OPTIMAL GRADIENT ESTIMATES OF HEAT KERNELS OF STABLE-LIKE OPERATORS

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ABSTRACT. In this note we show the optimal gradient estimate for heat kernels of stable-like operators by providing a counterexample.

1. INTRODUCTION

Let $\kappa : \mathbb{R}^d \to [0, \infty)$ be a bounded measurable function. For $\alpha \in (0, 2)$, consider the following nonlocal α-stable-like operator

$$\mathscr{L}_\kappa^{(\alpha)} f(x) := \int_{\mathbb{R}^d} (f(x+y) - f(x) - y^{(\alpha)} \cdot \nabla f(x)) \frac{\kappa(y)}{|y|^{d+\alpha}} \mathrm{d}y,$$

where

$$y^{(\alpha)} := 1_{\alpha\in(1,2)} y + 1_{\alpha=1} 1_{|y|\leqslant 1} y.$$

It is well known that if for some $K_0 \geqslant 1$,

$$K_0^{-1} \leqslant \kappa(y) \leqslant K_0, \ \ 1_{\alpha=1} \int_{r<|y|<R} \kappa(y)\mathrm{d}y = 0, \ 0 < r < R < \infty, \tag{1.1}$$

then there is a smooth fundamental solution $p_\kappa^{(\alpha)}(t, x)$ to the operator $\mathscr{L}_\kappa^{(\alpha)}$ satisfying (see [3, 4, 5])

$$\partial_t p_\kappa^{(\alpha)}(t, x) = \mathscr{L}_\kappa^{(\alpha)} p_\kappa^{(\alpha)}(t, \cdot)(x), \ t > 0, x \in \mathbb{R}^d.$$

Moreover, $p_\kappa^{(\alpha)}(t, x)$ enjoys the following two-sided estimates: for some $K_1 = K_1(\alpha, d, K_0) \geqslant 1$,

$$K_1^{-1} t(t^{1/\alpha} + |x|)^{-d-\alpha} \leqslant p_\kappa^{(\alpha)}(t, x) \leqslant K_1 t(t^{1/\alpha} + |x|)^{-d-\alpha},$$

and the gradient estimate: for some $K_2 = K_2(\alpha, d, K_0) \geqslant 1$,

$$|\nabla p_\kappa^{(\alpha)}(t, x)| \leqslant K_2 t^{1-1/\alpha} (t^{1/\alpha} + |x|)^{-d-\alpha}. \tag{1.2}$$

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The above estimates can be found in [4, 5]. Notice that for $\lambda > 0$, if we let $\kappa_\lambda(y) := \kappa(\lambda^{1/\alpha} y)$, then $p^{(\alpha)}_\kappa$ has the following scaling property:

$$p^{(\alpha)}_\kappa(\lambda t, \lambda^{1/\alpha} x) = \lambda^{-d/\alpha} p^{(\alpha)}_{\kappa_\lambda}(t, x).$$

Moreover, when $\kappa(y) \equiv 1$, it is well known that

$$\mathscr{L}^{(\alpha)}_1 = c_{\alpha,d}\, \Delta^{\alpha/2},$$

where $c_{\alpha,d} > 0$ and $\Delta^{\alpha/2} := -(-\Delta)^{\alpha/2}$ is the usual fractional Laplacian. In this case, it is also well known that the sharp gradient estimate takes the following form: for some $K_3 = K_3(\alpha, d) \geqslant 1$,

$$|\nabla_x p^{(\alpha)}_1(t, x)| \leqslant K_3 t (t^{1/\alpha} + |x|)^{-d-\alpha-1}. \tag{1.3}$$

Indeed, let $S^{(\alpha)}_t$ be the $\alpha/2$-subordinator and $\phi(t, x) = (2\pi t)^{-d/2} \mathrm{e}^{-|x|^2/(2t)}$ be the Gaussian heat kernel. By the subordination we have

$$p^{(\alpha)}_1(t, x) = \int_0^\infty \phi(s, x)\, \mathbb{P} \circ (S^{(\alpha)}_t)^{-1}(\mathrm{d}s). \tag{1.4}$$

Since $\mathbb{P} \circ (S^{(\alpha)}_t)^{-1}(\mathrm{d}s) \leqslant Cts^{-1-\alpha/2}\mathrm{e}^{-ts^{-\alpha/2}}\mathrm{d}s$, by elementary calculations, it follows that (see [1])

$$|\nabla_x p^{(\alpha)}_1(t, x)| \leqslant Ct|x| \int_0^\infty s^{-2-(d+\alpha)/2} \mathrm{e}^{-ts^{-\alpha/2}-|x|^2/(2s)} \mathrm{d}s \leqslant K_3 t (t^{1/\alpha} + |x|)^{-d-\alpha-1}.$$

Notice that the right hand side of (1.3) is smaller than the one in (1.2) when x goes to ∞. We mention that for a class of unimodal Lévy processes, Kulczycki and Ryznar [7] obtained the optimal gradient estimate (1.3). Moreover, gradient estimate (1.3) plays an important role in [6, Proposition 3.2] and [8, Theorem 1.1].

Here a natural question is that for general stable-like operator $\mathscr{L}^{(\alpha)}_\kappa$, is it possible to show the same gradient estimate (1.3)? We have the following negative answer. Thus (1.2) is optimal.

Theorem 1.1. *For $d = 1$, there is an even function $\kappa : \mathbb{R} \to [1, 2]$ such that*

$$\limsup_{|x|\to\infty} |x|^{1+\alpha} |\nabla_x p^{(\alpha)}_\kappa(1, x)| > 0.$$

Remark 1.2. Although we construct a symmetric example, it is in fact also easier to construct non-symmetric examples from the following proofs.

2. Proof of Theorem 1.1

Below we assume $d = 1$. For simplicity, we drop the superscript α in $p^{(\alpha)}_\kappa$ and write

$$p_\kappa(x) = p^{(\alpha)}_\kappa(1, x).$$

Suppose that κ satisfies (1.1) and $f : \mathbb{R} \to [0, 1]$ is a measurable function so that

$$\lambda := \int_{\mathbb{R}} f(x)|x|^{-1-\alpha}\mathrm{d}x < \infty.$$

Let N_t^λ be a Poisson process with intensity λ and $\xi_1, \cdots \xi_n, \cdots$ a sequence of i.i.d. random variables with common distribution density $q(x) := f(x)|x|^{-1-\alpha}/\lambda$, which are independent of N^λ. Let $\xi_0 := 0$ and

$$Y_t^f := \xi_0 + \xi_1 + \cdots + \xi_{N_t^\lambda}.$$

Then Y_t^f is a compound Poisson process with Lévy exponent

$$\int_{\mathbb{R}} (\mathrm{e}^{i\xi y} - 1) f(y)|y|^{-1-\alpha} \mathrm{d}y.$$

Let Z_t^κ be an independent Lévy process with Lévy measure $\kappa(y)|y|^{-1-\alpha}\mathrm{d}y$. The sum $Z_t^\kappa + Y_t^f$ is still a Lévy process with Lévy measure $(\kappa(y)+f(y))|y|^{-1-\alpha}\mathrm{d}y$. In particular, we have

$$p_{\kappa+f}(x) = \mathbb{E}p_\kappa(x - Y_1^f). \tag{2.1}$$

For the function f and real numbers $a, \varepsilon > 0$, we denote

$$f_a^\varepsilon(x) := f\Big(\frac{a+x}{\varepsilon}\Big) + f\Big(\frac{a-x}{\varepsilon}\Big).$$

Lemma 2.1. *Suppose that*

$$\mathrm{supp}(f) \subset [-1, 1], \quad \int_{\mathbb{R}} f(x)\,\mathrm{d}x = 1,$$

and for some $z_0 \in (-1, 0)$, $\varepsilon, \delta \in (0, 1)$ and $\gamma, A \geqslant 1$,

$$\inf_{x \in [z_0 - \varepsilon, z_0 + \varepsilon]} p'_\kappa(x) \geqslant \delta, \quad \|p'_\kappa\|_\infty \leqslant \gamma, \quad |p'_\kappa(x)| \leqslant \gamma|x|^{-2-\alpha} \text{ for } x \geqslant A. \tag{2.2}$$

Then there are constants $C = C(\alpha, \delta, \gamma) > 0$ and $A_0 = A_0(\delta, \gamma) \geqslant A + 1$ such that for all $a \geqslant A_0$,

$$p'_{\kappa + f_a^\varepsilon}(z_0 + a) \geqslant C a^{-1-\alpha}. \tag{2.3}$$

Proof. For $a > 1$, let

$$\lambda := \int_{\mathbb{R}} f_a^\varepsilon(x)|x|^{-1-\alpha}\mathrm{d}x = 2\int_{\mathbb{R}} f\Big(\frac{a+x}{\varepsilon}\Big)|x|^{-1-\alpha}\mathrm{d}x = 2\int_{\mathbb{R}} f\Big(\frac{a-x}{\varepsilon}\Big)|x|^{-1-\alpha}\mathrm{d}x.$$

By (2.1) and the definition of $Y_1^{f_a^\varepsilon}$, we have

$$\begin{aligned} p'_{\kappa + f_a^\varepsilon}(z_0 + a) &= p'_\kappa(z_0 + a)\mathbb{P}(N_1^\lambda = 0) + \mathbb{E}p'_\kappa(z_0 + a - \xi_1)\mathbb{P}(N_1^\lambda = 1) \\ &\quad + \mathbb{E}p'_\kappa(z_0 + a - Y_1^{f_a^\varepsilon} + \xi_1)\mathbb{P}(N_1^\lambda \geqslant 2) \\ &=: J_0 + J_1 + J_2. \end{aligned}$$

For J_0, by (2.2) we have

$$|J_0| \leqslant |p'_\kappa(z_0 + a)|\mathrm{e}^{-\lambda} \leqslant \gamma|z_0 + a|^{-2-\alpha}, \ a \geqslant A + 1.$$

For J_1, since $\mathrm{supp}(f) \subset [-1,1]$, we have

$$\begin{aligned} J_1 &= \mathrm{e}^{-\lambda} \int_{\mathbb{R}} p'_\kappa(z_0+a-y)\Big(f\Big(\frac{a+y}{\varepsilon}\Big)+f\Big(\frac{a-y}{\varepsilon}\Big)\Big)|y|^{-1-\alpha}\mathrm{d}y \\ &\geqslant \mathrm{e}^{-\lambda} \int_{\mathbb{R}} \left(\delta\cdot f\Big(\frac{a-y}{\varepsilon}\Big)-\gamma(2a-\varepsilon-|z_0|)^{-2-\alpha} f\Big(\frac{a+y}{\varepsilon}\Big)\right)|y|^{-1-\alpha}\mathrm{d}y \\ &= \frac{\lambda}{2}\,\mathrm{e}^{-\lambda}\Big(\delta-\gamma(2a-\varepsilon-|z_0|)^{-2-\alpha}\Big). \end{aligned}$$

For J_2, we have

$$|J_2| \leqslant \|p'_\kappa\|_\infty(1-\mathrm{e}^{-\lambda}-\lambda\mathrm{e}^{-\lambda}) \leqslant 2\gamma\lambda^2,\ \lambda\in(0,1).$$

Combining the above calculations, and thanks to

$$2\varepsilon(a+\varepsilon)^{-1-\alpha} \leqslant \lambda \leqslant 2\varepsilon(a-\varepsilon)^{-1-\alpha},$$

we obtain that for $a \geqslant (2\gamma/\delta)^{2+\alpha}\vee(A+1)$,

$$p'_{\kappa+f_a^\varepsilon}(z_0+a) \geqslant \lambda\mathrm{e}^{-\lambda}\delta/4-\gamma|z_0+a|^{-2-\alpha}-2\gamma\lambda^2 \geqslant Ca^{-1-\alpha}.$$

Thus we complete the proof. □

Recalling $p_1(x) = p_1^{(\alpha)}(1,x)$ given by (1.4), we may fix $z_0 \in (-1,0)$ and $\varepsilon \in (0,|z_0|/2)$ in the following so that

$$\delta := \inf_{x\in[z_0-\varepsilon,z_0+\varepsilon]} p'_1(x) \in (0,1). \tag{2.4}$$

Indeed, by (1.4), for $x\in[z_0-\varepsilon,z_0+\varepsilon]$, we have

$$\begin{aligned} p'_1(x) &= (2\pi)^{-d/2}\int_0^\infty s^{-d/2-1}(-x)\mathrm{e}^{-|x|^2/(2s)}\mathbb{P}\circ(S_1^{(\alpha)})^{-1}(\mathrm{d}s) \\ &\geqslant \frac{|z_0|-\varepsilon}{(2\pi)^{d/2}}\int_0^\infty s^{-d/2-1}\mathrm{e}^{-(|z_0|+\varepsilon)^2/(2s)}\mathbb{P}\circ(S_1^{(\alpha)})^{-1}(\mathrm{d}s) > 0. \end{aligned}$$

This together with the estimate (1.3) implies that the condition (2.2) is satisfied with $\kappa(y)\equiv 1$. So, by Lemma 2.1 we have

Corollary 2.2. *There are constants $C = C(\alpha,\delta,z_0) > 0$ and $A > 0$ such that for any $a > A$, one can find an even function $\kappa : \mathbb{R}\to[1,2]$ depending on a such that*

$$p'_\kappa(z_0+a) \geqslant Ca^{-1-\alpha}. \tag{2.5}$$

The above corollary implies that it is not possible to find a constant C that only depends on the bound of κ so that for all $\kappa : \mathbb{R}\to[1,2]$ and $x\in\mathbb{R}$,

$$|p'_\kappa(x)| \leqslant C(1+|x|)^{-2-\alpha}.$$

In fact, it has already given a negative answer to our question. Nevertheless, we are still interested in finding an even function $\kappa : \mathbb{R}\to[1,2]$ so that (2.5) holds for some sequence $a_n\to\infty$. We prepare the following lemma.

Lemma 2.3. *Let* $\kappa(x) = 1 + f(x)$*, where* $f : \mathbb{R} \to [0, 1]$ *is an even function so that*

$$\lambda := \int_{\mathbb{R}} f(x)|x|^{-1-\alpha}\mathrm{d}x < \infty.$$

(i) Under (2.4)*, there is a* $\lambda_0 = \lambda_0(\alpha, \delta) > 0$ *such that for all the above* f *with* $\lambda \leqslant \lambda_0$*,*

$$\inf_{x\in[z_0-\varepsilon, z_0+\varepsilon]} p'_\kappa(x) \geqslant \delta/2. \tag{2.6}$$

(ii) There exists a constant $\gamma = \gamma(\alpha, d) > 0$ *such that for all* $A \geqslant 1$ *and all the above* f *with* $\mathrm{supp}(f) \subset [-A, A]$ *and* $\lambda \leqslant 1$*,*

$$|p'_\kappa(x)| \leqslant \gamma|x|^{-2-\alpha}, \ \ |x| \geqslant A^2. \tag{2.7}$$

Proof. (i) By (2.1) with $\kappa = 1$ there, we have

$$\begin{aligned} p'_{1+f}(x) &= \mathbb{P}(N^\lambda_1 = 0)p'_1(x) + \sum_{k\geqslant 1} \mathbb{P}(N^\lambda_1 = k) \int_{\mathbb{R}} p'_1(x - y)q^{*k}(y)\mathrm{d}y \\ &\geqslant \mathrm{e}^{-\lambda}p'_1(x) - \|p'_1\|_\infty \sum_{k\geqslant 1} \mathbb{P}(N^\lambda_1 = k) \\ &= \mathrm{e}^{-\lambda}\Big(p'_1(x) - \|p'_1\|_\infty(\mathrm{e}^\lambda - 1)\Big), \end{aligned}$$

where $q(x) := f(x)|x|^{-1-\alpha}/\lambda$ and q^{*k} stands for the k-order convolution. In particular, one can choose λ_0 small enough so that

$$(1 \vee \|p'_1\|_\infty)(\mathrm{e}^{\lambda_0} - 1) \leqslant \delta/3.$$

Estimate (2.6) then follows by definition (2.4).

(ii) We make the following decomposition:

$$p'_\kappa(x) = \left(\sum_{k\leqslant\sqrt{|x|}/2} + \sum_{k>\sqrt{|x|}/2}\right)\mathbb{P}(N^\lambda_1 = k) \int_{\mathbb{R}} p'_1(x - y)q^{*k}(y)\mathrm{d}y =: J_1(x) + J_2(x).$$

For J_1, noticing that $\mathrm{supp}(q^{*k}) \subset [-kA, kA]$, by (1.3) and $|x| \geqslant A^2$, we have

$$\begin{aligned} |J_1(x)| &\leqslant \sum_{k\leqslant\sqrt{|x|}/2} \mathbb{P}(N^\lambda_1 = k) \int_{\mathbb{R}} |p'_1(x - y)|q^{*k}(y)\mathrm{d}y \\ &\leqslant K_3 \sum_{k\leqslant\sqrt{|x|}/2} \mathbb{P}(N^\lambda_1 = k)\Big(|x| - \sqrt{|x|}A/2\Big)^{-2-\alpha} \\ &\leqslant K_3 2^{2+\alpha}|x|^{-2-\alpha}. \end{aligned}$$

For J_2, by Stirling's formula and $\lambda \leqslant 1$, we have

$$|J_2(x)| \leqslant \|p'_1\|_\infty \mathrm{e}^{-\lambda} \sum_{k>\sqrt{|x|}/2} \lambda^k/k! \leqslant C|x|^{-2-\alpha}.$$

Combining the above calculations, we obtain (2.7). □

Now we are in a position to give

Proof of Theorem 1.1. Let $h : \mathbb{R} \to [0,1]$ be a measurable function with

$$\mathrm{supp}(h) \subset [-1,1], \quad \int_{\mathbb{R}} h(x)\mathrm{d}x = 1.$$

Let $A \geqslant A_0$ be a large number, whose value will be determined later, where A_0 is from Lemma 2.1. For $k \in \mathbb{N}$, define

$$A_1 := A, \quad A_{k+1} := (A_k + \varepsilon)^2, \ k \geqslant 1,$$
$$h_k(x) := h\Big(\frac{A_k - x}{\varepsilon}\Big) + h\Big(\frac{A_k + x}{\varepsilon}\Big), \quad \kappa(x) := 1 + \sum_{k=1}^{\infty} h_k(x).$$

Clearly, $\kappa(-x) = \kappa(x)$. We want to show that for some $C_0 = C_0(\alpha) > 0$,

$$p'_\kappa(z_0 + A_n) \geqslant C_0 A_n^{-1-\alpha}, \ \forall n \in \mathbb{N}. \tag{2.8}$$

First of all, by definition we have

$$\mathrm{supp}(h_k) \subset [A_k - \varepsilon, A_k + \varepsilon] \cup [-A_k - \varepsilon, -A_k + \varepsilon] \tag{2.9}$$

and

$$\begin{aligned} 2\varepsilon(A_k + \varepsilon)^{-1-\alpha} &\leqslant \int_{\mathbb{R}} h_k(x)|x|^{-1-\alpha}\mathrm{d}x \\ &= 2\int_{\mathbb{R}} h\Big(\frac{A_k - x}{\varepsilon}\Big)|x|^{-1-\alpha}\mathrm{d}x \leqslant 2\varepsilon(A_k - \varepsilon)^{-1-\alpha}. \end{aligned}$$

For $n \in \mathbb{N}$, define

$$\kappa_n(x) := 1 + \sum_{k=1}^{n} h_k(x) =: 1 + f_n(x).$$

By (1.2), there is a constant $M > 0$ such that for all $n \in \mathbb{N}$,

$$|p'_{\kappa_n}(x)| \leqslant M, \ \ x \in \mathbb{R}^d.$$

Noticing that $\mathrm{supp}(f_n) \subset \cup_{k=1}^n \mathrm{supp}(h_k) \subset [-A_n - \varepsilon, A_n + \varepsilon]$ and

$$\int_{\mathbb{R}} f_n(x)|x|^{-1-\alpha}\mathrm{d}x \leqslant 2\varepsilon \sum_{k=1}^{n} (A_k - \varepsilon)^{-1-\alpha} \leqslant C A^{-1-\alpha},$$

by Lemma 2.3, if $CA^{-1-\alpha} \leqslant \lambda_0$, then

$$\inf_{x \in [z_0 - \varepsilon, z_0 + \varepsilon]} p'_{\kappa_n}(x) \geqslant \delta/2, \tag{2.10}$$

and for some $\gamma = \gamma(\alpha) \geqslant M$,

$$|p'_{\kappa_n}(x)| \leqslant \gamma |x|^{-2-\alpha}, \ \ |x| \geqslant (A_n + \varepsilon)^2 = A_{n+1}. \tag{2.11}$$

Using these two estimates, and by Lemma 2.1 we derive that for some $C_1 = C_1(\alpha, \delta) > 0$,

$$p'_{\kappa_{n+1}}(z_0 + A_{n+1}) = p'_{\kappa_n + h_{n+1}}(z_0 + A_{n+1}) \geqslant C_1 A_{n+1}^{-1-\alpha}.$$

Finally, let $\tilde{f}_n(x) := \kappa(x) - \kappa_n(x) = \sum_{k>n} h_k(x)$ and

$$\tilde{\lambda} := \int_{\mathbb{R}} \tilde{f}_n(x)|x|^{-1-\alpha}\mathrm{d}x \leqslant 2\varepsilon \sum_{k=n+1}^{\infty} (A_k - \varepsilon)^{-1-\alpha} \leqslant C_2 A_{n+1}^{-1-\alpha}.$$

As above, we have

$$\begin{aligned} p'_\kappa(z_0 + A_n) &= \mathbb{P}(N_1^{\tilde{\lambda}} = 0) p'_{\kappa_n}(z_0 + A_n) + \sum_{k\geqslant 1} \mathbb{P}(N_1^{\tilde{\lambda}} = k) \int_{\mathbb{R}} p'_{\kappa_n}(z_0 + A_n - y)\tilde{q}^{*k}(y)\mathrm{d}y \\ &\geqslant \mathrm{e}^{-\tilde{\lambda}} p'_{\kappa_n}(z_0 + A_n) - M \sum_{k\geqslant 1} \mathbb{P}(N_1^{\tilde{\lambda}} = k) \\ &= \mathrm{e}^{-\tilde{\lambda}}(p'_{\kappa_n}(z_0 + A_n) - M(\mathrm{e}^{\tilde{\lambda}} - 1)) \\ &\geqslant \mathrm{e}^{-\tilde{\lambda}}(C_1 A_n^{-1-\alpha} - 2C_2 M A_{n+1}^{-1-\alpha}) \geqslant C_3 A_n^{-1-\alpha}, \end{aligned}$$

provided A large enough. Thus we get (2.8), which means

$$\limsup_{x\to\infty} x^{1+\alpha} p'_\kappa(x) \geqslant C_0 > 0.$$

By symmetry, we obtain the desired estimate. □

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