Delaying Product Introduction in a Duopoly: A Strategic Dynamic Analysis*

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Abstract

We analyze the optimal timing for the introduction of a new product in a duopoly. Two incumbent firms are active on a homogeneous product market and one of these firms has an option to additionally introduce a new product, thereby incurring costs of product adoption. We assume that the innovator can commit on the time of product introduction and numerically derive the optimal introduction time as well as the associated Markov-perfect equilibria for investment in production capacities. We find that depending on the initial capacities for the established product and the size of the adoption costs, three scenarios are possible for the innovator: innovating immediately, delaying introduction, and abstaining from product introduction. In case of delayed introduction, the innovator strategically reduces capacities on the established market prior to product introduction, whereas the dynamics of the non-innovator's capacity is ambiguous. Furthermore, in this case, the firm commits to a market introduction time such that at the time of market introduction it has incentives to further delay the product adoption.

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1 Introduction

Technological change is a crucial driver of industrial dynamics. Improved versions of products appear regularly. Furthermore, product innovations lead to differentiated products and new submarkets arise. According to an empirical investigation by Chandy and Tellis (2000), most of the product innovations has been achieved by established incumbents. Typical examples include Asus which has been active on the notebook market and has introduced netbooks in 2007 or Apple's introduction of the iPad in 2010 which generated a huge submarket for tablet computers. For a firm competing with others on a homogeneous market, a product innovation can be very valuable. Given that a product innovation has been made, the innovator has to decide whether to introduce a new product immediately, to delay the product introduction strategically, or not to introduce at all[1]. Wang and Hui (2012) provide examples where the market introduction of products has been delayed, e.g. DVD players and MP3-related products which could have been introduced earlier.

To analyze the question how an incumbent should optimally choose its market introduction strategy we consider two firms competing on an established homogeneous market, and assume that one of the firms has the option to introduce a new product, whereas the rival has to stick with producing the established product. Moreover, we assume that the new product is horizontally and vertically differentiated, in particular that it has a higher quality than the established product. Both firms are restricted by production capacities which they adjust over time. The setting after the introduction of the new product has been analyzed in Dawid et al. (2010). They find that not only the innovator benefits but the non-innovator is better off as well in most cases, in particular if the products are not too differentiated. The innovator strongly reduces capacities on the established market in order to increase demand for the established product.

Adjustments of capacities of established products *prior* to a product innovation has been studied in a stochastic setting in Dawid et al. (2017b) who consider a duopoly where both firms can also invest in R&D in order to increase the probability of product innovation (see Dawid et al. (2013) for an exogenous hazard rate). In contrast to those approaches, we assume that the innovation has been made already and the time of product introduction is an additional choice variable and hence is not directly linked to

[1]Several studies (Mansfield (1977), Åstebro (2003) and Åstebro and Simons (2003)) have found out that a large fraction of product innovations is not brought to the market.

the time of the successful completion of an R&D project. The separation of innovation and introduction has been employed by Dawid et al. (2009), however only in a three-stage model where continuous capacity adjustments are not taken into account and the timing of product introduction could not be addressed.

The game we are considering is a multi-mode differential game where one of the firms can induce a regime switch (in our context adding a second differentiated product to its product range) at any time. This is in contrast to models where a regime switch occurs when the state variable hits some critical threshold (see e.g. Reddy et al. (2015) and Masoudi and Zaccour (2013)) or is governed by a stochastic process as in Dawid et al. (2013, 2017b)).

In Gezer (2019) a related setting to that in this paper has been analyzed, however abstracting from competition. An incumbent monopolist has the option to introduce an new (substitute) product in addition to the one already offered. It is shown that the firm might delay product introduction if it incurs adoption costs. By delaying the product introduction, the monopolist benefits from discounted adoption costs, which has to be paid as a lump sum at the time of product introduction. Furthermore, the monopolist can increase the marginal value of the new product by decreasing established capacities. Similar effects are also present in the duopoly considered in this paper, however strategic interaction adds substantial new effects.

Optimal timing of innovation has been analyzed extensively in the optimal stopping and real options literature (see e.g. Dutta et al. (1995), Hoppe and Lehmann-Grube (2005) and Dixit and Pindyck (1994)). Recent contributions consider for stochastic demand, both, optimal timing and capacity choice simultaneously (see e.g. Huberts et al. (2019) and Huisman and Kort (2015)). The latter find in a setting with two firms who have the option to enter a new market that firms invest earlier compared to the monopoly setting. In particular, the first investor overinvests in order to delay market entry of the second investor. The innovation of the present paper relative to this literature is that it considers the dynamic adjustment of capacities before and after the innovation, whereas mostly one-time investments have been treated in the real options literature.

Optimal timing has been considered only in a few differential game models. Yeung (2000) derives feedback Nash equilibria for games with endogenous time horizon by restricting terminal values for state variables. Recently, Gromov and Gromova (2017) formalize the class of hybrid differential games and characterize a switching manifold in the time-state space which is determined by a switching condition. They argue that deriving feedback Nash equilibria for state-dependent switching is complicated and resort to open-loop Nash equilibria, which in certain games, parametrized by initial conditions yields feedback Nash equilibria.

In terms of timing, the most related contribution is Long et al. (2017) where in a differential game model with multiple regimes, the concept of piecewise-closed loop Nash equilibria (PCNE) is introduced. They derive necessary conditions for the optimal switching time in a two player setting, where both players can induce a change of the regime of the game. The timing decision is given implicitly by the state variable arriving at a certain state which is derived by optimality conditions. However, in their setting, it is assumed that firms commit to their switching time in the sense, that they would not alter that time even if the other firm would deviate from its equilibrium control path. Hence, the considered equilibrium is not fully Markov perfect with respect to the timing decision.

In our approach, we consider a case where the innovator can fully commit to its product introduction time. Hence, the competitor cannot influence the timing of the product introduction. An equilibrium is given if the choice of the product introduction time, T, maximizes the value of the game for the innovator while given this T, the investment strategies played by both players constitute a Markov-perfect Nash equilibrium in the classical sense. Note that the timing decision is made in the beginning of the game for given initial capacities and hence it is an open-loop strategy whereas the continuous control variables constitute a Markov perfect equilibrium using closed-loop strategies. Characterizing a fully closed-loop equilibrium in which the introduction of the new product is triggered if the state variable hits a switching manifold (to be optimally determined by the innovator) is technically challenging and might lead to non-existence of equilibria (see Long et al. (2017) for details).

From an economic perspective, the commitment to the product introduction time might be due to a preannouncement. There is a huge literature on preannouncements considering its effects on various interest groups such as consumers, competitors and others. Preannouncements are made for various purposes (cf. Lilly and Walters (1997)). They are used e.g. for building interest for the new product before the market launch (Bao et al. (2005)), in order to stimulate consumers to delay purchases, in particular to wait for a better product (Su and Rao (2010) or to deter entry of potential entrants or to induce a competitor to adjust capacities or to reposition (see Farrell and Saloner (1986) and Heil and Robertson (1991)).

We use dynamic programming for solving for the optimal capacity investment strategies and derive an optimality condition for the optimal timing which depends on the time-derivative of the corresponding value function at the outset of the game. In that respect our game might be interpreted as a two stage game, where in the first stage only the innovator decides on the introduction time and in the second stage both firms simultaneously choose their Markovian capacity investment strategies and apply them either starting with only the established product or with both products in case that the innovator introduces immediately.

We find that whenever it is optimal to delay the product introduction, the optimal introduction time is increasing in adoption costs. Furthermore, we find that the optimal introduction time increases in both initial capacities, i.e. the stronger the innovator or the non-innovator on the established market, the later the product introduction. The latter is in accordance with results of Dawid et al. (2017b) where R&D investments are negatively affected by both firms' capacities.

Additionally, we find that in a duopoly, the innovator introduces the product less often compared to a monopoly scenario and, in case of product introduction she introduces earlier compared to the monopoly. Thus, this paper contributes to the debate initiated by Schumpeter and Arrow in the sense that we show that market concentration facilitates product innovation but slows down the actual introduction of the new product.

In section 2, we provide the model and in section 3, we derive a general sufficient condition for delaying the product introduction. Furthermore, we derive general necessary conditions for optimal timing which has to hold at the outset of the game. In Section 4 we discuss the different dynamic patterns that can arise in equilibrium using numerical methods. In Section 5 we give some concluding remarks.

2 Model

We consider a duopoly where both firms, denoted by firm A and B, produce a homogeneous established product, denoted as product 1. Due to product innovation, firm A has the option to introduce a horizontally and vertically differentiated substitute product with higher quality, denoted as product 2. We call this firm the innovator whereas the other firm, firm B is called the non-innovator. The innovator incurs lumpy costs F at the time of introduction.

Both firms need to build and maintain production capacities, denoted by $K_{if}, i = 1, 2, f = A, B$, for every product they are offering. For simplicity, we assume that the innovator can only start to invest in the capacity of the new product after introduction, i.e. there are no capacities at the time of introduction for the new product, yet. In line with large parts of the literature (see e.g. Dockner et al. (2000); Huisman and Kort (2015)), it is assumed that capacities are always fully used. Production costs for given capacities are normalized to zero. There is no inventory, i.e. production equals sales.

Before product introduction, i.e. for all $t \leq T$, the linear inverse demand function for the established product is given by

$$p_1^{m_1}(K_{1A}(t), K_{1B}(t)) = 1 - K_{1A}(t) - K_{1B}(t), \tag{1}$$

whereas after product introduction, i.e. for all $t \geq T$, the inverse demand system is given by

$$p_1^{m_2}(K_{1A}(t), K_{1B}(t), K_{2A}(t)) = 1 - \big(K_{1A}(t) + K_{1B}(t)\big) - \eta K_{2A}(t), \tag{2}$$

and

$$p_2^{m_2}(K_{1A}(t), K_{1B}(t), K_{2A}(t)) = 1 + \theta - K_{2A}(t) - \eta\big(K_{1A}(t) + K_{1B}(t)\big), \tag{3}$$

where η with $0 < \eta < 1$ measures the degree of horizontal and $\theta > 0$, the degree of vertical differentiation of the strategic substitutes.

There are two modes in the game:

- mode 1 (m_1): The new product has been developed by the innovator and is ready for market introduction which is common knowledge. However, only the established product is sold.
- mode 2 (m_2): The new product has been introduced to the market. Both products are sold.

Capacity investment is costly with quadratic costs

$$C_1(I_{1f}(t)) = \frac{\gamma_1}{2} I_{1f}^2(t), \qquad f = A, B, \tag{4}$$

and

$$C_2(I_{2A}(t)) = \frac{\gamma_2}{2} I_{2A}^2(t). \tag{5}$$

The capacity dynamics in m_1 are

$$\dot{K}_{1f} = I_{1f} - \delta K_{1f}, \qquad f = A, B, \tag{6}$$

for initial capacities

$$K_{1f}(0) = K_{1f}^{ini}, \ f = A, B, \tag{7}$$

where $\delta > 0$ measures the depreciation rate of the capacities. In m_2, there is an additional state for the capacity of the new product which evolves in the same way according to

$$\dot{K}_{2A} = I_{2A} - \delta K_{2A}, \tag{8}$$

$$K_{2A}(t) = 0 \quad \forall t \leq T. \tag{9}$$

As in Dawid et al. (2010), we allow the firms to intentionally scrap capacities (i.e. investments might be negative) while capacities have to remain non-negative, i.e. $K_{1f} \geq 0 \ \forall \ t$, $f = A, B$, and $K_{2A} \geq 0 \ \forall \ t$.

The innovator wants to determine the optimal time of product introduction T, i.e. the time of transition from m_1 to m_2, and the optimal strategies for investment in capacities before and after product introduction, whereas the non-innovator only determines the optimal strategy for investing in her capacity for the established product. The discounted stream of profits of the innovator is given by

$$\begin{aligned} J_A = & \int_0^T e^{-rt}\big(p_1^{m_1}(\cdot)K_{1A} - C_1(I_{1A})\big)dt \\ & + \int_T^\infty e^{-rt}\big(p_1^{m_2}(\cdot)K_{1A} + p_2 K_{2A} - C_1(I_{1A}) - C_2(I_{2A})\big)dt - e^{-rT}F, \end{aligned} \tag{10}$$

which is maximized with respect to T, I_{1A} and I_{2A}. For the non-innovator, it is given by

$$J_B = \int_0^T e^{-rt}\big(p_1^{m_1}(\cdot)K_{1B} - C_1(I_{1B})\big)dt + \int_T^\infty e^{-rt}\big(p_1^{m_2}(\cdot)K_{1B} - C_1(I_{1B})\big)dt, \tag{11}$$

where the control variable of firm B is I_{1B}.

3 Equilibrium Strategies

In this section, we will derive some sufficient and necessary conditions for the optimal timing of the product introduction. It should be noted that those conditions hold generally for models where two firms' controls affect the dynamics of a continuously evolving state variable and one of the firms can induce a regime switch.

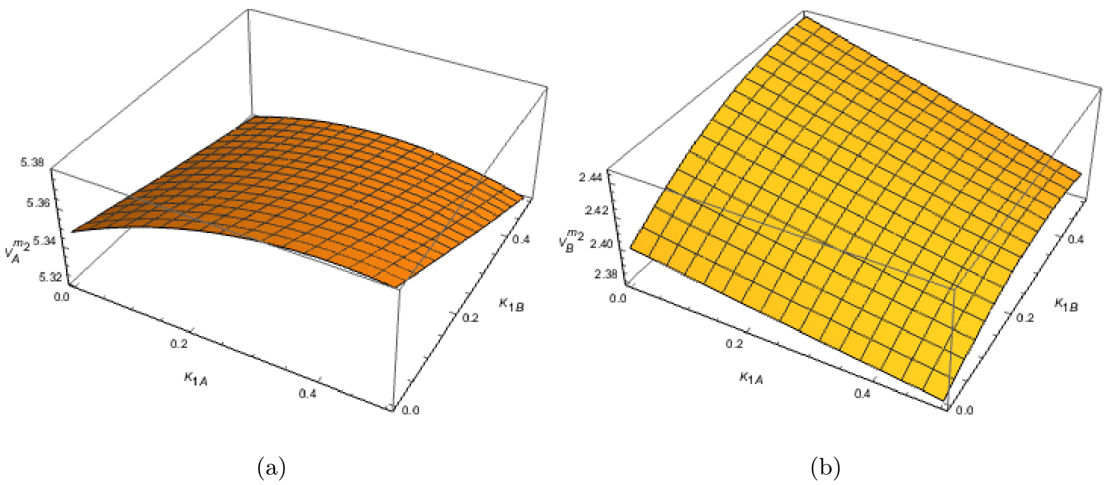


Figure 1: Value functions of m_2 for $K_{2A} = 0$. Parameters: $r = 0.04,\ \delta = 0.2,\ \eta = 0.5,\ \theta = 0.1,\ \gamma = 0.1$.

For the sake of brevity, denote the capacity pair (K_{1A}, K_{1B}) by K. Let

$$\phi_{if}(K, K_{2A}, t, m),\ \ f = A, i = 1, 2 \text{ and } f = B, i = 1$$

be the Markovian investment strategies of both firms in mode m and $T = \tau$ the timing strategy of the innovator. Then, a strategy vector of the innovator is a pair $\psi_A = ((\phi_{1A}, \phi_{2A}), \tau)$ whereas the strategy of the non-innovator is given by $\psi_B = \phi_{1B}$. A strategy profile (ψ_A, ψ_B) is an equilibrium if given τ, (ϕ_{1A}, ϕ_{1B}) constitutes a Markov perfect equilibrium and τ maximizes the objective functional of the innovator.

In the case that the innovator introduces the improved product at some finite time T, there will be a structural change of the model. Denote by $V_f^{opt}(K, K_{2A}, t, m)$ the value function of firm f in mode m where the switching time from m_1 to m_2 is selected *optimally* by the innovator. Furthermore, denote by $V_f^{m_1}(K)$ and $V_f^{m_2}(K, K_{2A})$, $f = A, B$, the value functions of the corresponding infinite horizon games where the mode is fixed and hence does not change. This immediately gives $V_f^{opt}(K(t), K_{2A}(t), t, m_2) = V_f^{m_2}(K(t), K_{2A}(t))$ for all t and $V_f^{opt}(K(T), K_{2A}(T), T, m_1) = V_f^{m_2}(K(T), K_{2A}(T)) - F$, $f = A, B$ for the switching time T since in m_2, the mode does not change anymore. The infinite horizon games are time-autonomous, and therefore we consider stationary strategies. Hence the value functions of the infinite horizon games with fixed mode do not explicitly depend on time. The subproblem of m_2 is of linear-quadratic type which can be solved easily by the dynamic programming approach. Due to the linear quadratic structure of the game, the value functions have the following form

$$
\begin{aligned}
V_f^{m_2} =& C_f^{m_2} + D_f^{m_2} K_{1A} + E_f^{m_2} K_{1A}^2 + F_f^{m_2} K_{1B} + G_f^{m_2} K_{1B}^2 + H_f^{m_2} K_{2A} + J_f^{m_2} K_{2A}^2 \\
& + L_f^{m_2} K_{1A} K_{1B} + M_f^{m_2} K_{1A} K_{2A} + N_f^{m_2} K_{1B} K_{2A}, \ f = A, B.
\end{aligned} \tag{12}
$$

Using this functional form, the HJB-equations can be reduced to a set of algebraic equations which has to be satisfied by the coefficients of the quadratic value functions. Coefficients can be found by standard numerical methods for a given parameter setting (cf. Dawid et al. (2010) for a similar model with slightly different inverse demand functions). Figure 1 illustrates the shape of the value functions in m_2. By regarding the value of the subproblem (minus adoption costs) as the salvage value of the finite time horizon problem in mode m_1, i.e.

$$
S(K_{1A}(T), K_{1B}(T)) = V_A^{m_2}(K_{1A}(T), K_{1B}(T), 0) - F \ , \tag{13}
$$

we can write the optimization problems of both firms in m_1 as

$$
\begin{aligned}
\max_{T, I_{1A}} \int_0^T & e^{-rt} \big(p_1 K_{1A} - C_1(I_{1A})\big) dt \\
& + e^{-rT} \Big(V_A^{m_2} \big(K_{1A}(T), K_{1B}(T), 0\big) - F \Big),
\end{aligned} \tag{14}
$$

and

$$
\max_{I_{1B}} \int_0^T e^{-rt} \big(p_1 K_{1B} - C_1(I_{1B})\big) dt + e^{-rT} V_B^{m_2} \big(K_{1A}(T), K_{1B}(T), 0\big). \tag{15}
$$

If an infinite time horizon is optimal, then the salvage value disappears and the value of the game is simply given by $V_f^{m_1}(\cdot)$ for $f = A, B$ and there is a unique stable steady state (see Jun and Vives (2004)).

As discussed above, we assume that the innovator announces the date of product introduction and has commitment power such that he cannot deviate from the announced date even though ex post it would be better to do so. Thus, the non-innovator takes T as given by the preannouncement and chooses his investment strategy in order to maximize the value of the game. Technically speaking, we employ Markov (feedback) strategies for the investment in capacities and open-loop strategies for the introduction time T.

Note that for any fixed T, the game in m_1 is still of linear quadratic structure. Since the problem in m_1 has a finite time horizon the coefficients in the value function depend on time and from the HJB-equations a set of Riccati equations for those coefficients is obtained. We solve this system using standard numerical solvers. The corresponding HJB-equations to be fulfilled are given in Appendix B. Denote the value function of the game starting in m_1 and switching to m_2 at a fixed T by $V_f(K, t; T)$, $f = A, B$, and the corresponding profile of Markovian strategies in equilibrium by $\phi_{1f}(K, t; T)$, $f = A, B$.

Since the game is time-autonomous, i.e. t appears explicitly only in the discounting term e^{-rt}, we can consider equilibrium strategies which depend only on the remaining time till T. This then also hold for the value function and we have $V_f(K, t; T) = V_f(K, 0; T - t)$, $f = A, B \; \forall \; K$ and $t \leq T$ (cf. Caulkins et al. (2015)).

In particular, we have $\phi_{1f}(K, T; T) = \phi_{1f}(K, 0; 0)$ and for finite T, we denote the right hand side of the HJB-equation of firm A (equation (40) in Appendix B) at the switching time by[2]

$$\begin{aligned} H(K) = p_1^{m_1}(K)K_{1A} - C(\phi_{1A}(K, 0; 0)) + V^{m_2}_{A,K_{1A}}(K, 0)(\phi_{1A}(K, 0; 0) - \delta K_{1A}) \\ + V^{m_2}_{A,K_{1B}}(K, 0)(\phi_{1B}(K, 0; 0) - \delta K_{1B}). \end{aligned} \tag{16}$$

Note that the optimal strategies ϕ_{1A} and ϕ_{1B} stem from m_1 whereas derivatives of the value function of m_2 are considered. We assume that $V(K, t; T)$ is sufficiently smooth, i.e. let $V(K, t; T)$ be continuously differentiable in K and t for all T. Then, the following lemma gives a sufficient condition for delaying the product introduction.

Lemma 1. *If for the initial capacities $(K_{1A}(0), K_{1B}(0)) = K^{ini}$ the inequality*

$$H(K^{ini}) > r(V_A^{m_2}(K^{ini}, 0) - F) \tag{17}$$

holds, then the optimal time of product introduction T^* is positive, possibly infinite.

Proof. Consider the value for the innovator to stay for the duration of ϵ in m_1 and afterwards to switch to m_2 under the equilibrium strategy $\phi = (\phi_{1A}, \phi_{1B})$:

$$V_A(K(0), 0; \epsilon) = \int_0^\epsilon e^{-rs} F_A^{m_1}(K(s), \phi(K(s), s; \epsilon)ds + e^{-r\epsilon}(V_A^{m_2}(K(\epsilon), 0) - F). \tag{18}$$

where $F_A^{m_1}(\cdot)$ is the instantaneous profit function of the innovator in m_1. For a finite time horizon, since we consider non-stationary strategies, altering the terminal time would yield different investments in m_1 and hence different values for the terminal capacities. Thus, for the sake of clarity, here we denote the capacity at t for terminal time T by $K_{1f}(t, T)$, $f = A, B$. $K_{1A}(\epsilon, \epsilon)$ can then be derived via the initial value $K_{1A}(0, \epsilon)$ and the investments from 0 until ϵ:

$$K_{1A}(\epsilon, \epsilon) = K_{1A}(0, \epsilon) + \int_0^\epsilon (\phi_{1A}(K(\tau, \epsilon), \tau; \epsilon) - \delta K_{1A}(\tau, \epsilon))d\tau. \tag{19}$$

[2]Actually, $H(K)$ is the Hamiltonian where the co-state variable is replaced by the state derivatives of the scrap value (cf. Pontryagin's maximum principle with finite time horizon e.g. in Dockner et al. (2000)).

Its derivative with respect to ϵ is then given by

$$\frac{\partial K_{1A}(\epsilon,\epsilon)}{\partial t} + \frac{\partial K_{1A}(\epsilon,\epsilon)}{\partial T} \tag{20}$$

$$= \phi_{1A}(K(\cdot),\tau;\epsilon) - \delta K_{1A}(\cdot) + \int_0^\epsilon \frac{\partial \phi_{1A}(K(\tau,\epsilon),\tau,\epsilon) - \delta K_{1A}(\tau,\epsilon)}{\partial T} d\tau. \tag{21}$$

In equation (18), subtracting $V_A(K(0),0;0)$ on both sides, dividing by ϵ and considering the limit $\epsilon \to 0$ yields

$$\begin{aligned}\frac{\partial V_A(K,0,0)}{\partial T} &= p_1^{m_1}(K)K_{1A}(0,0) - C(\phi_{1A}(K,0;0)) \\ &+ V^{m_2}_{A,K_{1A}}(K,0)\left(\dot{K}_{1A}(0,0) + \frac{\partial K_{1A}(0,0)}{\partial T}\right) \\ &+ V^{m_2}_{A,K_{1B}}(K,0)\left(\dot{K}_{1B}(0,0) + \frac{\partial K_{1B}(0,0)}{\partial T}\right) \\ &- r\left(V_A^{m_2}(K_{1A}(0,0),K_{1B}(0,0),0) - F\right)\end{aligned} \tag{22}$$

where no time derivatives of $V_A^{m_2}$ appear since we consider stationary strategies in m_2. Furthermore,

$$\frac{\partial K_{1f}(0,0)}{\partial T} = 0,\ \ f = A, B \tag{23}$$

and using inequality (17) we obtain

$$\frac{\partial V_A(K,0,0)}{\partial T} > 0, \tag{24}$$

which proves that delaying the product introduction marginally is better than introducing immediately. □

It follows from Lemma 1 that (17) being violated is a necessary condition for immediate product introduction. It should be noted that it is, however, not possible to derive a (local) sufficient condition for immediate introduction since marginally being worse-off does not necessarily imply that immediate introduction is optimal. For some $T > 0$, the corresponding value might still outweigh immediate introduction's value.

From optimal control theory, it is known that for $H(K^{ini}) > r(V_A^{m_2}(K^{ini},0) - F)$, the innovator prefers not introducing the product immediately but introducing whenever $H = r(V_A^{m_2} - F)$ holds. Here, $H = r(V_A^{m_2} - F)$ is satisfied on a *switching line* (see Appendix A). In an optimal control setting, the firm exerts control such that the state arrives at the switching line and the switch occurs. However, in a game, the other player can influence the time the switching line is reached because it controls the dynamics of its own capacity. In an equilibrium where the strategy determining when to introduce the new product is of feedback type, e.g. Markovian, this gives rise to intricate strategic effect to be considered. Here we assume however that firm A commits at $t = 0$ to the *time of product introduction* (which might be infinity if the firm decides not to introduce the product at all) rather than on a switching line in the state space and therefore these issues do not arise. Also, by choosing T, firm A influences the investment strategy of firm B. As it will turn out, this effect induces that in our setting in equilibrium the product in general is not introduced at the point in time when the state is on the switching line.

In order to characterize the optimal time of product introduction T, i.e. the choice of the time horizon of the game, which maximizes $V_A(K, 0, T)$, we proceed as follows. We consider a sufficiently large fixed time horizon and compute the optimal distance to the terminal time where the firm wants the game to start. For this, we use a *large* T, which is defined as follows.

Standard turnpike arguments (see McKenzie (1986) or Grüne et al. (2015)) yield that for $T \to \infty$, the change in the value function becomes small since it is converging to the (time-independent) value function of the infinite horizon game in mode m_1, $V_f^{m_1}$. For an ϵ with $0 < \epsilon \ll \left|V_A^{m_2}(K^{ini}, 0) - V_A^{m_1}(K^{ini})\right|$[3] and an initial capacity K^{ini}, a large T satisfies

$$\left|V_f(K^{ini}, 0; T) - V_f^{m_1}(K^{ini})\right| \leq \epsilon. \tag{25}$$

We denote by $T^l(\epsilon, K^{ini})$ the minimal T for which inequality (25) holds for all $T \geq T^l$. Among all capacities which yield positive prices, we select the maximal T^l which we denote by $T^L(\epsilon)$, i.e. $T^L(\epsilon) := T^l(\epsilon, K_{max})$ where $K_{max} = \arg\max_{K:p_1^{m_1}(K)\geq 0}(T^l(\epsilon, K))$.

Using this notation, in the following proposition we characterize firm A's choice of the optimal time of product introduction.

Proposition 1. *Let $V_f(K, t; T^L)$ be the value function of the game for a fixed large end time $T^L(\epsilon)$ for $f = A, B$. Let t^* be the time argument maximizing V_A for an initial pair $K^{ini} = (K_{1A}^{ini}, K_{1B}^{ini})$, i.e.*

$$t^*(K^{ini}) = \arg\max_{t\in[0,T^L]} V_A(K^{ini}, t; T^L). \tag{26}$$

If $t^*(K^{ini}) > 0$, then

$$T^*(K^{ini}) = T^L - t^*(K^{ini}), \tag{27}$$

[3]Note that for higher choices of ϵ, inequality (25) might be satisfied for all T and hence would not yield a *large* T.

is the optimal time of product introduction for $K(0) = K^{ini}$ and the value function in m_1 for $f = A, B$ and for initial capacities K^{ini} is given by

$$V_f^{opt}(K, 0, t, m_1) = V_f(K, t; T^*(K^{ini})). \tag{28}$$

Furthermore, if $t^*(K^{ini}) = 0$ for all $T \geq T^L(\epsilon)$ (i.e. for all $T^L(\tilde{\epsilon})$ with $\tilde{\epsilon} \leq \epsilon$), then

$$T^*(K^{ini}) = \infty, \tag{29}$$

is the optimal time of product introduction for $K(0) = K^{ini}$ and the value function is given by

$$V_f^{opt}(K, 0, t, m_1) = V_f^{m_1}(K), \; f = A, B. \tag{30}$$

Proof. Due to time invariance, the current value of the initial game defined on the time interval $[0, T^L]$ at t^* is equal to the current value at 0 of the game defined over $[0, T^*]$ where $T^* = T^L - t^*$. Hence, it is sufficient to derive the optimal distance to a fixed terminal time where the innovator wants the game to start.

If $t^*(K^{ini}) > 0$, i.e. $t^*(K^{ini})$ is interior in $[0, T^L]$, then for all $T \geq T^L$, according to inequality (25), $t^*(K^{ini})$ (shifted by $T - T^L$) is still an interior maximum. Hence, $T^L - t^*(K^{ini})$ is the optimal distance to the terminal time T^L.

If $t^*(K^{ini}) = 0$ for all $T \geq T^L(\epsilon)$, then the maximizing argument is at the left boundary. More precisely, for reducing ϵ and thereby increasing T^L, $t^* = 0$ remains optimal. Thus, $V_A(K^{ini}, t, T)$ is monotonously increasing in T. Hence, $T^* = \infty$ is optimal.

□

Essentially, from a family of value functions of the game for different values of T, i.e. for varying terminal times, the innovator has to select that one which maximizes his profits for the initial capacity. So, the optimal time of product introduction can be found via considering the value function for a fixed initial pair K^{ini} and a fixed sufficiently large terminal time and determining the optimal distance to the terminal time[4]. In the next corollary, we provide necessary conditions for the slope of the time derivative of the value function at the outset of the game.

[4]The idea of considering large values for the terminal time has been employed by several works, e.g. in Grass (2012).

Corollary 1. *i) If immediate product introduction, i.e. a corner solution $T^* = 0$ is optimal, then*

$$\lim_{T\to 0}\Big(\lim_{t\to T^-} V_{A,t}(K^{ini}, t; T)\Big) \geq 0, \tag{31}$$

and

$$H(K^{ini}) \leq r(V_A^{m_2}(K^{ini}, 0) - F). \tag{32}$$

ii) If no product introduction, i.e. $T^* = \infty$ is optimal, then

$$\lim_{T\to\infty} V_{A,t}(K^{ini}, 0; T) \leq 0, \tag{33}$$

iii) For an interior solution, i.e. $0 < T^* < \infty$ to be optimal we must have

$$V_{A,t}(K^{ini}, 0; T^*) = 0. \tag{34}$$

Proof. i) For a corner solution $T^* = 0$, the maximizing argument of (26) is on the right boundary, i.e. $t^* = T^L$. Thus,

$$\lim_{t\to T^-} V_{A,t}(K^{ini}, t; T) \geq 0,$$

holds for all $T > 0$, which implies (31) . Furthermore, the HJB-equation under $T^* = 0$ yields

$$rS(K^{ini}) - V_{A,t}(K^{ini}, 0; 0) = H(K^{ini}). \tag{35}$$

As the limit of $V_{A,t}$ stays positive, we obtain (32).

ii) For a corner solution $T^* = \infty$, the maximizing argument is on the left boundary, i.e. $t^* = 0$. This means $V_t(K^{ini}, 0; T^L) \leq 0$ for all T^L. Thus, $\lim_{T\to\infty} V_t(K^{ini}, 0; T) \leq 0$.

iii) follows directly from the first order condition if firm A.

□

Note that Corollary 1 yields necessary conditions only. In particular, condition (34) might be satisfied for local maxima which are not globally optimal. To get an intuition for this necessary optimality condition, consider the difference of the value of the game for a fixed state variable vector when time moves from t to $t + \Delta$, $\Delta > 0$:

$$V(K^{ini}, t + \Delta; T^L) - V(K^{ini}, t; T^L). \tag{36}$$

Assuming firm A is free to choose between $t + \Delta$ and t, (36) measures the change in the value function in current-value terms. If (36) is positive, it is (locally) optimal for the firm to choose a later starting point than t, and an earlier starting point, else. As K^{ini} is not affected by the choice of T^*, maximizing with respect to the second argument of the value function yields for fixed T^L the (globally) optimal time-span $T^* = T^L - t$ for firm A between the initial time and the time of product introduction, which corresponds to the optimal time of product introduction of the free end time game. The first order condition of the optimization of $V(K^{ini}, t; T^L)$ with respect to t yields (34). Since it is not feasible to provide an analytical characterization of the globally optimal choice of T it s also not possible to to derive results about the dependence of the introduction time and investment patterns in equilibrium. In order to get a more complete picture of the dependence of the optimal introduction time, as well as of the resulting equilibrium capacity dynamics, from initial capacities and key model parameters, in the following section we compare the actual equilibrium solutions under different parameter constellations using numerical methods.

4 Dynamics

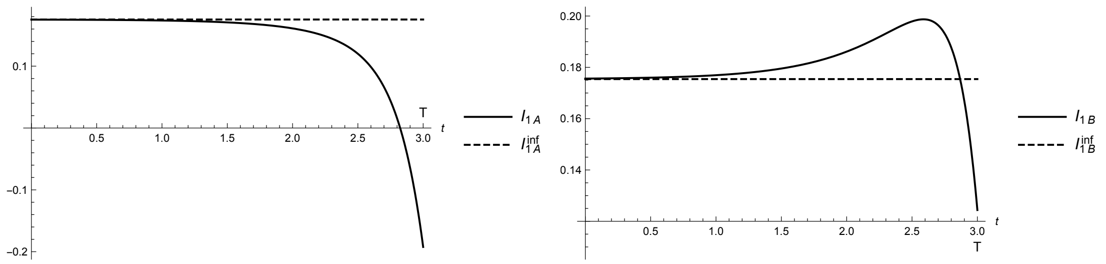
In this section, we first examine the behavior of the firms for an exogenously given product introduction time T. We then explore optimal timing and its dependence on adoption costs and initial capacities. In case of delay, we analyze how capacities evolve before introduction.

4.1 Exogenous Time Horizon

In order to depict optimal introduction time and the equilibrium investment paths, we use the following default parameter setting (similar to the parameter setting of Dawid et al. (2010))

$$r = 0.04,\ \delta = 0.2,\ \eta = 0.5,\ \theta = 0.1,\ \gamma_A = \gamma_B = 0.1, \tag{37}$$

We start by analyzing the equilibrium investment strategies $\phi_f(K, t; T)$, $f = A, B$, for a large fixed time horizon $T^L = 3$, fixed initial capacity $K^{ini} = (0.35, 0.35)$, and adoption costs $F = 1$. In Figure 2 the investment strategies $\phi_{1f}(K^{ini}, 0, t, m_1)$ in mode m_1 are depicted as functions of $t \in [0, T^L]$ The dashed line corresponds to the infinite horizon



(a) (b)

Figure 2: Optimal investments of both firms in mode m_1 at a fixed capacity $K^{ini} = (0.35, 0.35)$ for $F = 1$ and $T^L = 3$.

case in m_1. Obviously, T^L is large enough to resemble the infinite horizon investment strategy at $t = 0$. In panel (a), we see that the innovator reduces his investments as time approaches T^L which is due to the decreased marginal value of the established capacity when the innovator introduces the new product. For the non-innovator, we have an interesting investment strategy which is non-monotone in t. Note that the marginal value of its capacity is decreased in m_2 as well. Hence, eventually investments decline. The initial increase is due to the innovator's decreasing willingness to invest. Moreover, there is an intertemporal strategic effect, i.e. by increasing investment, via a higher capacity and lower price in the future, a firm can even further reduce the future investment of its competitor. As the innovator is affected on both markets by the established capacity while the non-innovator is affected only at the established market (since it is not producing product 2), the non-innovator has more influence on its competitor than the other way around.

Figure 2 is also suitable to assess the changes in investment incentives if the innovator (unexpectedly) preannounces the introduction of a new product at the capacity levels $(0.35, 0.35)$. Comparing the solid lines with the dotted ones, which correspond to investment level if no introduction of a new product is expected, we see that for the innovator, the expectation of future product introduction yields a reduction of its investment in capacities for the established product. For the non-innovator, it depends on the length T till the announced time of product introduction. For $T \lessapprox 0.15$, there is a negative effect of the preannouncement on investment, whereas for higher T, investment of firm B increases.

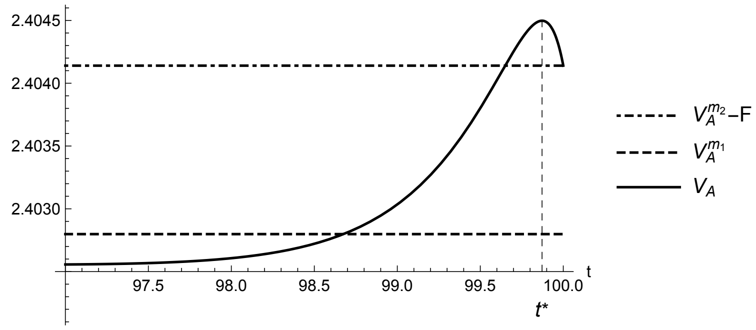
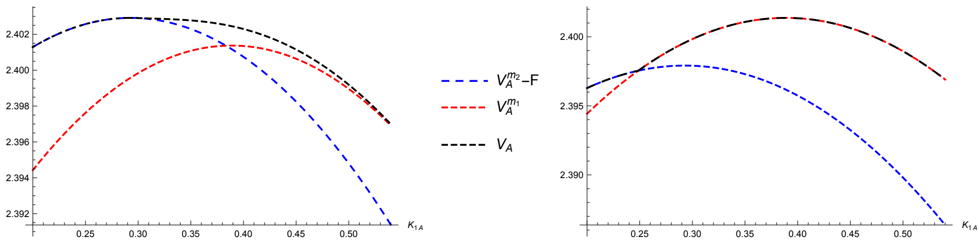


Figure 3: Value function for the innovator for $F = 2.94$, $K^{ini} = (0.35, 0.35)$ and for $T^L = 100$.



(a) $F = 2.94$.

(b) $F = 2.945$.

Figure 4: Value function for $K_{1B} = K_{1B}^{m_1,ss} \approx 0.3697$.

4.2 Endogenous Time Horizon

Employing the approach described in Section 3 and using Proposition 1, we are able to derive the optimal T to be preannounced by the innovator. In particular, we calculate the value function of firm A for a sufficiently large T^L and then determine the optimal distance to the terminal time. The approach is illustrated in Figure 3. It can be clearly seen that for the considered parameter the product is optimally introduced after a very short but positive delay of about $T^L - t^* \approx 0.12$. Furthermore, it can be seen that delaying the product introduction by more than 0.4 actually is dominated by immediate product introduction.

Using this approach we can obtain the equilibrium value of T for each pair of initial capacities and also the resulting value functions for both players. In Figure 4 we show the equilibrium value function at $t = 0$ for firm A (black line) as well as the value obtained under immediate introduction (blue line) and no introduction (red line) as a function of K_{1A}^{ini} for a given value of K_{1B}^{ini}. More precisely, we set K_{1B}^{ini} to the steady

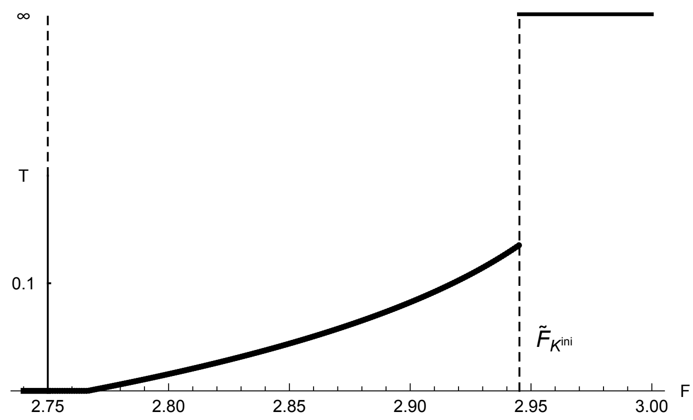


Figure 5: Optimal time to switch to m_2 ($K^{ini} = (0.35, 0.35)$).

state value of the infinite horizon game in m_1, which we denote by $K_{1B}^{m_1,ss}$. Assuming relatively low adoption costs (panel (a)) for low initial K_{1A}^{ini}, the innovator introduces immediately whereas for higher initial capacity, there is a gain by delaying the product introduction.[5] For even higher values of F not introducing becomes optimal for high capacities and hence infinite solutions for T occur. There arises an indifference point, where introducing after some delay and not introducing at all yield the same value for the innovator.[6] If adoption costs F become too high firm A either introduces immediately or never (see Figure 4(b)).

The pattern sketched above can be clearly seen in Figure 5, which shows the optimal product introduction time T as a function of adoption costs for given initial capacities. For low adoption costs the firm wants to introduce the new product immediately, whereas above some threshold $\bar{F}_{K^{ini}}$, the firm chooses to introduce after some delay. This delay is higher the higher F is. There is another threshold $\tilde{F}_{K^{ini}}$ where the innovator abstains totally from product introduction and stays with its established product. Thus, there is a jump from some finite T to infinity at this threshold. Note that the thresholds depend

[5] The value functions of immediate and no switching intersect at a point where the slopes of the value functions are very different and hence there is a kink. As usual in endogenous timing problems the option of delaying 'smoothes' the value function of firm A.

[6] In the literature, such indifference points separating different basins of attraction of the dynamics under optimal investments are called Skiba points, see e.g. Skiba (1978); Haunschmied et al. (2003). For a discussion of issues related to the existence of Skiba points under Markov Perfect Equilibria of differential games see Dawid et al. (2017a). In general, the value functions have a kink at such Skiba points since, depending on whether firm A plans to introduce the new product or not, equilibrium investments are different. This means that equilibrium investment strategies exhibit jumps at this point in the state space and accordingly the value functions exhibit a kink.

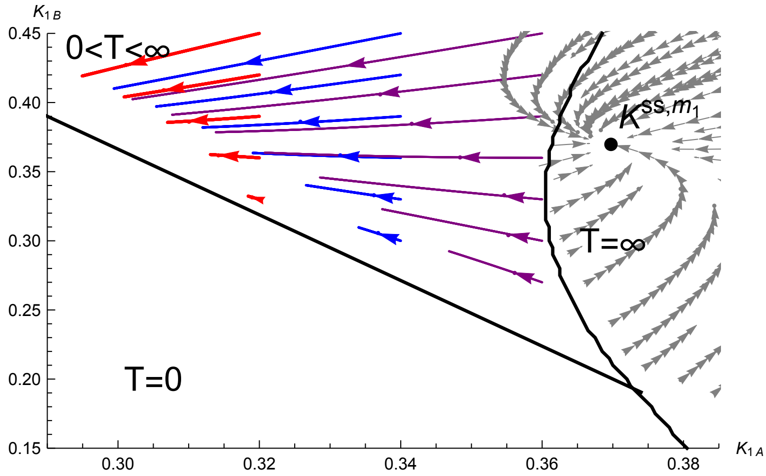


Figure 6: Optimal trajectories for different initial capacities.

on initial capacities.

A qualitative description of optimal timing for different levels of initial capacities of both firms is given in Figure 6. Each arrow in the figure depicts the equilibrium trajectory of the capacities for the corresponding initial condition taking into account the time of product introduction chosen by firm A in equilibrium. Here, the steady state of m_1 lies in the interior of the area where firm A decides not to introduce the new product (i.e. $T = \infty$). Still, as can be seen in the figure, a trajectory starting in the are where in equilibrium we have $T = \infty$ might for some time run through the area in the state space where $T < \infty$ would be chosen if the state at $t = 0$ were in this area. Clearly this is a feature of the open-loop strategy for the timing choice. Moreover, there are parameter settings where the steady state of m_1 does not lie in the corresponding area such that every trajectory starting in the $T = \infty$ area would end up in another $[0 < T < \infty]$ area where ex-post, the firm would like to introduce the product (possibly after some delay) if there were no commitment.

Furthermore, we are interested in how the optimal time of product introduction is influenced by the capacities of both firms. Regarding the capacity of the non-innovator, one might expect that if the non-innovator is stronger on the established market, the innovator has higher incentives to introduce the new product earlier in order to escape competition. But there is another effect as well, namely higher capacity of the non-innovator leads not only to a lower price of the established product but also to a lower price of the new product in m_2. In order to compensate for that, the innovator has incentives to decrease its own capacity on the established market in m_1 in order to be 'more prepared' when switching to m_2. Figure 6 suggests that the latter effect is stronger such that the stronger the competitor, the later the product introduction, i.e. T is increasing in K_{1B}. Moreover, the duration in m_1 is increasing in the innovator's capacity as well. Note that for the parameters considered here, the switching line, which separates the $0 < T < \infty$ from the $T = 0$ region, is never reached.

Another interesting observation is that for the innovator, for every initial capacity in the delaying region, it is optimal to reduce capacity whereas for the non-innovator, the dynamics of its capacity depends on initial capacities, in particular on K_{1B}. If K_{1B} is relatively low, then its capacity increases, otherwise it decreases as well. Note that the steady state value of the non-innovators capacity in m_2 is higher than in m_1. Thus, it is very natural, that the non-innovator might increase its capacity already in m_1.

Considering initial conditions with $0 < T < \infty$ it could be expected that the change in mode at T leads to a discontinuous adjustment of the investment of firm B, since at $t = T$ the there is a discontinuous change in its instantaneous profit function and it has no influence on the choice of the product introduction time T. However, the investment trajectory of the non-innovator, is continuous at all $t \geq 0$, including at $t = T$, when the new product is introduced. Intuitively, in our setting, where the switching time T is fixed and known already at $t = 0$, firm B anticipates the marginal effect of investment on profits in m_2 even before T and therefore investment incentives do not jump at $t = T$.

Finally, we like to mention that in comparison to the monopoly case where the non-innovator does not exist, which has been analyzed in Gezer (2019), we find the following interesting pattern: The innovator introduces earlier, i.e. the delay in product innovation is shorter but at the same time innovation occurs for a smaller range of costs of product introduction, i.e. for some F the innovator would innovate in monopoly but not in presence of a competitor even though the competitor is only active on the established market. Thus, we see a connection between the Schumpeterian and Arrowian perspective where market concentration facilitates innovation but decreases its speed.

5 Conclusion

In this paper, assuming commitment of the innovator with respect to the product introduction time, we have characterized how adoption costs and initial capacities for the established product influence the optimal timing of new product introduction in a dynamic duopoly market. In the interesting case of delay of product introduction, the innovator reduces capacities of the established product before the new product is introduced, whereas the dynamics of the non-innovator's capacity depends on initial capacities. Furthermore, in our setting the innovator would always like to further delay product introduction at the point in time where according to its initial commitment, the new product is brought to the market. More generally, our analysis indicates conditions for determining optimal mode transitions in multi-mode games under the assumption of open-loop determination of the transition times combined with Markov perfect equilibrium profiles within each mode. A challenging and interesting topic for future research clearly is the investigation of fully closed loop equilibria, where also the made transitions are determined by feedback strategies of one (or several) players.

Appendix

Appendix A

As derived in Lemma 1, the innovator is indifferent between waiting marginally and introducing the new product if and only if $H = rS$, which reduces to

$$\frac{1}{2\gamma_2}\Big(\frac{\partial V_A^{m_2}}{K_{2A}}\Big)^2 = rF. \tag{38}$$

Rearranging equation (38) yields the switching line

$$K_{1B} = \frac{\sqrt{2r\gamma_2 F} - H_A^{m_2} - M_A^{m_2} K_{1A}}{N_A^{m_2}}. \tag{39}$$

Appendix B

Given the terminal time T, the HJB-equations for non-stationary Markovian investment strategies are given by

$$\begin{aligned} rV_A(K_{1A}, K_{1B}, t) - \frac{\partial V_A(K_{1A}, K_{1B}, t)}{\partial t} = \max_{I_{1A}} \Big[& p_1 K_{1A} - C_1(I_{1A}) + \frac{\partial V_A}{\partial K_{1A}}(I_{1A} - \delta K_{1A}) \\ & + \frac{\partial V_A}{\partial K_{1B}}(I_{1B}^* - \delta K_{1B}) \Big] \end{aligned} \tag{40}$$

and

$$\begin{aligned} rV_B(K_{1A}, K_{1B}, t) - \frac{\partial V_B(K_{1A}, K_{1B}, t)}{\partial t} = \max_{I_{1B}} \Big[& p_1 K_{1B} - C_1(I_{1B}) + \frac{\partial V_B}{\partial K_{1A}}(I_{1A}^* - \delta K_{1A}) \\ & + \frac{\partial V_B}{\partial K_{1B}}(I_{1B} - \delta K_{1B}) \Big] \end{aligned} \tag{41}$$

with the transversality conditions

$$V_f(K_{1A}(T), K_{1B}(T), T) = V_f^{m_2}(K_{1A}(T), K_{1B}(T), T), f = A, B. \tag{42}$$

Maximizing the right hand side of the HJB-equations yields

$$I_{1f} = \frac{1}{\gamma}\frac{\partial V_f}{\partial K_{1f}}, \quad f = A, B. \tag{43}$$

Additionally, firm A has to select the optimal value for T maximizing its discounted stream of profits. Due to the linear-quadratic structure of the game, we impose the following form for the value function:

$$V_f = C_f(t) + D_f(t)K_{1A} + E_f(t)K_{1A}^2 + F_f(t)K_{1B} + G_f(t)K_{1B}^2 + L_f(t)K_{1A}K_{1B},\ f = A, B. \tag{44}$$

Due to the finite time horizon, we consider non-stationary Markovian strategies and hence coefficients depend on time. Comparison of coefficients yields the following system of 12 riccati differential equations which are solved by standard numerical methods:

$$\begin{aligned}
rC_A(t) &= \frac{D_A(t)^2 + 2F_A(t)F_B(t) + 2\gamma_1 C_A'(t)}{2\gamma_1}\\
rD_A(t) &= \frac{\gamma_1 + D_A(t)(-\gamma_1\delta_1 + 2E_A(t)) + F_B(t)L_A(t) + F_A(t)L_B(t) + \gamma_1 D_A'(t)}{\gamma_1}\\
rE_A(t) &= \frac{2E_A(t)(-\gamma_1\delta_1 + E_A(t)) + L_A(t)L_B(t))}{\gamma_1} - 1 + E_A'(t)\\
rF_A(t) &= \frac{2F_B(t)G_A(t) + F_A(t)(-\gamma_1\delta_1 + 2G_B(t)) + D_A(t)L_A(t) + \gamma_1 F_A'(t))}{\gamma_1}\\
rG_A(t) &= \frac{G_A(t)(-4\gamma_1\delta_1 + 8G_B(t)) + L_A(t)^2 + 2\gamma_1 G_A'(t))}{2\gamma_1}\\
rL_A(t) &= \frac{2(-\gamma_1\delta_1 + E_A(t) + G_B(t))L_A(t) + 2G_A(t)L_B(t) + \gamma_1(-1 + L_A'(t))}{\gamma_1}\\
rC_B(t) &= \frac{2D_A(t)D_B(t) + F_B(t)^2 + 2\gamma_1 C_B'(t)}{2\gamma_1}\\
rD_B(t) &= \frac{D_B(t)(-\gamma_1\delta_1 + 2E_A(t)) + 2D_A(t)E_B(t) + F_B(t)L_B(t) + \gamma_1 D_B'(t)}{\gamma_1}\\
rE_B(t) &= \frac{(-4\gamma_1\delta_1 + 8E_A(t))E_B(t) + L_B(t)^2 + 2\gamma_1 E_B'(t))}{2\gamma_1}\\
rF_B(t) &= \frac{\gamma_1 + F_B(t)(-\gamma_1\delta_1 + 2G_B(t)) + D_B(t)L_A(t) + D_A(t)L_B(t) + \gamma_1 F_B'(t)}{\gamma_1}\\
rG_B(t) &= \frac{2G_B(t)(-\gamma_1\delta_1 + G_B(t)) + L_A(t)L_B(t))}{\gamma_1} - 1 + G_B'(t)\\
rL_B(t) &= \frac{2E_B(t)L_A(t) + 2(-\gamma_1\delta_1 + E_A(t) + G_B(t))L_B(t) + \gamma_1(-1 + L_B'(t))}{\gamma_1}
\end{aligned} \tag{45}$$

with transversality conditions $C_f(T) = C_f^{m_2}, D_f(T) = D_f^{m_2}, E_f(T) = E_f^{m_2}, F_f(T) = F_f^{m_2}, G_f(T) = G_f^{m_2}, L_f(T) = L_f^{m_2}, f = A, B$.

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