. □

A.5

Lemma 5. $\exists!\ \tilde{F} > 0$ *such that* $\forall F \geq \tilde{F}$, $\exists\ K_1$ *with* $T^*(K_1) = \infty$, *i.e.* $V(K_1) = V^{m_1}(K_1)$ *and* $\forall F < \tilde{F}, \nexists\ K_1$ *with* $T^*(K_1) = \infty$.

Proof. The value function of m_1 without the option to switch to m_2 is independent of F whereas the value function of the control problem with introduction option is decreasing in F due to the decreasing salvage value. Thus, there is some $\tilde{F}$ where the value function of the control problem with introduction option hits the value function of m_1 for the first time which is greater than 0 since for $F = 0$, switching is costless and in m_2, there is the option of producing the new product which has a higher quality $(\theta > 0)$[25]. □

[25]Even without vertical differentiation, introducing the new product is beneficial as the market

This result leads to the following corollary.

Corollary 1. *For $F < \tilde{F}$, $T^*(K_1)$ is finite for all initial capacities and Lemma 4 applies.*

Proof. Follows directly from Lemma 5. □

Denote by $\tilde{K_1}$ the lowest value of initial capacity where an infinite solution T^* exists for $\mathcal{P}(\tilde{K_1})$:

$$\tilde{K_1} = \min\{K_1 \mid T^*(K_1) = \infty\}. \tag{62}$$

Note that $\tilde{K_1}$ exists for $F \geq \tilde{F}$. The following lemma and proposition characterize the situation at $\tilde{F}$.

Lemma 6. *At $F = \tilde{F}$,*

$$K_1^{lb} \leq \tilde{K_1} \tag{63}$$

holds.

Proof. Let $F = \tilde{F}$. Assume $\tilde{K_1} < K_1^{lb}$. Then, for $\tilde{K_1}$, $H < rS$, which yields that the unique solution is to switch to m_2 which contradicts $F = \tilde{F}$. □

Proposition 2. *At $F = \tilde{F}$,*

$$\tilde{K_1} = K_1^{ss,m_1}, \tag{64}$$

and the free end-time problem $\mathcal{P}(\tilde{K_1})$ has a unique solution with $T^* = \infty$.

We first state the following lemma which is necessary for the proof of Proposition 2.

Lemma 7. *The dynamics at the terminal pair $\left(K_1^{lb}, \lambda(T)\right)$ are not $\dot{K_1} > 0$ and $\dot{\lambda} > 0$ simultaneously.*

Proof. The terminal pair is determined by $H = rS$ and $\lambda(T) = S_{K_1}$. The line $\lambda(T) = S_{K_1} = b + cK_1$ has a positive ordinate ($b > 0$) as K_1's marginal value is positive if there are no capacities installed. One might think that this line could

is expanded and the firm is able to split the total quantity among the two products which yields a higher price (cf. Dawid et al. (2015)).

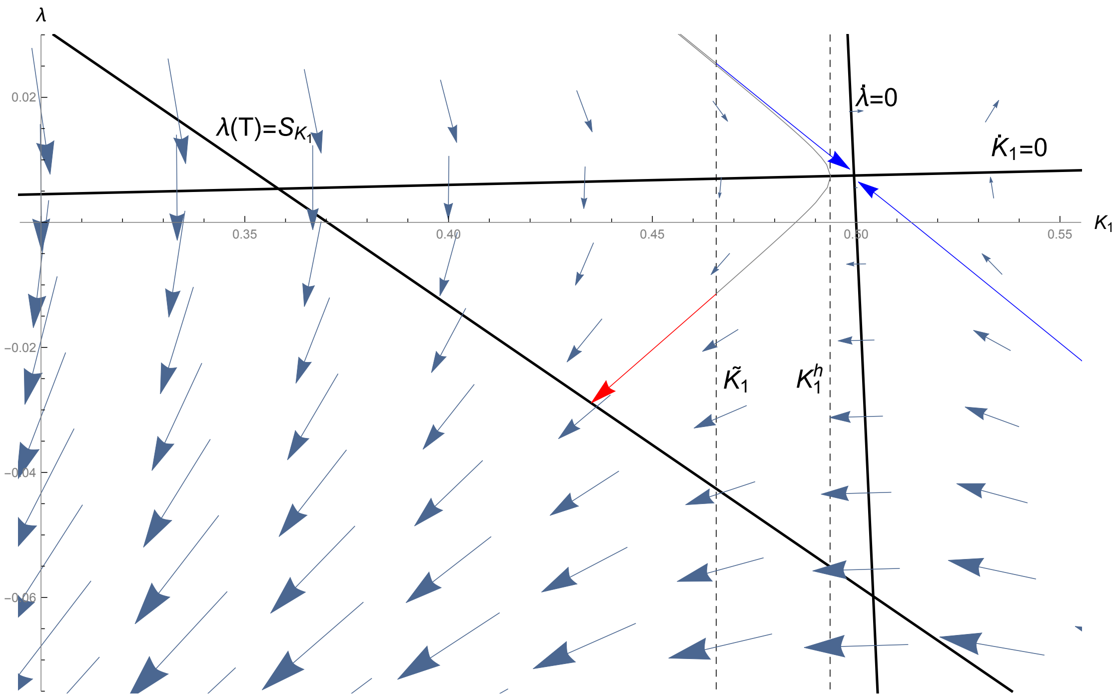


Figure 11: Vector plot. Parameters: $r = 0.04, \delta = 0.1, \eta = 0.9, \theta = 0.1, \gamma = 0.15, F = 1.275(> \tilde{F})$.

pass through the area to the right-upper of the intersection point of $\dot{K}_1 = 0$ and $\dot{\lambda} = 0$ where $\dot{K}_1 > 0$ and $\dot{\lambda} > 0$ hold. This would yield different dynamics than studied so far. However, one can easily show that for terminal pairs in that area, there is no candidate for an optimal solution with $0 < T^* < \infty$. In particular, for $K_1^{ini} > K_1^{lb}$, there are either no candidate paths or only non-monotone paths arriving at the terminal pair which cannot be optimal[26]. Converging to the steady state of m_1 along the stable manifold is not optimal as well as time consistency is violated since for $K_1 < K_1^{lb}$, $H < rS$ holds. Thus, there are no optimal paths for $K_1^{ini} > K_1^{lb}$ which yields a contradiction and proves that this situation cannot occur.

□

Proof of Proposition 2. As the steady state of m_1 is a saddle-point, there is a

[26]Non-monotone paths imply a set of Skiba points which generates fluctuating paths for $T^* = \infty$, which contradicts to the uniqueness property of the steady state of the infinite horizon problem.

stable and unstable manifold. If T^* is finite but not zero, then the switching pair $\big(K_1(T), \lambda(T)\big)$ in the (K_1, λ) space is derived from the condition $H = rS$ and the transversality condition $\lambda(T) = S_{K_1}$. As F increases and K_1^{lb} decreases, there is an F, where $\big(K_1^{lb}, \lambda(T)\big)$ is on the unstable manifold with $\dot{K}_1 < 0$ and $\dot{\lambda} < 0$[27]. Denote that F by F^{uns}. For arriving at that pair, the initial pair has to be on the unstable manifold. Thus, for all $K_1 \geq K_1^{ss,m_1}$, there is no optimal path which leads to $(K_1^{lb}, \lambda(T))$, i.e. for all $K_1 \geq K_1^{ss,m_1}$, $T^*(K_1) = \infty$.

Next, we prove that $\tilde{F} = F^{uns}$. Obviously, $\tilde{F} \leq F^{uns}$ [28]. Assume $\tilde{F} < F^{uns}$. Then, by Lemma 1, at $\tilde{F}$, the terminal pair is to the right of the unstable manifold. Denote for all possible terminal values $K_1(T)$ the value of the path which leads to the terminal pair by $V^{term}(K_1(t), K_1(T), F)$ which in this case exists for all $K_1 \geq K_1(T)$ and for all $F < F^{uns}$ and is continuous in F.

In order to avoid confusion, for an F, we denote the corresponding K_1^{lb} by $K_1^{lb}(F)$. For $K_1^{ini} > \tilde{K}_1$,

$$V^{term}(K_1^{ini}, K_1^{lb}(\tilde{F}), \tilde{F}) < V^{m_1}(K_1^{ini}), \tag{65}$$

holds[29]. Hence, $\exists\ F^l < \tilde{F}$ with

$$V^{term}(K_1^{ini}, K_1^{lb}(F^l), F^l) = V^{m_1}(K_1^{ini}), \tag{66}$$

which contradicts the minimality of $\tilde{F}$. Hence, the assumption $\tilde{F} < F^{uns}$ was wrong and $\tilde{F} = F^{uns}$ holds.

Now, we prove that $\tilde{K}_1$ is not less than K_1^{ss,m_1} again by contradiction. Assume that $\tilde{K}_1 < K_1^{ss,m_1}$. Then, consider K_1^{int} for which $\tilde{K}_1 < K_1^{int} < K_1^{ss,m_1}$ holds. For

[27]As shown in Lemma 7, the dynamics at the terminal pair are not $\dot{K}_1 > 0$ and $\dot{\lambda} > 0$ simultaneously.

[28]Note that for F^{uns} infinite solutions for T exist. As $\tilde{F}$ is the minimal value of adoption costs for which infinite solutions exist, $\tilde{F} \leq F^{uns}$ holds.

[29]It cannot be $V^{term}(K_1^{ini}, K_1^{lb}(F), F) = V^{m_1}(K_1^{ini})$ since for $K_1^{ini} \geq \tilde{K}_1$, trajectories of the finite and infinite solution move in the same direction (as due to Lemma 6, $K_1^{lb} \leq \tilde{K}_1$) and according to Proposition 1 in Caulkins et al. (2015), in that case, the trajectories have to coincide for all $t \in [0, T^*(K_1^{ini})]$ which is apparently not true. Moreover, $V^{term}(K_1^{ini}, K_1^{lb}(F), F) > V^{m_1}(K_1^{ini})$ cannot hold either since this leads to another solution for the problem without introduction option via moving to $\tilde{K}_1$ along the path corresponding to the finite solution of T and switching at $\tilde{K}_1$ to the solution of the problem without introduction option.

$F = \tilde{F}$, we have[30]

$$V^{term}(K_1^{int}, K_1^{lb}(\tilde{F}), \tilde{F}) < V^{m_1}(K_1^{int}). \tag{67}$$

Again, by continuity of V^{term} in F, there exists an $F^l < \tilde{F}$ with

$$V^{term}(K_1^{int}, K_1^{lb}(F^l), F^l) = V^{m_1}(K_1^{int}), \tag{68}$$

which contradicts the minimality of $\tilde{F}$. Thus, $\tilde{K}_1 = K_1^{ss,m_1}$ and it is a threshold point[31] where the firm is not indifferent.

□

A.6

Corollary 2. *At $\tilde{F}$, for $K_1 < \tilde{K}_1$,*

$$T^*(K_1) < \infty, \tag{69}$$

holds and for $\tilde{K}_1 \leq K_1$,

$$T^*(K_1) = \infty, \tag{70}$$

holds.

Proof. Due to the definition of $\tilde{K}_1$, for $K_1 < \tilde{K}_1$ only finite solutions are optimal. According to the proof of Proposition 2, for $\tilde{K}_1 \leq K_1$, only infinite solutions are optimal. □

For characterizing the evolution of $\tilde{K}_1$, we denote by $\bar{F}$ the value of adoption costs for which

$$V^{m_1}(K^{lb}) = V^{m_2}(K^{lb}) - F \; (= S(K^{lb})) \tag{71}$$

holds, i.e. where the firm is indifferent between introducing immediately and delaying infinitely at K_1^{lb}.

[30]Note that in this case, V^{term} exists for $K_1 < \tilde{K}_1$. Moreover, as this problem is time invariant and trajectories of the finite and infinite solution move in opposite directions and due to the monotonicity of the trajectory of the infinite solution (see Hartl (1987)), the trajectory of the finite solution is monotone as well and there cannot be an overlap region, i.e. there is no interval of Skiba points (cf. Caulkins et al. (2015)). Thus, at $\tilde{F}$ for K_1^{int}, the infinite solution is the unique optimal solution.

[31]Here, a threshold point is characterized by having finite and infinite solutions for T in every neighborhood (cf. Caulkins et al. (2015)).

Proposition 3. *$\tilde{K_1}$ is decreasing in F and for all $\tilde{F} < F < \bar{F}$, the free end-time problem $\mathcal{P}(\tilde{K_1})$ has two different solutions with optimal terminal times $0 < T^f < \infty$ and $T^\infty = \infty$, i.e. $\tilde{K_1}$ is a Skiba point where the firm is indifferent between introducing the product after some delay and not at all.*

Proof of Proposition 3. As K_1^{lb} decreases with F, for $\tilde{F} < F < \bar{F}$, the terminal pair $\big(K_1(T), \lambda(T)\big) = \big(K_1^{lb}, \lambda(T)\big)$ is to the left of the unstable manifold (cf. proof of Proposition 2 in Appendix A.5). There, the dynamics are given by $\dot{K}_1 < 0$ and $\dot{\lambda} < 0$. Starting at the terminal pair $\big(K_1^{lb}, \lambda(T)\big)$ and moving backwards along the arc leading to it, i.e. considering V^{term} introduced in Appendix A.5 (cf. Figure 11), we can identify candidates for the optimal starting point for different K_1^{ini}. This arc hits the $\dot{K}_1 = 0$ line at some K_1^h. This is the highest K_1 for which a finite candidate T exists since following the arc further gives further candidates for $K_1^{lb} \leq K_1 < K_1^h$ as there is $\dot{K}_1 > 0$, which implies non-monotone paths for K_1 which cannot be optimal (cf. Appendix A.5). Hence, V^{term} is well defined. For any $K_1 < K_1^{ss,m_1}$, it is also possible to converge to the steady state of m_1 by following the stable arc of the steady state. Comparing values of both candidates by taking the upper curve of the value functions corresponding to both options we obtain the value function and the optimal strategies of the control problem with introduction option. Hence, there is an indifference point $0 < \tilde{K_1} \leq K_1^h$ where the firm is indifferent moving to the steady state along the stable manifold and moving to K_1^{lb}. Thus, $\tilde{K_1}$ is a Skiba point. As F increases, K_1^{lb} and K_1^h decreases. Next, we prove that $\tilde{K_1}$ decreases as well by contradiction. For $F^a, F^b \in (\tilde{F}, \bar{F})$, with $F^a < F^b$, denote the corresponding indifference points by $\tilde{K_1}^a$ and $\tilde{K_1}^b$ and assume that $\tilde{K_1}^a \leq \tilde{K_1}^b$, i.e. $\tilde{K_1}$ is nondecreasing in F. Then,

$$V^{m_1}(\tilde{K_1}^b) = V^{term}(\tilde{K_1}^b, K_1^{lb}(F^b), F^b) < V^{term}(\tilde{K_1}^b, K_1^{lb}(F^a), F^a) \leq V^{m_1}(\tilde{K_1}^b) \tag{72}$$

which yields a contradiction[32] . Hence, $\tilde{K_1}$ is decreasing in F. □

Now, we show that at $\bar{F}$ the waiting region vanishes and only immediate or

[32]The last inequality is due to the following: $\tilde{K_1}^a \leq \tilde{K_1}^b$ and for $K_1 \geq \tilde{K_1}^a$, infinite solutions are optimal.

infinite solutions for T remain.

Corollary 3. *For $\bar{F} \leq F < \bar{\bar{F}}$, there exists a $\tilde{K_1} > 0$ such that for all $K_1 < \tilde{K_1}$ the firm introduces the new product immediately whereas for all $K_1 > \tilde{K_1}$ the firm never introduces the new product. At $\tilde{K_1}$, the incumbent is indifferent, in particular the free end-time problem $\mathcal{P}(\tilde{K_1})$ has two different solutions with $0 = T^f < T^\infty = \infty$.*

Moreover, at $\bar{F}$, $\tilde{K_1} = K_1^{lb}$.

Proof. By definition of $\bar{F}$, the firm is indifferent between immediate and infinite product introduction. By Proposition 3, $\tilde{K_1}$ is decreasing and hits K_1^{lb} at $\bar{F}$ where solutions with $0 < T < \infty$ vanish. □

A.7

Quarterly data on worldwide HDD sales beginning in Q1 in 2015 and ending in Q4 in 2017 is given by

$$(125, 111, 118.7, 115.1, 100.5, 98.7, 113.8, 111.5, 98.8, 96.4, 104.1, 104.8), \tag{73}$$

while SSD sales in the same period are given by

$$(23.19, 23.86, 26.22, 29.53, 30.78, 33.69, 38.25, 45.03, 39.78, 42.09, 40, 42.49) \tag{74}$$

The corresponding value function in mode m_2 is given by

$$V^{m_2}(K_1, K_2) = 7.57695 + 0.946976K_1 - 1.14284K_1^2 + 1.58041K_2 - 1.70404K_1K_2 - 1.14284K_2^2. \tag{75}$$

References

Adaku, E., Amoatey, C. T., Nornyibey, I., Famiyeh, S., and Asante-Darko, D. (2018). Delays in new product introduction: Experiences of a food processing company in a developing economy. *Journal of Manufacturing Technology Management*, 29(5):811–828.

Avlonitis, G. J. (1983). The product-elimination decision and strategies. *Industrial Marketing Management*, 12:31–43.

Billington, C., Lee, H., and Tang, C. (1998). Successful strategies for product rollovers. *Sloan Management Review*, 39(3).

Caulkins, J. P., Feichtinger, G., Grass, D., Hartl, R. F., Kort, P. M., and Seidl, A. (2015). Skiba points in free end-time problems. *Journal of Economic Dynamics and Control*, 51:404–419.

Chandy, R. K. and Tellis, G. J. (2000). The Incumbent's Curse? Incumbency, Size, and Radical Product Innovation. *Journal of Marketing*, 64(3):1–17.

Chicu, M. (2012). Dynamic Investment and Deterrence in the U.S. Cement Industry. *Job Market Paper*.

Dawid, H., Keoula, M. Y., Kopel, M., and Kort, P. M. (2015). Product innovation incentives by an incumbent firm: A dynamic analysis. *Journal of Economic Behavior & Organization*, 117:411–438.

Dixit, A. K. and Pindyck, R. S. (1994). Investment under uncertainty. *Princeton: Princeton University Press*.

Doraszelski, U. (2004). Innovations, improvements, and the optimal adoption of new technologies. *Journal of Economic Dynamics and Control*, 28(7):1461–1480.

Farzin, Y. H., Huisman, K. J. M., and Kort, P. M. (1998). Optimal timing of technology adoption. *Journal of Economic Dynamics and Control*, 22(5):779–799.

Fudenberg, D. and Tirole, J. (1985). Preemptive and Rent Equalization in the Adoption of New Technology. *The Review of Economics Studies*, 52(3):383–401.

Goyal, M. and Netessine, S. (2007). Strategic Technology Choice and Capacity Investment Under Demand Uncertainty. *Management Science*, 53(2):192–207.

Grass, D., Caulkins, J., Feichtinger, G., Tragler, G., and Behrens, D. (2008). *Optimal control of nonlinear processes: With applications in drugs, corruption, and terror*. Springer.

Hartl, R. F. (1987). A Simple Proof of the Monotonicity of the State Trajectories in Autonomous Control Problems *. *Journal of Economic Theory*, 215:211–215.

Hendricks, K. B. and Singhal, V. R. (1997). Delays in New Product Introductions and the Market Value of the Firm: The Consequences of Being Late to the Market. *Management Science*, 43(4):422–436.

Hinloopen, J., Smrkolj, G., and Wagener, F. (2013). From mind to market: A global, dynamic analysis of R&D. *Journal of Economic Dynamics and Control*, 37(12):2729–2754.

Hinloopen, J., Smrkolj, G., and Wagener, F. (2017). Research and Development Cooperatives and Market Collusion: A Global Dynamic Approach. *Journal of Optimization Theory and Applications*, 174(2):567–612.

Hoppe, H. C. (2002). The timing of new technology adoption: Theoretical models and empirical evidence. *Manchester School*, 70(1):56–76.

Huisman, K. J. M. and Kort, P. M. (2015). Strategic capacity investment under uncertainty. *RAND Journal of Economics*, 46(2):376–408.

Igami, M. (2017). Estimating the innovator's dilemma: Structural analysis of creative destruction in the hard disk drive industry, 1981–1998. *Journal of Political Economy*, 125(3):798–847.

Kamien, M. I. and Schwartz, N. L. (1972). Some Economic Consequences of Anticipating Technical Advance. *Western Economic Journal*, 10(2):123–138.

Katana, T., Eriksson, A., Hilletofth, P., and Eriksson, D. (2017). Decision model for product rollover in manufacturing operations. *Production Planning and Control*, 28(15):1264–1277.

Kiseleva, T. and Wagener, F. O. (2010). Bifurcations of optimal vector fields in the shallow lake model. *Journal of Economic Dynamics and Control*, 34(5):825–843.

Koca, E., Souza, G. C., and Druehl, C. T. (2010). Managing product rollovers. *Decision Sciences*, 41(2):403–423.

Kreps, D. M. and Scheinkman, J. a. (1983). Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes. *The Bell Journal of Economics*, 14(2):326–337.

Li, H., Graves, S. C., and Rosenfield, D. B. (2010). Optimal planning quantities for product transition. *Production and Operations Management*, 19(2):142–155.

Long, N. V., Prieur, F., Tidball, M., and Puzon, K. (2017). Piecewise closed-loop equilibria in differential games with regime switching strategies. *Journal of Economic Dynamics and Control*, 76:264–284.

Reinganum, J. F. (1981). On the Diffusion of New Technology: A Game Theoretic Approach. *Review of Economic Studies*, 48(3):395–406.

Wagener, F. O. O. (2003). Skiba points and heteroclinic bifurcations, with applications to the shallow lake system. *Journal of Economic Dynamics and Control*, 27(9):1533–1561.

Wang, Q. H. and Hui, K. L. (2012). Delayed product introduction. *Decision Support Systems*, 53(4):870–880.