Dynamic Investment Strategies and Leadership in Product Innovation *

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August 2020

Abstract

We study the inter-temporally optimal innovation strategies of incumbent manufacturing firms that compete in an established market and can extend their product line through product innovation. Firms invest in production capacity and R&D knowledge stock, where the R&D knowledge stock and the current R&D investment determine the hazard rate of innovation. Our findings show that the firms' optimal R&D strategies are driven by a subtle interplay between the relative positions of their R&D knowledge stocks and their current relative positions on the established market. First, we find that under symmetric investment costs the knowledge leader should spend more on R&D than the knowledge laggard only if it has a substantially smaller market share on the established market. If the knowledge leader's market share is sufficiently large, its optimal investment in R&D is so small that its innovation rate is lower than the knowledge laggard's. Second, optimal investment in R&D knowledge is negatively affected by the opponent's production capacity on the established market if the competitor has not innovated yet. However, we find that this effect is reversed after the competitor has successfully introduced the new product on the submarket. Third, the manufacturing firm with higher costs of adjusting production capacity for the established product has a higher incentive to engage in product innovation and might even achieve a higher total discounted profit than its more efficient competitor.

Key Words: Product Innovation Strategy, Dynamic Competition, Capacity Investment, Markov Perfect Equilibrium

*The authors thank Jürgen Mihm, a Senior Editor, and three anonymous referees for providing thoughtful comments that improved the paper considerably. Dawid, Kopel and Kort gratefully acknowledge financial support from the German Research Foundation (DFG) through the Collaborative Research Centre 1283 "Taming uncertainty and profiting from randomness and low regularity in analysis, stochastics and their application". Keoula has been supported by German Science Foundation (DFG) under grant DA 763/4-1. An earlier version of this paper was circulated in SSRN as Bielefeld Working Papers in Economics and Management No. 03-2017.

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1 Introduction

A considerable fraction of product innovations in related submarkets is accomplished by established incumbents (e.g. Chandy and Tellis (2000), Sood and Tellis (2011), King and Tucci (2002), Franco et al. (2009), Buensdorf (2016)). This observation raises the question how an incumbent's optimal strategy in an R&D race depends on its strength on its established market. To illustrate, in 2010 when Apple and Samsung introduced Tablet PCs and thus created a new submarket that coexisted with the established market of portable computers, they had relatively small market shares on the Laptop market (3.4% and 2.8% respectively) compared to Hewlett Packard and Dell (18% and 12% respectively). Interestingly, HP and Dell entered the Tablet market much later in 2013.[1] In the market for smartphones, Nokia, as the clear market leader in the early 2000s (market share 2005: 32.5%), introduced its first touchscreen phone in 2011, while its initially smaller competitor Samsung (market share 2005: 12.7%) introduced its first smartphone with a touchscreen in 2008. As a result, Samsung achieved a higher market share in 2012 and also exhibited strongly positive dynamics of profit in the smartphone market compared to Nokia.

Why do dominant incumbent firms leave emerging innovative submarkets to their competitors? The operations management literature provides several explanations for an observed pattern of *action-reaction* where big incumbents are often slower than smaller rivals (or entrants) in pioneering newly created submarkets. First, Christensen (1997) argues with his Innovator's Dilemma theory that big incumbent firms rely on a dominant managerial logic, organizational capabilities and cognitive frames, which lead these firms to miss new opportunities sometimes even after investing large sums (see also, e.g., Henderson (1993, 2006), Tripsas (1997), Tripsas and Gavetti (2000)).

Second, incumbents underinvest in the development of new technologies or products due to the fear of cannibalizing their existing businesses. However, Nault and Vandenbosch (1996) argue that incumbent firms have to 'eat their own lunches before somebody else does'. Third, large firms lack the potential of small firms to motivate their engineering staff. The associated bureaucracy and incentive effects lead to lower innovation intensity (e.g. Zenger (1994)). Fourth and finally, incumbent firms are less successful in innovation since there are diseconomies of scope which arise from key assets that have to be shared across businesses (Bresnahan et al. (2012)).

In contrast to these explanations that focus on the suboptimality of firms' decisions, our paper provides a rational argument for why the innovator's dilemma happens in certain markets. We in fact show that large incumbents might enter a new market relatively late as a result of intertemporally optimal behavior of all firms in the market (see also Pacheco-de-Almeida (2010), Swinney et al. (2011)).

However, smaller incumbent firms do not always leapfrog their dominant rivals. Frequently, *increasing dominance* emerges and dominant firms introduce their new products earlier than their competitors. For example, as noted by Chandy and Tellis (2000), Hattori-Seiko – which was a dominant producer of mechanical watches at the time – was the first firm to introduce the analog quartz watch. Likewise the Hamilton Company, another incumbent firm, was an early mover in the digital quartz watch submarket. So, what drives dominant manufacturing firms to stay in the leadership position? As an explanation why some dominant firms invest aggressively in innovation while others do not, Chandy et al. (2003) argue that some

[1] HP made a short - but unsuccessful - premature attempt of offering Tablet PCs in 2010.

managers might have different expectations about a new technology's impact on existing products than other managers. In contrast, our paper also provides a rational explanation for the outcome of increasing dominance.

In particular, we employ a dynamic duopoly model in which manufacturing firms compete in an established market and can extend their product line through product innovation by investing in R&D.[2] Firms invest in production capacity and R&D knowledge stock, where the R&D knowledge stock jointly with the firm's current R&D investment determines the hazard rate of innovation. After a firm has successfully innovated, it is active on the established and the new market. We capture the strength of a firm on the established market by the firm's product capacity for the established product and its capability to successfully develop a new product by its R&D knowledge stock. We find that the occurrence of an action-reaction pattern or an increasing dominance pattern not only depends on the strength on the established product market, but that instead it is driven by an interplay between the firms' relative positions of their production capacities for the established product and their relative positions of their current R&D knowledge stocks.

The main contribution of this paper is that it provides new insights into the interplay between firms' production capacities and their R&D investments under dynamic duopoly competition. First, we consider a market where manufacturing firms are symmetric except for their production capacities on the established market. We find that the knowledge leader invests more in R&D than its competitor only if it has a substantially smaller market share on the established market. In contrast, if the knowledge leader's market share is sufficiently large, it invests so little in R&D that its innovation rate is lower than the knowledge laggard's. Intuitively, the fear of cannibalization arising after the new product introduction reduces the R&D incentives of a knowledge leader with a large market share. Hence, whether the knowledge leader should invest more or less in R&D than its competitor depends on the relative strengths of the two firms on the established market. In our setting, both patterns of R&D behavior, increasing dominance of the knowledge leader or leapfrogging by the knowledge laggard, can occur under the same mode of competition. This is in contrast to the existing literature, which highlights that the particular pattern strongly depends on whether firms compete in quantities or prices (see Vickers (1986)).

Second, we consider a market where one manufacturing firm has higher investment costs than its rival. We find that the firm with the cost *disadvantage* on the established market invests more in building up knowledge for product innovation than its competitor and, therefore, expects to innovate faster. It might even turn out that this firm can actually have a higher expected discounted profit stream than its more cost-efficient opponent. Intuitively, the cost disadvantage acts as a commitment device for the less efficient firm to innovate faster.

Third, we exploit the opportunity offered by our dynamic setup to study a manufacturing firm's optimal product innovation strategy in the period *before* the innovation, where competition only occurs on the established market, and the period *after* the competitor has created a new submarket with its product innovation. We find that there are significant qualitative differences in the optimal innovation strategy. Innovation activity is negatively affected by the opponent's production capacity on the established market if the opponent has not innovated yet. However, this influence is reversed *after* the opponent has successfully

[2] The monopoly version of this model has been analyzed in Dawid et al. (2015).

innovated. Intuitively, as long as a firm has a chance to become the sole manufacturer of the new product, a large production capacity of the competitor on the established market reduces the profitability of the innovation. As soon as the competitor is already active on the new market, an increase of its production capacity on the established market reduces its activity on the new market, which makes this market more profitable for the focal firm. Furthermore, the optimal level of a firm's investment in R&D exhibits a downward jump at the time when the opponent creates the new submarket, since this precludes the firm from ever becoming the sole producer of the new product. Hence, our paper extends the literature by showing that the properties of a firm's optimal R&D strategy are crucially affected by the competitor's product range.

A methodological contribution of our paper is that it develops a modeling approach which captures dynamic strategic interaction in a market environment characterized by the interplay of continuously evolving state variables (production capacities, R&D stocks) and endogenous discrete changes which are governed by arrival rates depending on knowledge stocks (i.e. state variables) and R&D investments (i.e. control variables). Such situations occur frequently in real-world markets, for example, if suppliers invest in capabilities to subsequently encroach in the downstream manufacturer's retail market or if manufacturing firms invest in process R&D and in case of success have access to better technologies. Characterizing Markov Perfect Equilibria in such games requires the solution of a system of coupled Hamilton-Jacobi-Bellman equations. We put forward a numerical procedure based on sparse grid methods, which allows us to obtain (approximate) solutions to such a model even for relatively high dimensions of the state space. The application of such a method to managerial strategy analysis is new to the literature and highlights the potential of our approach for applications in the field of production and operations management.

The paper is organized as follows. The next section gives an overview of the related literature. Section 3 introduces the model and details its assumptions. Section 4 provides a formal characterization of the equilibrium investment strategies. Section 5 presents our main insights about the properties of firms' optimal R&D strategies. In Section 6 we analyze the effect of competition on R&D investments. In Section 7 we investigate the robustness of our results and show that our qualitative insights remain valid for a large range of all our model parameter values. Section 8 concludes and discusses some possible extensions of our analysis. Appendix A contains the proof of Proposition 1 and Online Appendices provide a detailed description of our numerical method (Appendix B) and additional robustness tests (Appendix C).

2 Literature

A rich stream of literature has addressed the question why leading incumbents often are slower to enter newly created submarkets than their smaller rivals or entrants. Game-theoretic analyses of this issue, however, have to a large extent relied on static models (see e.g. Schmidt and Porteus (2000a,b), Druehl and Schmidt (2008), Huang and Sosic (2010)). We complement this literature by considering a dynamic game theoretic setting in which firms' strengths on the existing market as well as the hazard rates of the innovations are endogenously determined by the firms' investment strategies.

A paper close to ours is Dawid et al. (2013). They study a static game-theoretic model, and their main insight is that the larger incumbent has less incentive to innovate. In our genuinely dynamic model, we confirm their result in the special case when the firms start out with identical knowledge stocks. In addition, we study situations where firms differ in their initial knowledge stocks which allows us to identify situations where the larger firm invests either less or more in R&D and, depending also on its knowledge stock, has a lower or higher probability to be the first innovator. Jointly considering the firms' production capacities on the established market and their relative positions in R&D knowledge stock enables us to pin down the drivers for action-reaction or increasing dominance in more detail. In addition to Dawid et al. (2013), we further investigate the effect of asymmetric investment costs. We find that under certain conditions the less efficient firm has a higher expected discounted profit stream. Moreover, our dynamic framework allows us to investigate situations before and after the competitor innovates and identify managerial implications for a firm's innovation strategy contingent on the competitor's product range.

The literature so far has not considered the determinants of dynamic *product innovation strategies* of incumbent firms which intend to extend their product range.[3] For example, work on dynamic R&D competition between incumbents has focused on the dependence of a firm's R&D investment on its (relative) level of technology (e.g. Aghion et al. (2001), Canbolat et al. (2012), Grishagin et al. (2001)). In these models, a firm typically carries out R&D in order to improve its technology, which determines its current profit and also the expected return from innovation.

Additionally, patent race models with *exogenously* given value of innovation (see Reinganum (1989)) have studied the dependence of R&D investment on different factors. Within a framework of an n-firm race, Canbolat et al. (2012) show how the set of firms active in the race and their (one-time) spending on the innovation project depends on their technological and marketing efficiency. Extending memoryless patent race models, a variety of contributions have analyzed models with multiple innovation stages (e.g. Fudenberg et al. (1983), Harris and Vickers (1985), Grossman and Shapiro (1987)). Concerning the R&D leader's and the R&D follower's innovation incentives, it has been analyzed if the current R&D leader will become increasingly dominant or if R&D leadership will change in a process of action-reaction. A general insight from these contributions is that firms invest most in R&D when they are neck-and-neck with their competitors and that R&D laggards trailing the R&D leader by too many steps have incentives to drop out of the race. Hence, multi-stage patent race models tend to predict increasing dominance.

Another paper close to ours is Doraszelski (2003). Like us, he considers a single-step race between two firms, in which the innovation rate of a firm depends both on current R&D effort and its R&D knowledge stock accumulated through past R&D activities. A main difference however is that Doraszelski (2003) analyzes a patent race with *exogenously* given innovation value, whereas in our paper the innovation payoff is *endogenously* determined. Intuitively, in our setting the innovator's payoff depends negatively on the R&D knowledge stock of the R&D laggard since the larger the R&D laggard's knowledge stock, the sooner the laggard will catch up with the innovator. This reduces the innovator's value. The implication is that our results crucially differ from Doraszelski (2003) in two important ways. *First*, due to the negative dependence of the innovator's value and the R&D laggard's knowledge stock, we find a negative relationship between R&D investment and the competitor's R&D knowledge stock. In contrast, Doraszelski (2003) obtains that

[3] An exception is Igami (2017), who considers a dynamic industry model in which incumbents take binary decisions to extend their product range. Differently from our setup, this model captures neither the uncertainty of the innovation process nor the dynamic adjustment of R&D knowledge and production capacity levels.

the effect of an increase of the opponent's knowledge stock on a firm's R&D investments is positive if either the own knowledge stock or the payoff of winning the race is sufficiently large. As a *second* important difference, in our dynamic model both patterns – action-reaction *and* increasing dominance – can result from optimal R&D strategies. Consequently, we are able to identify both patterns of innovation behavior observed in real-world markets within the same model setting by highlighting the crucial role of the firms' relative positions in terms of their R&D knowledge stocks and their relative strengths on the established product market. Quite in contrast, Doraszelski (2003) finds that as long as the hazard rate depends in a (weakly) concave way on the knowledge stock (which also holds in our case), the laggard always invests more in R&D than the knowledge leader and an action-reaction scenario arises.

Dynamic innovation models, in which the value of innovation is endogenously determined by some form of market competition have been considered e.g. by Hörner (2004) and Ludkovski and Sircar (2016).[4] In these models, the flow profits of firms depend on their (relative) number of successful innovations, where innovation probabilities are determined by current R&D effort. Hörner (2004) analyzes a setting in which only the sign but not the size of the difference in the number of successful innovation steps of the two competitors determines their profits. He demonstrates that investment is highest in situations in which the gap between firms is large rather than when competitors are neck-and-neck. Ludkovski and Sircar (2016) consider a setting with Cournot competition where a successful innovator improves on its own previous technology. They show that in this setting increasing dominance occurs. Our setup shares with this literature that the value of innovation for the firms is determined endogenously. However, we combine manufacturing firms' investments in production capacities in a Cournot setting with a race for obtaining a product innovation. This allows us to disentangle the effect of the relative position on the established market and the relative position in the innovation race and enables us to characterize the occurrence of increasing dominance versus action-reaction in terms of the interplay of these forces.

Our paper also builds on the literature on capital accumulation games (e.g. Jun and Vives (2004), Reynolds (1991)), in which capacity investments of single-product firms engaged in oligopolistic competition have been characterized both in the framework of Open-Loop and Markov Perfect Equilibria. Besanko and Doraszelski (2004) introduce a capacity accumulation game in discrete time to characterize an evolving industry structure and to explain the persistence of differences in firm size (cf. also Doraszelski and Pakes (2007), Escobar (2013)). Besanko et al. (2010) study capacity accumulation patterns in a discrete-time dynamic duopoly game with strategic uncertainty (about the rival's cost). This literature, however, has not dealt with investments in capacity of multi-product firms. Furthermore, we extend this literature by analyzing the interplay of dynamic investments in production capacities with the firms' incentives to invest in R&D to enlarge their product range.[5]

With respect to the numerical method, our paper builds on work using a Chebyshev collocation method for the determination of Markov Perfect Equilibria of differential games (e.g. Vedenov and Miranda (2001), Doraszelski (2003), Dawid et al. (2017)). Sparse-grids (Smolyak bases) have been employed for the solution of large dynamic equilibrium models in macroeconomics (see Maliar and Maliar (2014) and the survey in

[4]See Vickers (1986) for a seminal contribution to this stream of literature.

[5]Dynamic models of product introduction strategies of capacity-constrained firms have for example been analyzed by Bilginer and Erhun (2015).

Fernandez-Villaverde et al. (2016)). To our knowledge, the present paper is the first application of a sparse grid method to an Operations Management problem with dynamic strategic interaction. In particular, we are the first to provide a numerical characterization of a Markov Perfect Equilibrium in a multi-mode differential game with a high-dimensional state space. Since differential games with evolving structure are capable of capturing a variety of applications in management science and operations management, we believe that our paper is of interest to scholars beyond the area of firm strategy and innovation.

3 The Model

We consider a dynamic duopoly in continuous time with evolving market structure.[6] Initially, two incumbent manufacturing firms A and B produce a homogeneous product called product 1. We refer to product 1 as the established product. Both firms invest in the accumulation of an R&D knowledge stock which is essential to develop a new and differentiated product, called product 2. Let τ_A and τ_B be the stochastic completion times of the firms' R&D projects. Once one of the firms has innovated, a new submarket can be created.[7] At this stage, only the innovator can sell the established product and the new product. The innovation laggard is selling the established product while simultaneously investing in R&D knowledge stock to eventually enter the new submarket as well. Once both firms have innovated, the established product 1 and the new product 2 are supplied by both firms. In the sequel, we refer to both firm A and B by subscript f $(f = A, B)$ but keep the distinction whenever it is necessary.

To enable production, firms A and B need production capacities. These capacities are denoted by $K_{if}(t), f = A, B$, where subscript $i = 1, 2$ refers to product i. In the period prior to the creation of the new submarket, called mode m_1, both firms invest in their production capacity $K_{1f}(t)$ for product 1. Additionally, they also invest in the accumulation of their firm-specific R&D knowledge stock $K_{Rf}(t)$. In the period after the new submarket has been created, the innovator (i.e. the firm that has first completed the R&D project) can invest in production capacities for both products. The innovation laggard which has not innovated yet continues to invest in production capacity for product 1 and in its R&D knowledge stock. We will denote the scenario where firm A (firm B) innovates first mode m_2 (mode m_3). In the last phase or mode m_4, which emerges after the new product has also been launched by the innovation laggard, firms compete on both markets and (dis)invest in both production capacities.

At each point in time t, the status of firm $f = A, B$ is characterized by production capacities and the R&D knowledge stock $(K_{1f}(t), K_{2f}(t), K_{Rf}(t))$ and the mode of the game $m(t) \in M = \{m_1, m_2, m_3, m_4\}$, that captures which of the firms has already innovated. The state of the game consists of the three stock variables of both firms and is therefore six-dimensional. Production capacities and R&D knowledge stocks

[6]Formulating the game in continuous time implies that no scenarios with simultaneous innovations by both firms within one time period have to be considered, thereby reducing the computational burden of the analysis, see also Doraszelski and Judd (2012).

[7]The submarket is actually created once the innovator invests in production capacity for the new product and hence this product is manufactured and sold on the market.

accumulate according to

$$\dot{K}_{if}(t) = I_{if}(t) - \delta_i K_{if}(t) \qquad\qquad i = 1, 2, R, \quad f = A, B, \tag{1}$$

where $I_{if}(t)$ is the investment of firm f in stock K_{if} at time t. These standard dynamics account for the fact that accumulation of production capacities and stock of knowledge take time, but also that depreciation of production capacities takes place where $\delta_i > 0, i = 1, 2$ denote the (symmetric across firms) depreciation rates (see, e.g., Besanko et al. (2010) and Dierickx and Cool (1989)). With regard to the depreciation of R&D knowledge stock, organizational forgetting (see Doraszelski (2003) and references therein) is captured by $\delta_R > 0$.[8]

Concerning production capacities, we allow the firms to intentionally disinvest, i.e. $I_{if} \in \mathbb{R}$. With regard to the R&D knowledge stocks, we make the sensible assumption that knowledge investments are non-negative, i.e. $I_{Rf} \geq 0$. The firms cannot invest in production capacity of the second product before the R&D project has been completed, which implies that $I_{2f}(t) = 0$ for all $t < \tau_f$.

Furthermore, all stocks have to be non-negative:

$$K_{if}(t) \geq 0 \;\; \forall t \geq 0 \;\; i = 1, 2, R, f = A, B. \tag{2}$$

The probability that firm f successfully innovates is determined by a firm's hazard rate. The hazard rate in mode m_1 is positively affected by the current investment $I_{Rf}(t)$ in the R&D knowledge stock, as well as by accumulated knowledge through past R&D investments captured by the firm's R&D knowledge stock $K_{Rf}(t)$ itself. We employ an additive form of the hazard rate (see Doraszelski (2003)) given by

$$\lambda(I_{Rf}(t), K_{Rf}(t)) = \alpha I_{Rf}(t) + \beta(K_{Rf}(t))^{\psi}, \;\; \alpha \geq 0, \;\; \beta \geq 0, \;\; \psi > 0, \quad f = (A, B), \tag{3}$$

which captures the described determinants of the hazard rate in the most simple way. The parameters α and β determine, respectively, the marginal impact of the current investment and the accumulated knowledge on the hazard rate. In what follows, we will focus on scenarios where $\psi = 1$, i.e. where the firm's hazard rate is linear in its R&D knowledge stock. In Section 7 we check the robustness of our findings also for concave ($\psi < 1$) and convex ($\psi > 1$) hazard rates. The hazard rate in modes m_2 or m_3, where the competitor has already innovated, is given by $\omega\lambda(I_{Rf}(t), K_{Rf}(t))$ with $\omega > 0$. In the main part of our analysis, we assume that the hazard rate does not change between modes and set $\omega = 1$. However, in our robustness section we also consider a scenario where innovation by the laggard firm is impeded by patents granted (or other forms of intellectual property protection) to the innovator ($\omega < 1$). Additionally, we study the case where due to imitation effects the product innovation of a laggard firm becomes easier once the new product has already been introduced by the competitor ($\omega > 1$).

Formally, the changes between the modes of the game are described by a Markov process $m(t)$ on the

[8] These standard dynamics are also used in empirical work to obtain estimates for the depreciation rates of physical capital stock, e.g. Nadiri and Prucha (1996), as well as of R&D capital stock, e.g. Li and Hall (2020).

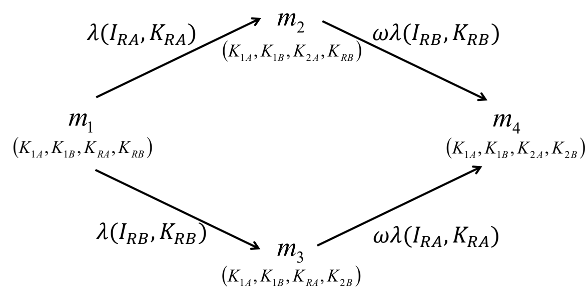


Figure 1: The transition rates between the different modes of the game. Below each mode we indicate in brackets the vector of relevant state variables in that mode.

set of modes M where the transition rates are given by

$$\lim_{\Delta \to 0} \frac{1}{\Delta} Prob\{m(t+\Delta) = m_j \mid m(t) = m_i\} = \begin{cases} \lambda(I_{RA}, K_{RA}) & (i,j) = (1,2), \\ \lambda(I_{RB}, K_{RB}) & (i,j) = (1,3), \\ \omega\lambda(I_{RA}, K_{RA}) & (i,j) = (3,4), \\ \omega\lambda(I_{RB}, K_{RB}) & (i,j) = (2,4), \\ 0 & \text{else.} \end{cases} \tag{4}$$

This formulation embodies the idea that before the creation of the new submarket (i.e. in mode m_1), there are positive probabilities of transition either to mode m_2 with firm A as innovator or to mode m_3 with firm B as innovator. From modes m_2 or m_3, the transition results in a switch to mode m_4, where both firms offer both products. The expected time of this final transition depends on the hazard rate of the innovation laggard. Once both firms offer both products, no more transitions are possible. We illustrate the possible transitions between the different modes as well as the state variables that are relevant in each mode in Figure 1. In each mode only four out of the six states are relevant. The reason is that before a firm has innovated, it cannot invest in production capacity of the new product and therefore $K_{2f} = 0$. After firm f has innovated, both production capacities (K_{1f}, K_{2f}) are relevant, but the R&D knowledge stock K_{Rf} is not since no further innovation is possible.

At any point in time t, firms compete in quantities, where it is assumed that current production capacities for the two products are always fully used. This assumption is commonly made in the literature on capacity-constrained oligopoly competition (e.g. Anand and Girotra (2007), Goyal and Netessine (2007) and Huisman and Kort (2015)). For example, Goyal and Netessine (2007) argue that firms may find it difficult to produce below capacity due to fixed costs associated with, for example, labor inputs, commitments to suppliers, or production ramp-up. Given this assumption, prices are given by the linear inverse demand system (see e.g.

Lus and Muriel (2009)):[9]

$$p_1(t) = 1 - (K_{1A}(t) + K_{1B}(t)) - \eta(K_{2A}(t) + K_{2B}(t)) \tag{5}$$

$$p_2(t) = 1 + \theta - \eta(K_{1A}(t) + K_{1B}(t)) - (K_{2A}(t) + K_{2B}(t)). \tag{6}$$

In this setting, the parameter η ($-1 < \eta < 1$) determines the degree of horizontal differentiation between the established product and the new product. In line with our main research questions we restrict attention to scenarios where the new product is a partial substitute of the established product, corresponding to $0 < \eta < 1$. Accordingly, a firm's dynamic strategy of building up capacities for the established product and the new product is inextricably linked through its impact on the prices of the products on the two markets. The parameter θ determines the degree of vertical differentiation and measures the difference in quality between the new product and the established product. The assumption that product 2 is at least of the same quality as product 1 translates into $\theta \geq 0$.

Investment costs are assumed to have the linear-quadratic form

$$\Gamma_{if}(I_i(t)) = \mu_i I_{if}(t) + \frac{\gamma_{if}}{2} I_{if}(t)^2 \quad i = 1, 2, R. \tag{7}$$

For products $i = 1, 2$ the parameter μ_i represents the unit price of capacity and $\gamma_{if} > 0$ the adjustment cost parameter for firm $f = A, B$.[10] Increasing R&D knowledge stock is associated with a convex cost function, i.e. $\gamma_{Rf} > 0$, in line with standard arguments that building up knowledge takes time and therefore fast R&D stock accumulation is more costly compared to slower accumulation. In our default setting, all cost parameters are symmetric across firms. Asymmetry between manufacturing firms arises due to heterogeneity of the initial production capacity on the established market. However, in order to be able to study the impact of structural (dis-)advantages of a firm on a certain market, in Section 5.2 we further consider asymmetric adjustment costs between firms. For simplicity, marginal production costs are normalized to zero.

Firms choose their investment strategies in order to maximize their expected infinite horizon discounted profit stream. Formally, we have

$$\begin{aligned} J_f = \quad & \mathbb{E}\left\{\int_0^\infty e^{-rt}\left[(1 - (K_{1A} + K_{1B}) - \eta(K_{2A} + K_{2B}))K_{1f}\right.\right. \\ & +(1 + \theta - \eta(K_{1A} + K_{1B}) - (K_{2A} + K_{2B}))K_{2f} \\ & \left.\left. -\mu_1 I_{1f} - \tfrac{\gamma_{1f}}{2} I_{1f}^2 - \mu_2 I_{2f} - \tfrac{\gamma_{2f}}{2} I_{2f}^2 - \mu_R I_{Rf} - \tfrac{\gamma_{Rf}}{2} I_{Rf}^2\right] dt\right\}, \end{aligned} \tag{8}$$

where the expectation is taken with respect to the mode dynamics. The first two lines capture the instantaneous sales revenue for the established product and the new product. The third line contains the current

[9] This demand system does not take into account product diffusion as in, e.g., Bilginer and Erhun (2015) and Balakrishnan and Pathak (2014). This means that in our setting, willingness to pay for the new product is already high from the moment the product is launched. There are examples for that, like tablets and smartphones. Still, in our model, the speed of market development is restricted by the capacity build-up.

[10] We abstract from potential differences between sale and resale price of capital. However, due to adjustment costs returns from selling are lower than costs of buying. Furthermore, we assume adjustment costs to be the same for increasing or decreasing capacity, whereas they could differ substantially in reality. However, it should be noted that disinvestment hardly plays any role in our analysis; see, e.g., Figure 2(b) in Section 5.

costs of investment in production capacity and R&D knowledge stock. It should be noted that in modes where a firm has not introduced the new product yet, both investment and production capacity for that product are zero, such that the corresponding terms vanish in the instantaneous profit function.

This gives rise to a piecewise deterministic differential game with objective functions (8), state dynamics (1) and mode dynamics (4). Since a firm can build up production capacity for the new product only after the new submarket has been created and it has added the new product to its product line, the following constraints hold in the different modes:

$$
\begin{aligned}
I_{2f}(t) &= 0,\ \forall t \text{ s.t. } m(t) = m_1, \quad f = A, B \\
I_{2B}(t) &= 0,\ \forall t \text{ s.t. } m(t) = m_2, \\
I_{2A}(t) &= 0,\ \forall t \text{ s.t. } m(t) = m_3.
\end{aligned}
$$

Non-negativity constraints for production capacities, the R&D knowledge stock, and the investments in R&D knowledge stock have to be satisfied. To study how the anticipation of the emergence of a new submarket impacts the firms' current R&D investments, we assume that the game starts before the new submarket has been created. That is, $m(0) = m_1$ and the initial values of production capacities and the R&D knowledge stock are given by $K_{1f}(0) = K_{1f}^{ini},\ K_{2f}(0) = 0,\ K_{Rf}(0) = K_{Rf}^{ini}$.

4 Dynamic Investment Strategies

In order to analyze optimal strategies for investing in production capacities and R&D knowledge stocks, we consider stationary Markov Perfect Equilibria (MPE) of the game described in the previous section.[11] A stationary Markovian strategy of firm f is given by a triple $(\phi_{1f}, \phi_{Rf}, \phi_{2f})$ such that each of the feedback strategies ϕ_{if} and ϕ_{Rf} describe the optimal dynamic investment for accumulating production capacity and R&D knowledge, respectively, as a function of the states and the current mode of the game. Stationary strategies however do not explicitly depend on time.[12] More precisely, each of these feedback strategies has the form $\phi_{if} : [0,1]^2 \times [0, \bar{K}_R]^2 \times [0, 1+\theta]^2 \times M \mapsto \mathbb{R}$ for $i \in \{1, 2\}$ and $\phi_{Rf} : [0,1]^2 \times [0, \bar{K}_R]^2 \times [0, 1+\theta]^2 \times M \mapsto \mathbb{R}_0^+$. The upper bound $\bar{K}_R$ of the R&D knowledge stock is assumed to be sufficiently large to ensure that the stable steady states characterized in the following analysis are interior. Using strategies ϕ_{if}, $i = 1, 2, R$, firm $f = A, B$ at each point in time invests $I_{if}(t) = \phi_{if}(K_{1A}(t), K_{1B}(t), K_{2A}(t), K_{2B}(t), K_{RA}(t), K_{RB}(t), m(t))$.

Although formally the feedback strategies have the general form with six states and one mode as arguments, some arguments are irrelevant in some modes (see Figure 1). To ease notation, in what follows we therefore drop all irrelevant arguments in the corresponding modes and write the feedback strategies in each mode only as functions of the four relevant states. As a notational convention, the first two arguments

[11] For other applications of this equilibrium concept for the analysis of optimal firm strategies in Operations Management problems, see Dockner and Fruchter (2014), Chevalier-Roignant et al. (2019) or Huberts et al (2019).

[12] Since under the considered stationary Markovian strategies investment at t depends on the mode $m(t)$, which follows a stochastic process, the actual investment at time t for a given state (K_{1f}, K_{Rf}, K_{2f}), $f = A, B$ is stochastic. Since the probability that $m(t) = m_i, i = 1, 2, 3$ changes with t, from an ex-ante perspective, i.e. based on the information available at $t = 0$, the distribution of investment at t therefore changes with time t although the considered strategies are stationary.

in the feedback functions of both firms always refer to the production capacities on the established market, followed by the second relevant state variables of firm A and firm B, which differ across modes. The vector of the relevant states in mode m is denoted by $\vec{K}^m$. Furthermore, due to the investment constraints in the different modes, $\phi_{2f} = 0$ has to hold in mode m_1 for both firms $f = A, B$, since no investment in production capacity of the new product is possible before the product innovation is successful. For the non-innovator the same holds in modes m_2 or m_3. Additionally, we have $\phi_{RA} = 0$ ($\phi_{RB} = 0$) in mode m_2 (m_3) and $\phi_{Rf} = 0, f = A, B$, in m_4 since no more innovations are possible. In accordance with the literature (see Dockner et al. (2000)) we only consider non-anticipating strategies, i.e. strategies where firms cannot condition their action on realizations of the time of mode transitions which lie in the future.

A Markov Perfect Equilibrium of the game is a profile of stationary Markovian strategies, where each manufacturing firm uses a strategy that maximizes expected profit given the strategy of the opponent. The following proposition characterizes the firms' optimal investment strategies. Appendix A contains the proof of the proposition and provides the explicit formulations of the Hamilton-Jacobi-Bellman (HJB) equations in the different modes that characterize the equilibrium value functions.

Proposition 1. *Denote by $V_f(\vec{K}^m, m)$ the value function of firm $f = A, B$ in mode $m \in M$ satisfying the Hamilton-Jacobi-Bellman equations (14) - (17) given in Appendix A. Then, the feedback strategies in a Markov Perfect Equilibrium of the game are given by*

$$\phi^*_{if}(\vec{K}^m, m) = \frac{1}{\gamma_{if}}\left(\frac{\partial V_f(\vec{K}^m, m)}{\partial K_{if}} - \mu_i\right), \; i = 1, 2, f = A, B, m \in \{m_1, .., m_4\}, \tag{9}$$

$$\begin{aligned}\phi^*_{RA}(\vec{K}^{m_1}, m_1) = \frac{1}{\gamma_{RA}}\Bigg(&\frac{\partial V_A(\vec{K}^{m_1}, m_1)}{\partial K_{RA}} - \mu_R + \\ &+\alpha(V_A(K_{1A}, 0, K_{1B}, K_{RB}, m_2) - V_A(\vec{K}^{m_1}, m_1))\Bigg),\end{aligned} \tag{10}$$

$$\begin{aligned}\phi^*_{RB}(\vec{K}^{m_1}, m_1) = \frac{1}{\gamma_{RB}}\Bigg(&\frac{\partial V_B(\vec{K}^{m_1}, m_1)}{\partial K_{RB}} - \mu_R + \\ &+\alpha(V_B(K_{1A}, K_{RA}, K_{1B}, 0, m_3) - V_B(\vec{K}^{m_1}, m_1))\Bigg),\end{aligned} \tag{11}$$

$$\begin{aligned}\phi^*_{Rf}(\vec{K}^m, m) = \frac{1}{\gamma_{Rf}}\Bigg(&\frac{\partial V_f(\vec{K}^m, m)}{\partial K_{Rf}} - \mu_R + \\ &+\alpha(V_f(K_{1A}, K_{2A}, K_{1f}, 0, m_4) - V_f(\vec{K}^m, m))\Bigg) \begin{array}{c} f = A, B, \\ m = m_2, m_3 \end{array}.\end{aligned} \tag{12}$$

Optimal investment in production capacity of the established product is proportional to the difference between the marginal effect of an increase in capacity on the firm's value function and the unit price of capacity (see (9)). Concerning a firm's investment in R&D knowledge stock, an additional effect arises because such an investment increases the hazard rate of making the transition to a different mode where the firm is active on both markets. This mode transition induces a jump in the value function and the corresponding effect on R&D incentives is captured by the last terms in equations (10) to (12).

The main technical and computational challenge for analyzing the firms' optimal investment strategies is to determine the value functions $(V_A(\vec{K}^m, m), V_B(\vec{K}^m, m))$ which satisfy the system of Hamilton-Jacobi-

Bellman equations (14) - (17) containing one equation for each firm in each mode. To illustrate the technical challenges associated with finding the solutions of the system of HJB equations we show in equation (13) the schematic form of the HJB equation for firm A in mode m_1,[13]

$$\begin{aligned} rV_A(\vec{K}^{m_1}, m_1) \;&=\; max_{I_{1A}, I_{RA}} \Big[\pi_A(\vec{K}^{m_1}, m_1) \\ &+ \frac{\partial V_A(\vec{K}^{m_1})}{\partial K_{1A}} \dot{K}_{1A} + \frac{\partial V_A(\vec{K}^{m_1})}{\partial K_{RA}} \dot{K}_{RA} + \frac{\partial V_A(\vec{K}^{m_1})}{\partial K_{1B}} \dot{K}_{1B} + \frac{\partial V_A(\vec{K}^{m_1})}{\partial K_{2R}} \dot{K}_{2R} \\ &+ \lambda(I_{RA}, K_{RA})(V_A(\vec{K}^{m_2}, m_2) - V_A(\vec{K}^{m_1}, m_1)) \\ &+ \lambda(\phi_{RB}(\vec{K}^{m_1}, m_1), K_{RB})(V_A(\vec{K}^{m_3}, m_3) - V_A(\vec{K}^{m_1}, m_1)) \Big], \end{aligned} \tag{13}$$

where $\pi_A(\vec{K}^{m_1}, m_1)$ denotes the instantaneous profit of firm A in mode m_1 given by the market profit minus firm A's costs of investing in production capacity for the established product and in R&D. Hence, the first two lines of the right hand side of (13) are standard expressions capturing the current flow of profits and investment costs for firm A as well as the effect of the state dynamics on firm A's value function. Since in our model specification profit and cost functions are quadratic functions of the state and control variables and all state dynamics are linear, the form of this part of the HJB equation corresponds to that in a linear-quadratic game. From a technical perspective, the terms in the last two lines, which capture the implications of a possible change of the mode to m_2 or m_3 for the value function of firm A, make the task of finding a solution more challenging. First, the value functions for firm A in modes m_2 and m_3 appear in these terms, implying that the HJB equations of both players are linked across modes and hence cannot be solved separately. Second, due to the fact that in these terms the value function $V_A(\vec{K}^{m_1}, m_1)$ is multiplied by the hazard rates of the transition to modes m_2 and m_3, which are both functions of the state vector, further implies that (13) does not allow for a closed-form (polynomial) solution. Similar statements apply for the HJB equations in modes m_2 and m_3. Only mode m_4 allows for quadratic solutions of the HJB equations of the two firms, since no further mode transition is possible from mode m_4. The solutions for mode m_4 can be found using the standard approach of comparison of coefficients (see e.g. Dockner et al. (2000)).

For the computation of the value functions in modes m_1 - m_3 we rely on numerical collocation methods. The general approach underlying this method is to determine polynomial approximations of the value functions of both firms in these modes with the property that, after inserting these approximate value functions and the corresponding feedback functions into the HJB equation of the corresponding mode, the (absolute) value of the difference between the left and the right hand side of the HJB equation is sufficiently small on an appropriate grid of points in the state space (see Vedenov and Miranda (2001), Dawid et al. (2017)). The large dimension of the state space in our model requires the application of sparse grid methods for constructing the considered grid of points in the state space and the set of basis functions to be used in the collocation. For all results presented in the following sections this approach has been used and it has then been checked that the HJB equations hold up to a numerical error below a given error bound on the entire state space and that the transversality conditions are satisfied in all modes. Details of

[13] The complete expression of this equation is given in (14) in Appendix A.

Symbol	Definition	Constraint	Baseline
α	Effectiveness of current R&D	≥ 0	0.2
β	Effectiveness of knowledge stock	≥ 0	0.2
ψ	Exponent of know. stock in innov. rate	≥ 0	1
ω	Coeff. of innovation rate in mode m_2/m_3	≥ 0	1
η	Horizontal differentiation	$\lvert\eta\rvert < 1$	0.65
θ	Vertical differentiation	≥ 0	0.2
μ_1, μ_2	Unit costs of prod. capacity $i = 1, 2$	≥ 0	0
μ_R	Unit costs of R&D investment	≥ 0	0.1
γ_{1A}, γ_{1B}	Adjustment costs for product 1	> 0	3
γ_{2A}, γ_{2B}	Adjustment costs for product 2	> 0	3
γ_{RA}, γ_{RB}	Adjustment costs for knowledge stock	> 0	0.1
δ_1, δ_2	Depreciation rates for capacities K_1, K_2	> 0	0.2
δ_R	Depreciation rate of knowledge stock	> 0	0.3
r	Discount rate	$0 < r \leq 1$	0.04

Table 1: Baseline Parameter Setting of the Model

the numerical method underlying our analysis are given in Appendix B. The application of a sparse grid collocation approach for calculating optimal firm strategies in a dynamic Operations Management problem with strategic interactions is an innovative methodological contribution of our paper.

In what follows, we present numerical results obtained by this approach for the baseline parameter setting given in Table 1. The unit prices of production capacity, μ_i, are normalized to zero. The values of the adjustment cost parameters, γ_{if}, are positive. To capture the interplay between the markets, the degree of horizontal differentiation is assumed to be $\eta = 0.65$. We deal with a scenario where the difference in qualities between the new product and the established product is moderate, which is reflected by $\theta = 0.2$. Depreciation rates and discount rate are set to values which are in line with empirical estimates (see, e.g., Nadiri and Prucha (1996) and Li and Hall (2020)). As discussed above, we assume that the hazard rate is a linear function of the firm's R&D knowledge stock ($\psi = 1$). The coefficients α and β of the hazard rate as well as the parameters μ_R, γ_{Rf} of the R&D investment costs have been chosen such that expected innovation times in equilibrium are less than 1.5 years. This is in accordance with innovation clockspeed in high-technology industries, like for instance the computer industry (see also Pacheco-de-Almeida 2010). In Section 7 we provide an extensive robustness analysis in which we demonstrate that the main qualitative findings presented in this paper continue to hold for a large range of parameter values around our baseline setting.

5 Strategy Analysis

As a first step of our analysis, we consider a situation where firms are symmetric except for the initial production capacity on the established product market. Figure 2 shows the evolution of production capacities, investments in capacities for the established product, R&D investments and R&D knowledge stocks, and the probability for each firm to win the innovation race if both firms follow their optimal investment

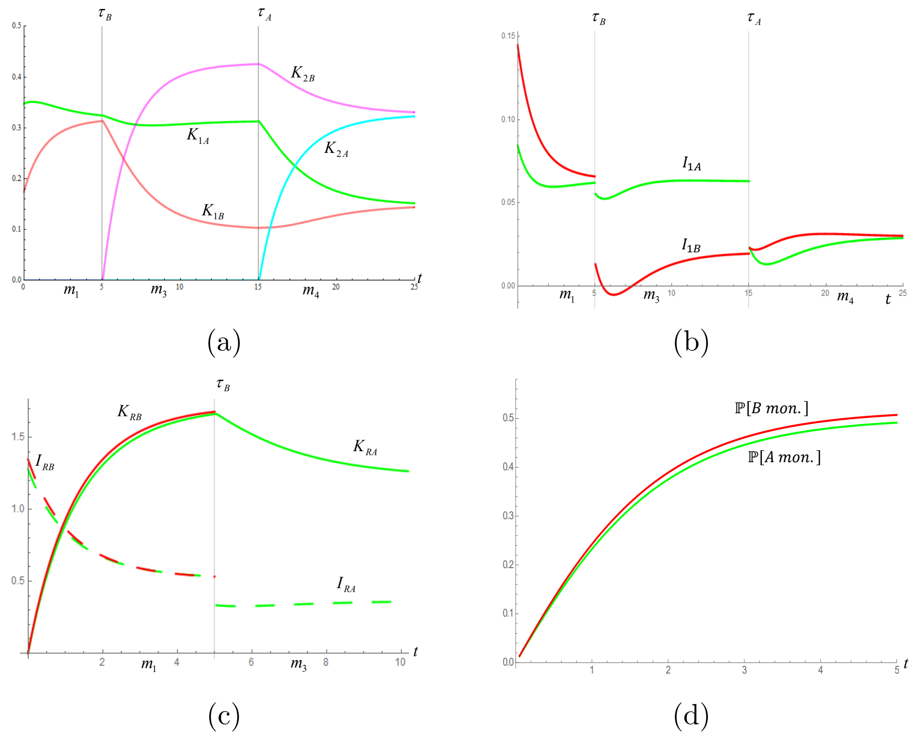


Figure 2: Equilibrium dynamics under the baseline parameter setting for asymmetric initial production capacities on the established market. (a) Production capacities. (b) Investments in production capacities for the established product. (c) R&D knowledge stocks and R&D investments. (d) Probabilities to become temporary monopolist on the new market. In panels (a) - (c) it is assumed that the smaller firm B (shown in red) innovates earlier than firm A (shown in green). The innovation time of firm B is indicated as τ_B.

strategies.[14] Both firms start with zero R&D knowledge stocks. Firm A's initial production capacity for the established product corresponds to the steady state level in a scenario where firms do not account for the option to develop a new product. This corresponds to a dynamic setting where the firms just invest in established product capacity. Firm B starts with half of firm A's production capacity. For the purpose of illustration, it is assumed in panels (a) - (c) of Figure 2 that firm B enters the new market first.[15] To check the qualitative robustness of our findings reported below, Figure 11 in Appendix C shows the dynamics of stocks and investments for the the case where firm A enters the new market first. Comparison of Figure 11 with panels (a) - (c) of Figure 2 ensures that our observations about the change in the optimal investment strategies of the innovator and innovation laggard after the first introduction of the new product do not depend on whether the larger or the smaller firm on the established market is the first innovator.

At first sight it might be surprising to see that the production capacities of both firms in mode m_1 stay below the steady state level of the corresponding game without innovation option (i.e. the initial capacity

[14] Although, in general, we cannot expect uniqueness of Markov Perfect Equilibria of the considered game, we have always found only a single MPE in our numerical explorations. To ensure robustness, we have carried out the analysis for a wide range of initializations of the collocation algorithm without finding other MPEs. All figures show the trajectories resulting from this single MPE.

[15] Panel (d) of Figure 2 confirms that firm B entering the new market first has higher probability than firm A being the first innovator.

of firm A). Intuitively, this is because firms in equilibrium anticipate the introduction of the new product at a future point in time, which then reduces the value of their production capacity on the established market.[16] The inter-temporally optimal reactions of both firms to this reduction in the future expected value of the production capacity of the established product is to reduce the investment in this capacity. As both firms accumulate more R&D knowledge over time (see panel (c)), their hazard rates increase. This reduces the expected time until the introduction of the new product. Accordingly, the anticipation effect becomes stronger during mode m_1 which leads to a decreasing pattern of investments in production capacities on the established market (see Figure 2(b)). Furthermore, panel (b) of the figure shows that the optimal reactions of both firms to the introduction of the new product at time τ_B at the beginning of mode m_3, is to cut their investments in the production capacities of the established product. For firm A, the reason is that the launch of the new product causes additional competition to its established product. Firm B produces both products in mode m_3 and even dis-invests to reduce its product capacity for the established product. The reason for firm B's disinvestment is that besides the intensified competition for the established product, sales of the established product reduce the price of the new product for which firm B is now the sole manufacturer. As soon as mode m_4 is reached, firm A has also innovated and launched the new product. To reduce the competitive effect of the established submarket on the new product, it is now optimal for firm A to severely reduce the investment in its established product capacity. This, in turn, triggers a slight increase of firm B's investment in its established product 1.

Concerning firms' R&D investments, Figure 2(c) shows that during the innovation race in mode m_1, the smaller firm B should invest more and accumulate a larger knowledge stock than what is optimal for firm A. Hence, firm B is the R&D knowledge leader throughout this mode. The hazard rate of the smaller firm B is larger. Consequently, as can be seen in panel (d), the probability that firm B becomes (temporarily) the sole manufacturer of the new product by winning the race is larger than firm A's chances of winning. Therefore, under optimal R&D strategies of both competitors, the firm with a relatively weak position on the established market has a higher probability to become the first firm to be active on the new submarket. Furthermore, at the moment of the introduction of the new product by firm B (mode m_3), the optimal R&D investment of firm A exhibits a downward jump. Although R&D investment is increasing after firm B has launched the new product, this implies that both the R&D knowledge stock and the hazard rate of firm A decrease over time throughout mode m_3. This points to a qualitative difference of the optimal R&D investment strategy in the modes before and after the competitor's innovation. A more detailed discussion of the reasons for this difference is postponed until Section 5.3.

In Figure 3(a), we show the dynamics of profit flows for both firms under the optimal investment trajectories depicted in Figure 2. Following their inter-temporally optimal investment paths, both firms have to accept initial losses generated by their R&D investments. Interestingly, the product innovation by Firm B generates an immediate upward jump of the profit of the innovation laggard (Firm A) in the beginning of m_3. This sudden increase in firm A's profit is due to a reduction in firm A's R&D investment,

[16] Although we assume that firms use non-anticipating strategies, i.e. they do not know at which exact time in the future the new product will be introduced, they know the arrival rate of the new product under the equilibrium investments. Therefore, they know the distribution of the time left until the introduction of the new product happens. In this sense, the firms anticipate that the new product will eventually be launched in the market.

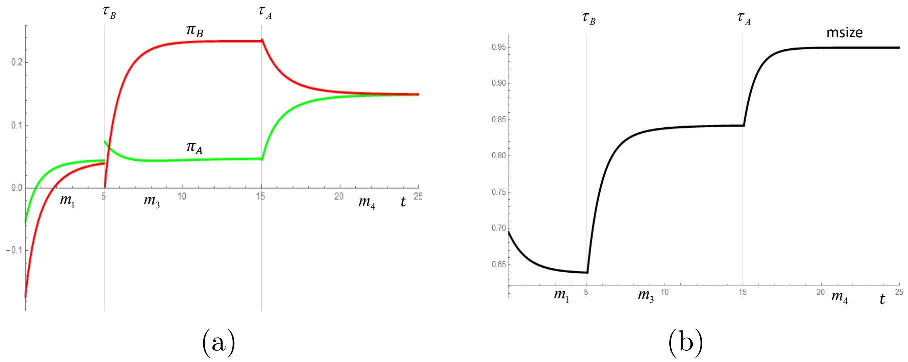


Figure 3: Dynamics of (a) profits of firms A and B and (b) total market size (i.e. sum of all production capacities on both markets) under the baseline parameter setting.

the reason of which is explained in Subsection 5.1. Once firm B has built sufficient production capacity for the new product, its competitive advantage is however quite significant. Since both firms have identical cost structure, the long run profit in mode m_4 (i.e. after both firms have innovated) is the same for both competitors. Panel (b) of Figure 3 highlights that the introduction of the new product leads to an expansion of the overall market size since the new product is of better quality (vertical differentiation) and only a partial substitute for the established product. Hence, total sales of both firms increase when they launch the new product.

Overall, Figure 3 illustrates that innovation managers in firms face a serious challenge during the innovation race. They have to balance the costs of R&D effort with the expected future returns, which are not only subject to technical uncertainty (time of completion of the firm's own R&D project) but also to strategic uncertainty (time of market introduction of the new product by the competitor). In the next section we analyze in more detail how this trade-off influences the properties of the optimal firm strategy. In subsection 5.1 we assume that firms have identical R&D and production technologies and we study how a firm's optimal R&D investment is affected by the interplay between production capacities and R&D knowledge stocks of both firms. In subsection 5.2, we study how a firm's R&D strategy should be adjusted if the firm has an investment cost (dis-)advantage on the established market.

5.1 R&D Incentives and the Interplay of Knowledge Leadership and Market Position

In an R&D race with knowledge accumulation and exogenously given fixed profits for the innovator and the losing firm, the strategic interaction pattern of action-reaction emerges in equilibrium, see Doraszelski (2003).[17] This means that during the race the knowledge laggard, i.e. the firm with the smaller R&D knowledge stock, invests more in R&D than the knowledge leader. As Figure 4(a) shows, this observation does not necessarily carry over to our setting of an R&D race with *endogenously* determined flow profits. In the green area the optimal R&D investment of firm A is higher than that of firm B. In the yellow area, the opposite holds. It is evident from the figure that the boundary between these two areas does not coincide with the line $K_{RA} - K_{RB} = 0$. Therefore, for the part of the green area where $K_{RA} < K_{RB}$, the

[17]To be precise, this relationship is shown if a firm's hazard rate is linear (or concave) with respect to the firm's knowledge stock, which is the baseline case considered also in this paper.

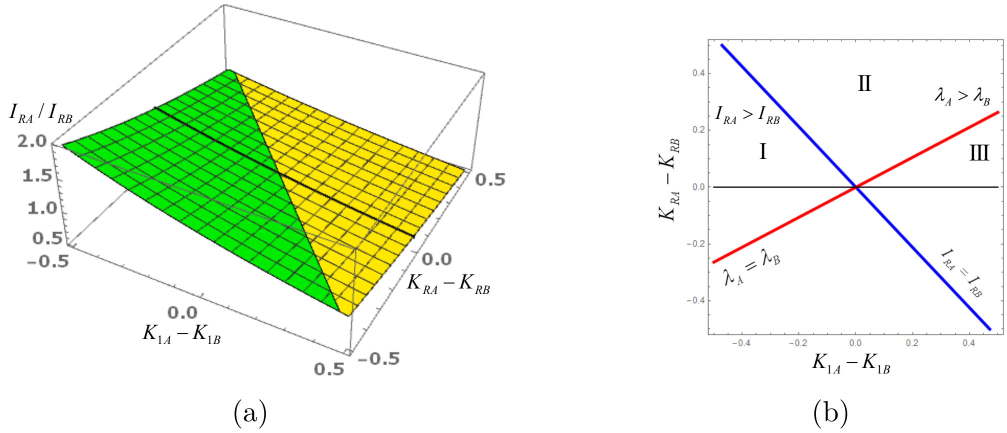
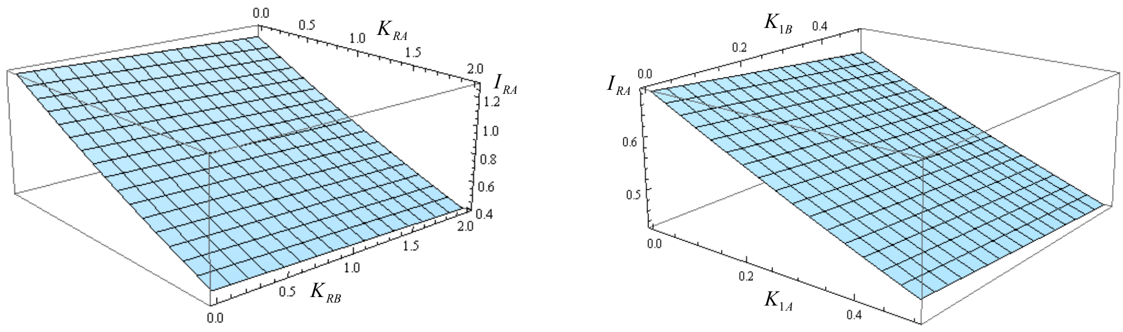


Figure 4: (a) The ratio of equilibrium R&D investments of firms A and B depending on the difference in capacities on the established market and on the difference in knowledge stocks. The green (yellow) region indicates where firm A (firm B) invests more in R&D than its competitor. (b) Regions in the state space in which firm A has higher R&D investment than firm B (below the blue line) and in which firm A has a higher hazard rate than firm B (above the red line). In both panels, the black line indicates the boundary between the regions in which firm A is the knowledge leader or knowledge laggard. The center point (i.e. the point for which $K_{1A} - K_{1B} = K_{RA} - K_{RB} = 0$) corresponds to the steady state values of the corresponding state variables in mode m_1.

optimal R&D investment of the firm with the lower knowledge stock is higher than the investment of the knowledge leader. Hence, we find an action-reaction pattern. In the part with $K_{RA} > K_{RB}$, the knowledge leader invests more. Hence, we observe an increasing dominance pattern. Analogous conclusions can be drawn for the yellow area in Figure 4(a). From a managerial point of view, these arguments highlight that in the common situation where the firms' profits are endogenously determined by the firms' decisions, the optimal R&D strategy crucially depends on the firms' relative positions on the established market and the firms' relative positions with regard to their accumulated R&D knowledge. In what follows we discuss the economic mechanisms underlying these observations.

Effect of R&D Knowledge Stocks on R&D Investments

For symmetric production capacities on the established market, i.e. $K_{1A} = K_{1B}$, it is always optimal for the R&D knowledge laggard to invest more in R&D than its competitor. This observation can be explained by considering Figure 5(a), which shows the R&D investment of firm A in equilibrium as a function of the R&D knowledge stock of both firms (for symmetric production capacities on the established market). The firm's optimal R&D investment depends negatively on its own R&D knowledge stock as well as on the knowledge stock of its competitor. Note that the negative dependence of the firm's own knowledge stock is much more pronounced. Hence, the R&D knowledge leader invests less in R&D. The intuition for both of these negative relationships is that larger R&D knowledge stocks reduce the expected duration of mode m_1 and any transition to another mode (m_3 or m_2) reduces the value of the knowledge stock. A transition to mode m_3, where firm B has innovated, reduces the value of firm A's R&D knowledge stock. In fact, the



(a) (b)

Figure 5: Investment in knowledge stock of firm A depending on (a) R&D knowledge stocks, and (b) production capacities on the established market of the two firms. The values of the arguments of the feedback functions not varied are set at the steady state levels in mode m_1.

increase in the value function generated by a potential innovation of firm A is substantially larger as long as firm B has not introduced the new product yet since the price of the new product is lower if the opponent is already active on the new market. If the transition is to mode m_2 and firm A itself is the innovator, the value of the firm's R&D knowledge stock completely vanishes because no further innovation is possible by the firm.[18] Doraszelski (2003) discusses the negative effect of the own knowledge stock on a firm's R&D incentives in his patent race setting and denotes it as the '*pure knowledge effect*'.

It should be pointed out that our prediction of a negative dependence of a firm's optimal R&D investment on the *competitor's* R&D knowledge stock differs qualitatively from the observations reported in the literature on patent races with exogenously given innovation value. Doraszelski (2003) shows that in such a setting the effect of an increase of the opponent's knowledge stock on a firm's R&D investments is positive if either the own knowledge stock or the payoff of winning the race is sufficiently large. Regarding this qualitative difference in the results, we note that it is crucial that in our setup we explicitly capture the market evolution after the introduction of the new product. The opponent's R&D knowledge stock has a negative effect on the additional profit a firm expects from introducing the new product since the expected duration of the period in which the innovator is the sole manufacturer of the new product negatively depends on the opponent's knowledge stock. This effect is absent in a patent race setting, in which the payoff of the innovator is exogenously given. More formally, from (10) it follows that the optimal R&D investment of firm A in mode m_1 decreases with respect to K_{RB} if and only if

$$\frac{\partial^2 V_A(\vec{K}^{m_1}, m_1)}{\partial K_{RA}\, \partial K_{RB}} + \alpha \frac{\partial V_A(K_{1A}, 0, K_{1B}, K_{RB}, m_2)}{\partial K_{RB}} - \alpha \frac{\partial V_A(\vec{K}^{m_1}, m_1)}{\partial K_{RB}} < 0.$$

The second of these three terms, which is always negative, is absent in patent race models in which the post

[18] In a setting with repeated innovations of each firm, which is beyond the scope of this paper, the knowledge stock would not completely lose its value upon successful innovation. However, also in such a setting, due to the discounting of profits from future innovation events, the value of the knowledge stock would exhibit a downward jump at the time of the innovation breakthrough.

innovation payoff does not depend on the innovation activities of the opponent. Hence, the endogeneity of the post-innovation profit reinforces the submissive response of a firm to an increase in the competitor's knowledge. For all parameter settings considered in our analysis this effect turns out to be dominant so that our model predicts a negative dependence of I_{RA} on K_{RB}.

Since a firm's hazard rate of innovation does not only depend on its current R&D investment, but also on its knowledge stock, investing more in R&D does not imply that the hazard rate is larger than that of the competitor. As can be inferred from Figure 4(b), for symmetric capacities on the established market, the knowledge laggard actually always has a lower hazard rate than its competitor although it invests more in R&D (see region II).

Effect of Production Capacities on R&D Investments and Innovation Leadership

For asymmetric capacities on the established market, it might be optimal for the R&D knowledge leader to invest more than the R&D laggard. In particular, our model shows that this happens if the knowledge leader has a substantially smaller capacity on the established market (see region I in Figure 4(b)). Figure 5(b) shows that the optimal R&D investment of a firm negatively depends on both firms' production capacities on the established market, where the negative dependence is much stronger for a firm's own production capacity.[19] Therefore, a smaller production capacity on the established market makes a firm more aggressive with respect to R&D effort.

The introduction of the new product in the new (but linked) submarket results in a decrease of the price for the established product. The larger a firm's production capacity on the established market, the more strongly it is affected by this decrease (*cannibalization effect*).[20] Furthermore, the incentive to invest in the R&D knowledge stock is influenced by a *size effect*. A larger total production capacity on the established market induces a smaller price for the new product after it has been launched. This reduces the profitability of the new product and thereby the incentive to speed up the introduction of the new product. Note that, in contrast to the cannibalization effect, for the size effect it is irrelevant whether the capacity expansion on the established market is due to an increase of the competitor's production capacity or the firm's own production capacity. The joint influence of the cannibalization effect and the size effect explains why an increase of the firm's own production capacity decreases the firm's own incentives to invest in R&D more strongly than an increase in the competitor's production capacity.

Therefore, if the knowledge leader is the much smaller firm on the established market, it should optimally invest more in R&D might than the knowledge laggard. As a consequence, in equilibrium a pattern of increasing dominance might arise. In this case, the gap in knowledge stock grows over time (region I in Figure 4(b)). The negative effect of R&D knowledge leadership on R&D investment, however, starts to dominate if the knowledge gap becomes sufficiently large. This induces that the R&D knowledge leader then invests less than the R&D knowledge laggard (transition from region I to region II in Figure 4(b)).

[19]The range of capacities K_{1A}, K_{1B} considered in the figure covers the whole interval between 0 and the monopoly output level on the established market.

[20]Formally, the difference in value functions $V_A(\vec{K}^{m_2}, m_2) - V_A(\vec{K}^{m_1}, m_1)$ decreases with respect to K_{1A} and hence the gains from innovation are weaker for the larger firm with a higher current profit. In this sense, this observation is related to the well-known Arrow replacement effect (Arrow (1962)).

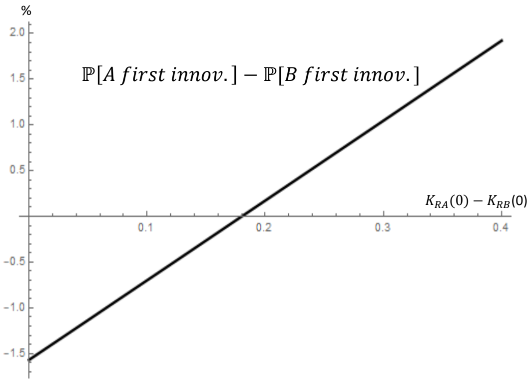
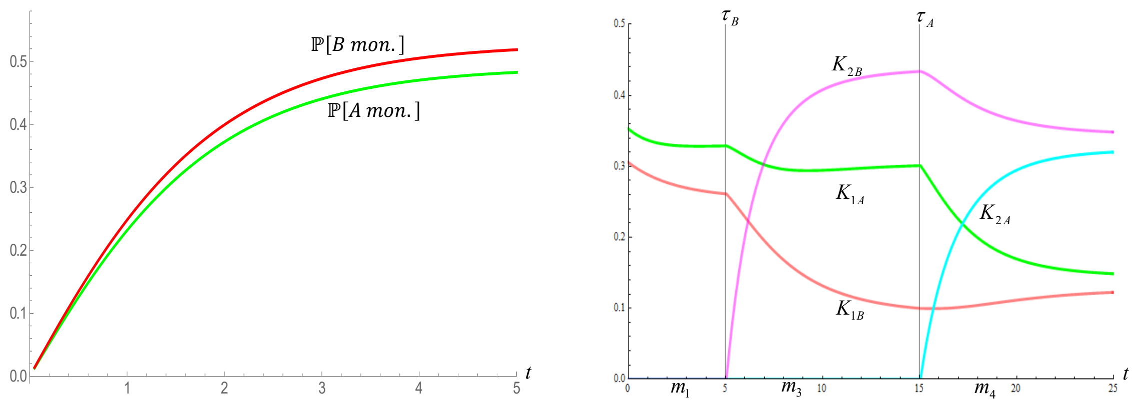


Figure 6: Difference between the probabilities of winning the innovation race in mode m_1 for firm A and firm B. Initial capacities of both firms on the established market are given by our baseline setting with $K_{1A}(0) > K_{1B}(0)$.

The discussion above shows that both R&D knowledge leadership and dominance on the established market have a negative impact on the relative incentive of a firm to invest in product innovation. In situations in which the same firm dominates its competitor both in terms of its knowledge stock and its product capacity on the established market, this has the clear-cut implication that the strategic interaction pattern of action-reaction emerges. In this case, it is optimal for the R&D knowedge laggard to invest more in R&D. This effect can be so strong, that the hazard rate of the knowledge leader falls below the hazard rate of the R&D knowledge laggard (region III in Figure 4(b)). In other words, our model highlights that R&D knowledge leadership does not coincide with innovation leadership. Indeed, the knowledge leader's probability of winning the innovation race might be smaller than R&D knowledge laggard's probability of winning. To illustrate this point, we show in Figure 6 the difference between the probabilities of winning the race for firms A and B in a scenario where, as in Figure 2, initially firm A's production capacity on the established market is twice as large as firm B's production capacity and both firms use their optimal investment strategies. On the horizontal axis, we vary the difference in the initial R&D knowledge stocks of the two firms. The figure shows that if initial knowledge stocks are close to each other the smaller firm is more likely to win the race. This case corresponds to initial conditions in region III of Figure 4(b)). Note that also in this case, due to the stochastic nature of the innovation process, there is a positive probability that the dominant firm on the established market innovates first, although it has a lower hazard rate than its competitor. If the initial R&D knowledge stock of firm A is substantially larger than firm B's knowledge stock, the probability of winning the race is larger for firm A. This case corresponds to initial conditions in region II of Figure 4(b)). The initial advantage of firm A with respect to its knowledge stock dominates the effect that its R&D investment is smaller than firm B's R&D investment. Consequently, the dominant firm A does not only have an initial advantage with respect its R&D knowledge stock but also an overall higher probability of winning the race.

In Figure 2, we have illustrated the dynamics for initial conditions $K_{1A} > K_{1B}$ and $K_{RA} = K_{RB} = 0$. In terms of Figure 4(b), this initial condition is positioned in region III on the line $K_{RA} - K_{RB} = 0$. Hence, firm A optimally invests less in R&D than firm B and also has a lower hazard rate of innovating first. The



(a) (b)

Figure 7: Probabilities to become temporary monopolist on the new market (a) and equilibrium dynamics of production capacities (b) in a scenario with asymmetric capacity adjustment costs on the established market ($\gamma_{1A} = 3, \gamma_{1B} = 9$).

lower R&D investment of firm A implies that firm A's R&D knowledge stays lower than firm B's, i.e. for $t > 0$ we always have $K_{RA} - K_{RB} < 0$. Since the difference in the production capacities on the established market, $K_{1A} - K_{1B}$, decreases over time, the state moves downwards and to the left in the diagram of Figure 4(b), and eventually approaches the blue line along which $I_{RA} = I_{RB}$. This explains why the difference in R&D investments between the two firms vanishes and R&D knowledge leadership of firm B shrinks during the innovation race in mode m_1 (see Figure 2(b)).

5.2 Competitive Disadvantage Turns Into Innovation Leadership

Our analysis so far has assumed that the two competitors (only) possess different initial production capacities on the established market. In this context, we have been studying the strategic implications of transitory differences for the R&D investments of otherwise symmetric firms. This section turns to the case where firms differ in their competitiveness on the established market. In particular, we impose that investment costs of firm B on the established market are higher than the investment costs of firm A. We adjust our baseline parameter setting given in Table 1 by increasing the adjustment cost parameter of firm B on the established market to $\gamma_{1B} = 9$, but keep firm A's adjustment cost parameter at the baseline value $\gamma_{1A} = 3$.

The dynamics emerging in equilibrium in such a setting is depicted in Figure 7. Like in Figure 2, initial R&D knowledge stocks of both firms are assumed to be zero. The initial production capacities on the established market correspond to the steady state values of this game without product innovation option. Due to its competitive disadvantage with respect to capacity adjustment costs, the initial production capacity of firm B on the established market is, therefore, lower than firm A's initial production capacity. Figure 7 highlights two important implications of this asymmetry between firms. First, panel (a) reveals that throughout mode m_1, firm B has a higher probability of winning the innovation race than its competitor, which can be explained by its higher R&D investment and subsequently its higher hazard rate.

Second, under the assumption that firm B innovates first, it has a strictly smaller production capacity on the established market, but a strictly larger production capacity on the new submarket throughout all modes

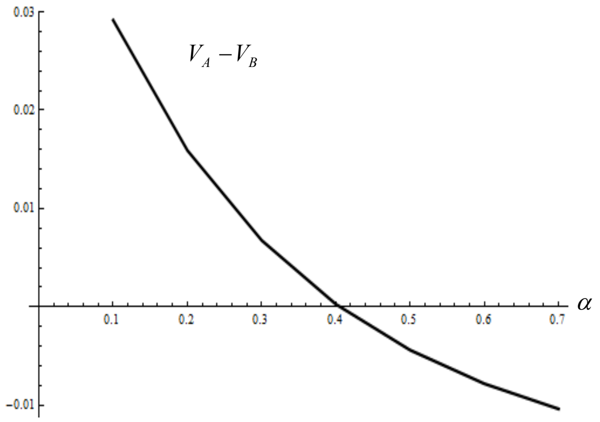


Figure 8: Difference in the value functions of firms A and B for different values of the parameter α and symmetric initial conditions.

where these markets exist (see panel (b)). Consequently, the disadvantage of higher capacity adjustment costs on the established market acts as a commitment device for firm B to be more aggressive during the innovation race and also to be a tougher competitor on the new submarket. Innovation leadership then follows from the negative relationship between the firm's own production capacity on the established market and its incentive to invest in R&D knowledge stock, as discussed in Section 5.1.

These arguments show that the disadvantage of firm B with respect to investment costs on the established market has two counteracting implications for the firm's profit. The direct effect is that the profit of firm B on the established market is negatively affected by its larger capacity adjustment costs. The indirect effect is that the competitor, firm A, invests less in its R&D knowledge stock because firm A takes into account that firm B has a stronger incentive to invest in R&D. This raises firm B's profit. The second effect becomes more important if the expected time until the creation of the new submarket is shorter. In the framework of our model, this aspect is closely related to the parameter α, which measures the impact of current R&D investment on the hazard rate. Figure 8 illustrates that, for sufficiently large values of α, the firm with a cost disadvantage on the established market can indeed have a larger expected discounted payoff than its more efficient competitor.[21] The conclusion is that a firm's burden of having higher higher capacity adjustment costs in an established market might eventually turn out to be a blessing since this firm has to focus its attention on innovative submarkets and, surprisingly, might even turn into the innovation leader achieving a higher expected profit.

5.3 Impact of Competitor's Innovation on a Firm's R&D Strategy

The creation of the new submarket by a competitor has substantial implications for the optimal dynamic innovation strategy of the firm that has not innovated yet. Figure 2(b) demonstrates that the R&D investment of firm A exhibits a downward jump at $t = \tau_B$ when firm B's innovation project is successful and

[21]In order to isolate the effect of an asymmetry in capacity adjustment costs (represented by γ_{1f}) on the value functions of firms A and B, the difference in value functions in Figure 8 is calculated for symmetric initial conditions $K_{1A}^{ini} = K_{1B}^{ini} = 0.353$. This value corresponds to the steady-state capacity of firm A in the game without innovation option.

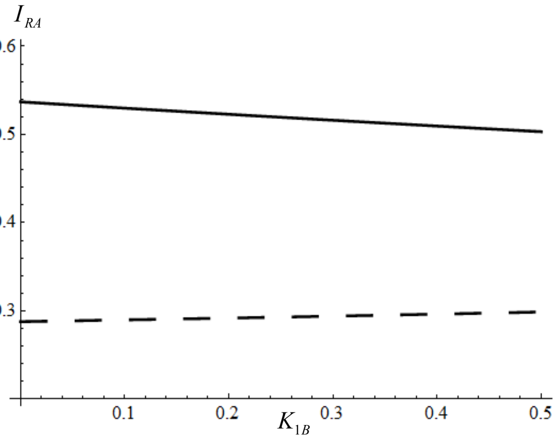


Figure 9: Investment in R&D knowledge of firm A depending on production capacity of firm B on the established market before (solid line) and after (dashed line) firm B has innovated. The figure is based on the baseline parameter setting with symmetric investment costs on the established market. The values of the variables apart from K_{1B} are given by their steady-state value in the respective mode.

the new product is launched in the submarket. Correspondingly, firm A's hazard rate exhibits a downward jump at $t = \tau_B$ as well. A direct implication of this observation is that the expected waiting time until firm A's project is successful exhibits an upward jump after the opponent's innovation.

Firm A's expected value of introducing the new product decreases considerably at the time firm B creates the new submarket since at this point the possibility to become temporarily the sole manufacturer of the new product and reap the benefits, vanishes. For this reason, it is optimal for firm A to reduce its investment in the R&D knowledge stock at $t = \tau_B$. This induces a downward jump of its hazard rate. The reduction in the level of investment in R&D induces firm A's R&D knowledge stock to decrease over time. Taking into account the pure knowledge effect, this decrease in R&D knowledge stock has a positive impact on the level of firm A's R&D investment. Figure 2(c) demonstrates that this effect results in an increasing pattern of firm A's R&D investment I_{RA} in mode m_3 even though the quick build-up of production capacity for the new product by firm B further reduces the attractiveness of the innovation for firm A.

The discussion above highlights that the innovation laggard's (firm A) optimal level of investment in its R&D knowledge stock differs between modes m_1 and m_3. However, there are also qualitative changes in the properties of the firm's innovation strategy. In particular, the transition from mode m_1 to m_3 implies that the sign of the relationship between investment in R&D knowledge stock and the opponent's production capacity for the established market changes (see Figure 9). In mode m_1, an increase in incumbent B's production capacity K_{1B} decreases firm A's incentive to invest in R&D (bold line, see also Figure 5(b)), whereas in mode m_3 it implies an increase in firm A's R&D activities (dashed line). The reason for this qualitative change is that once firm B is active on the new submarket, an increase of its production capacity on the established market induces a reduction of firm B's future investment on the new market. This, in turn, makes the new submarket more attractive for firm A and, therefore, results in an increase of firm A's optimal investment in the R&D knowledge stock.[22] Although this effect is already present in mode m_1, there

[22]Our robustness analysis in Section 7 shows that this property no longer holds if the adjustment speed of the knowledge

the effect is weighted with the probability that the opponent wins the race and is discounted according to the expected innovation time. Furthermore, from the perspective of mode m_1 there is also a positive probability that firm A innovates first. In this case, a large production capacity K_{1B} on the established market reduces the value of innovation (size effect). The size effect is stronger and dominates in mode m_1, which yields the negative dependence of R&D investment on the opponent's capacity, as discussed above. In other words, the opponent's product range determines whether firm A should increase or decrease its R&D investment as a response to the competitor's capacity expansion on the established market. In our discussion we have assumed that the smaller firm B innovates first. However, in Appendix C we show that also if the larger firm innovates first, the slope of the innovation laggard's optimal R&D investment function with respect to the competitor's capacity on the established market changes sign after the competitor's innovation (see Figure 12). Consequently, the property that a firm's optimal R&D strategy crucially depends on the rival's product range is robust in this respect.

6 Effect of Competition on R&D Investment

Our discussion in Section 5 has highlighted that each firm can induce a sudden drop in the optimal R&D activities of its incumbent competitor in the established market by launching a new differentiated product. Even before the product is launched, a firm can reduce the competitor's R&D investment by increasing its own R&D knowledge stock. Hence, compared to a single-firm setting, in a duopoly market competition induces additional (strategic) incentives to undertake investments in R&D. Therefore, from a managerial perspective the question arises how R&D investments should be adjusted in the face of changing intensity of competition.

To address this issue, in panel (a) of Figure 10 we vary the marginal impact of current R&D investment on the hazard rate, captured by α, and compare the optimal initial R&D investments of firms A and B in our standard duopoly setting with the scenario in which one of the firms is a monopolist.[23] R&D investments are increasing in α, because a larger value of α implies that a given R&D investment results in a higher innovation probability. The figure shows that the monopolist always invests more in R&D. The main driving force of this finding is that in a monopoly the innovator does not need to take into account the later entry of a competitor into the new submarket. Consequently, the expected intertemporal rent is larger than in the duopoly. Put more formally, in a duopoly the value function of the innovator is negatively affected if eventually the other firm also launches the new product. In contrast, under monopoly the innovator can extract the monopoly rent for the new product indefinitely.

Although the incentive to invest in R&D for the individual firm is larger in a monopoly market compared to duopoly, panel (b) of Figure 10 shows that nevertheless innovation occurs faster in duopoly. Despite the

stock is too large relative to that of the production capacities, i.e. for very large values of γ_{1f}, γ_{2f} or very small values of γ_{Rf}. Intuitively in such a scenario the expected effect of an increase in K_{1A} on the future values of I_{2A} becomes less important than the instantaneous negative impact of such an increase on the price of the new product.

[23] More formally, we consider the dynamic optimization problem of firm A if firm B is absent from the established market and also cannot innovate; see Dawid et al. (2015) for a formal definition of the monopoly problem. Furthermore, to make the monopoly scenario comparable to the duopoly, we assume that the initial production capacity of the monopolist on the established market is given by the sum of the initial production capacities of the two firms in the duopoly scenario.

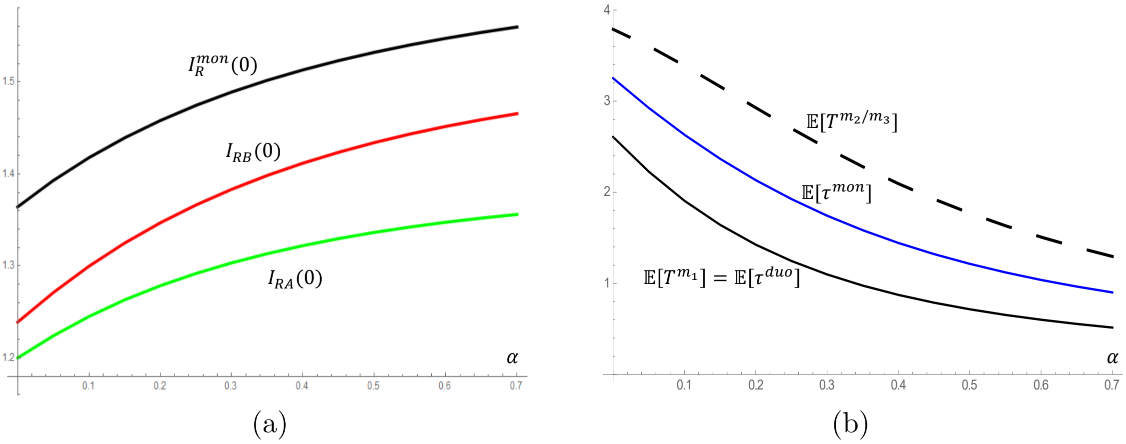


Figure 10: (a) R&D investments at time zero of firms A and B in duopoly and in monopoly for different values of the parameter α. (b) Expected innovation time in duopoly (black solid) and in monopoly (blue) as well as expected duration of the innovator's monopoly on the new market in the duopoly model (black dashed) for different values of the parameter α.

lower hazard rate for each individual firm, the expected time until the creation of the new submarket is smaller in duopoly since both firms are working independently to achieve the innovation breakthrough. Figure 10(b) also demonstrates that the expected time the innovation laggard needs to enter the new market after the competitor has successfully innovated is larger than the expected innovation time in monopoly. This holds true although the innovation laggard has already accumulated some R&D knowledge at the time of the competitor's innovation, whereas we assume that the initial knowledge stock of the firm under monopoly is zero. However, compared to a monopolist, the innovation laggard's advantage of accumulated R&D knowledge is outweighed by the smaller expected return from introducing the new product to the market where the competitor is already active. That a higher value of α reduces the expected time to innovate follows from the fact that R&D investment is increasing in α, and also because a given level of R&D investment increases the innovation probability more if α is larger. Since the duration of mode m_2 or m_3 also corresponds to the length of the time window in which the innovator is the sole manufacturer of the new product, Figure 10(b) also demonstrates that the expected duration of monopoly on the new market decreases with α.

7 Robustness

In this section, to attest the robustness of our findings, we identify intervals around the baseline values of the model parameters for which all the qualitative properties of the R&D investment functions identified in Sections 5.1 to 5.3 hold. In order to carry out this robustness check, each considered parameter is varied in the given interval and the value functions for these parameter variations are calculated. The analysis is then based on the consideration of the properties of the Markov-perfect equilibrium strategies associated with these value functions at the steady states in the corresponding modes. In Table 2 we give a description of the properties that we have checked. To account for potential numerical errors stemming from our method,

	Section	**Mode**	**Description**	**Formal Requirement**
(i)	5.1	m_1	R&D investment decreases w.r.t. own old market capacity	$\frac{\partial \phi_{RA}(.;m_1)}{\partial K_{1A}} < -5.10^{-4}$
(ii)	5.1	m_1	R&D investment decreases w.r.t. competitor's old market capacity	$\frac{\partial \phi_{RA}(.;m_1)}{\partial K_{1B}} < -5.10^{-4}$
(iii)	5.1	m_1	Negative effect of own old market capacity on R&D is stronger than that of competitor's capacity ($\Rightarrow$ larger firm invests less in R&D)	$\lvert\frac{\partial \phi_{RA}(.;m_1)}{\partial K_{1A}}\rvert / \lvert\frac{\partial \phi_{RA}(.;m_1)}{\partial K_{1B}}\rvert > 1 + 5.10^{-4}$
(iv)	5.3	$m_1,\ m_3$	R&D investment exhibits a downward jump when competitor innovates	$\phi_{RA}(.;m_1) > \phi_{RA}(.;m_3)$
(v)	5.3	m_3	R&D investment increases w.r.t. competitor's old market capacity after opponent's innovation	$\frac{\partial \phi_{RA}(.;m_3)}{\partial K_{1B}} > 5.10^{-4}$

Table 2: Qualitative properties checked in the robustness analysis.

a margin of $5 \cdot 10^{-4}$ has been used to check for the signs of the involved expressions.

Table 3 gives the intervals of the model parameters for which we have checked and found that the key properties listed in Table 2 hold. All properties (i) - (v) are robust and, consequently, we can be confident that the insights reported in Section 5 are robust with respect to changes in the parameter setting.

Even under simultaneous variation of more than one parameter from the baseline value, properties (i)-(iv) seem robust. However, Property (v), which describes that equilibrium R&D investment after the competitors innovation is an increasing function of the competitor's production capacity on the established market, fails to hold for some of the considered parameter variations generated in this way. In particular, if an increase in the vertical product differentiation is complemented with additional parameter changes, property (v) might no longer hold. In Appendix C, we show in Table 4 the robustness of properties (i)-(v) with respect to parameter variations if the quality difference between the established product and the new product is larger, $\theta = 0.4$, rather than its baseline value $\theta = 0.2$. It is highlighted there that property (v) no longer holds for all parameters on the entire tested interval, but that the combination of strong vertical differentiation of the new product with weak horizontal differentiation or strong depreciation of production capacities induces that the innovator's production capacity on the established market no longer has any influence on the R&D investments of the innovation laggard. This is quite intuitive, because in a scenario with strong vertical and weak horizontal differentiation a large production capacity of the innovator on the established market will be quickly reduced and hence has no significant effect on future values of the innovator's production capacity on the new market. Hence, the strategic effect, due to which it becomes more attractive for the innovation laggard to launch the new product if the innovator is strong on the established market, disappears.

	Description	Baseline	Tested interval	Robust (i) - (v)
α	Effectiveness of current R&D	0.2	$[0.1, 0.3]$	✓
β	Effectiveness of knowledge stock	0.2	$[0.1, 0.3]$	✓
ψ	Exponent of know. stock in innov. rate	1	$[0.5, 2]$	✓
ω	Coeff. of innovation rate in mode m_2/m_3	1	$[0.5, 1.5]$	✓
η	Horizontal differentiation	0.65	$[0.3, 0.8]$	✓
θ	Vertical differentiation	0.2	$[0, 0.4]$	✓
μ_R	Unit costs of R&D investment	0.1	$[0.05, 0.3]$	✓
γ_{1A}, γ_{1B}	Adjustment costs for product 1	3	$[1, 5]$	✓
γ_{2A}, γ_{2B}	Adjustment costs for product 2	3	$[1, 5]$	✓
γ_{RA}, γ_{RB}	Adjustment costs for knowledge stock	0.1	$[0.05, 0.5]$	✓
δ_1	Depreciation rate for capacity K_1	0.2	$[0.1, 0.3]$	✓
δ_2	Depreciation rate for capacity K_2	0.2	$[0.1, 0.3]$	✓
δ_r	Depreciation rate of knowledge stock	0.3	$[0.1, 0.5]$	✓
r	Discount rate	0.04	$[0.02, 0.06]$	✓

Table 3: Range of parameter values for which the different qualitative properties listed in Table 2 are satisfied. A checkmark indicates that these properties are satisfied on the entire tested interval.

8 Conclusions

In real-world markets, action-reaction or increasing dominance patterns of innovation can be observed. Under action-reaction, the smaller incumbent manufacturing firm in the established market becomes the innovation leader in the new market. Under increasing dominance, the dominant incumbent manufacturing firm in the established market also dominates the new market. In this paper, we develop a stochastic duopoly framework in order to highlight some of the drivers that endogenously lead to one of these patterns of innovation in equilibrium. In particular, we study the factors that determine the incentives of incumbent firms to invest in the development of a new product which extends their product range. Our dynamic setting particularly emphasizes the interplay between the firms' relative positions in terms of their R&D knowledge stocks and their relative strengths on the market for the established product. We explicitly take into account that the adjustment of production capacities as well as the build-up of R&D knowledge are costly and take time.

The first main insight from our analysis is that the interplay between the firms' R&D knowledge stocks and their positions on the established market allows a more fine-grained analysis than existing work and can provide additional answers concerning the question how an incumbent firm should invest in R&D and which firm will dominate innovative submarkets. In particular, we show that the knowledge leader might have a smaller innovation rate than its competitor if it has a sufficiently large market share on the established market. Furthermore, for a firm with a sufficiently small capacity on the established market it is optimal to invest more in R&D than its competitor even if it already has a knowledge advantage. These insights have managerial implications for optimal R&D investment and also provide a theoretical explanation for the empirically observed pattern that larger incumbents are often late to enter emerging new submarkets. It particularly highlights that the "innovator's dilemma", i.e. that dominant incumbents are frequently late in moving into newly emerging submarkets, might not be due to myopic decisions of firm management, but instead might be fully in line with intertemporally optimal firm behavior.

Our second main finding addresses the impact of a firm's structural disadvantage, captured by higher costs of adjusting production capacity for the established product, on the firm's incentive to invest in R&D knowledge. What we find is that the burden of having higher adjustment costs can actually be a blessing in the innovation race in the long run, as it acts as a commitment device for the disadvantaged firm to invest more aggressively in product innovation. We identify scenarios where this effect can be so substantial that despite the higher capacity adjustment costs on the established market, the cost follower can end up with a higher overall expected discounted profit than the more efficient cost leader.

A third innovative contribution that our dynamic approach allows is the characterization of the firms' optimal innovation strategies before and after a product innovation of the competitor. We show that it is optimal for an incumbent to reduce its investment in product innovation once the opponent has successfully launched its new product in the submarket. We also demonstrate that the relationship between the optimal innovation effort and the capacities on the established market crucially depends on whether the competitor is already active on the new submarket or not. The main take-away for a firm's optimal R&D strategy is that the rival's product range must be taken into account.

A limitation we share with most of the literature on innovation incentives under oligopolistic competition is that firms do not face any financial constraints. As a consequence, firms are always able to fully implement their planned investment strategies. However, a rich empirical literature indicates that many firms encounter difficulties in obtaining external funding for investments in R&D, and, therefore, have to rely on internal sources for financing their R&D activities. This opens up an additional channel which influences the interaction between a firm's accumulated profits resulting from established products and a firm's incentive to invest in product innovation. Examining the impact of financial constraints and internal funding of R&D activities on the firm's optimal dynamic innovation strategy is a challenging avenue for future research.

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Appendix A: Proof of Proposition 1

Standard theory for piecewise deterministic games establishes that in each mode the value function of each firm in a Markov Perfect Equilibrium is characterized by a Hamilton-Jacobi-Bellman (HJB) equation. In particular we obtain the following HJB equations in the different modes (see Dockner et al. (2000)):

Mode $\mathbf{m}_1$: the HJB equations of both firms are symmetric and are given by

$$
\begin{aligned}
& r\ V_f(K_{1f}, K_{Rf}, K_{1(-f)}, K_{R(-f)}, m_1) \\
= \quad & \max_{I_{1f}, I_{Rf}} \Big[(1-(K_{1A}+K_{1B}))K_{1f} - \mu_{1f} I_{1f} - \frac{\gamma_1}{2} I_{1f}^2 - \mu_{Rf} I_{Rf} - \frac{\gamma_R}{2} I_{Rf}^2 \\
& + \frac{\partial V_f(\cdot, m_1)}{\partial K_{1f}} (I_{1f} - \delta K_{1f}) + \frac{\partial V_{f,(m_1)}}{\partial K_{Rf}} (I_{Rf} - \delta_R K_{Rf}) \\
& + \frac{\partial V_f(\cdot, m_1)}{\partial K_{1(-f)}} (\phi_{1(-f)} - \delta K_{1(-f)}) + \frac{\partial V_{f,(m_1)}}{\partial K_{R(-f)}} (\phi_{R(-f)} - \delta_R K_{R(-f)}) \\
& + (\alpha I_{Rf} + \beta K_{Rf}^{\psi})\,(V_f(\cdot, m_2) - V_f(\cdot, m_1)) \\
& + (\alpha \phi_{R(-f)} + \beta K_{R(-f)}^{\psi})\,(V_f(\cdot, m_3) - V_f(\cdot, m_1)) \Big].
\end{aligned} \tag{14}
$$

The last two terms on the right hand side of the HJB-equation have to be added to capture the effect of the future jump either to mode m_2 or mode m_3 on the value function in mode m_1. The right hand side (RHS) of equation (14) is strictly concave in (I_{1f}, I_{Rf}). Consequently, the first order conditions for the maximization of the RHS are necessary and sufficient and yield expressions (9), (10) and (11) for I_{1f} and I_{Rf}.

Mode $\mathbf{m}_2$: in modes m_2 (and m_3) the HJB equations of the innovator and the laggard differ substantially. In mode m_2, the HJB equation of the innovator firm A reads

$$
\begin{aligned}
& r\ V_A(\vec{K}^{m_2}, m_2) \\
& = \max_{I_{1A}, I_{2A}} \Big[(1-(K_{1A}+K_{1B}) - \eta K_{2A})K_{1A} - \mu_1 I_{1A} - \frac{\gamma_{1A}}{2} I_{1A}^2 \\
& + (1+\theta - K_{2A} - \eta(K_{1A}+K_{1B}))K_{2A} - \mu_2 I_{2A} - \frac{\gamma_{2A}}{2} I_{2A}^2 \\
& + \frac{\partial V_A(\vec{K}^{m_2}, m_2)}{\partial K_{1A}} (I_{1A} - \delta_1 K_{1A}) + \frac{\partial V_A(\vec{K}^{m_2}, m_2)}{\partial K_{1B}} (\phi_{1B}(\vec{K}^{m_2}, m_2) - \delta_1 K_{1B}) \\
& + \frac{\partial V_A(\vec{K}^{m_2}, m_2)}{\partial K_{2A}} (I_{2A} - \delta_2 K_{2A}) + \frac{\partial V_A(\vec{K}^{m_2}, m_2)}{\partial K_{RB}} (\phi_{RB}(\vec{K}^{m_2}, m_2) - \delta_R K_{RB}) \\
& + (\alpha \phi_{RB}(\vec{K}^{m_2}, m_2) + \beta K_{RB}^{\psi})(V_A(K_{1A}, K_{2A}, K_{1B}, 0, m_4) - V_A(\vec{K}^{m_2}, m_2)) \Big].
\end{aligned} \tag{15}
$$

For the laggard firm B we obtain

$$
\begin{aligned}
&r\ V_B(\vec{K}^{m_2}, m_2) \qquad (16)\\
&= \max_{I_{1B}, I_{RB}} \Big[(1-(K_{1A}+K_{1B})-\eta K_{2A})K_{1B} - \mu_1 I_{1B} - \frac{\gamma_{1B}}{2} I_{1B}^2 - \mu_R I_{RB} \\
&\quad -\frac{\gamma_{RB}}{2} I_{RB}^2 + \frac{\partial V_B(\vec{K}^{m_2}, m_2)}{\partial K_{1B}}(I_{1B}-\delta_1 K_{1B}) + \frac{\partial V_B(\vec{K}^{m_2}, m_2)}{\partial K_{RB}}(I_{RB}-\delta_R K_{RB}) \\
&\quad +\frac{\partial V_B(\vec{K}^{m_2}, m_2)}{\partial K_{1A}}(\phi_{1A}(\vec{K}^{m_2}, m_2)-\delta_1 K_{1A}) + \frac{\partial V_B(\vec{K}^{m_2}, m_2)}{\partial K_{2A}}(\phi_{2A}(\vec{K}^{m_2}, m_2)-\delta_2 K_{2A}) \\
&\quad +(\alpha I_{RB} + \beta K_{RB}^{\psi})(V_B(K_{1A}, K_{2A}, K_{1B}, 0, m_4) - V_B(\vec{K}^{m_2}, m_2))\Big].
\end{aligned}
$$

In this mode, the last term captures the effect of the future jump to mode m_4 on the current value function.

Like in mode m_1, the derivation of the expressions of the investment functions (9) and (12) by the first order conditions is straightforward.

Symmetric equations are obtained for mode m_3, where firm B is the innovator and firm A is the laggard.

Mode $\mathbf{m}_4$: in this mode the HJB equations are again symmetric across firms and read

$$
\begin{aligned}
&r\ V_f(\vec{K}^{m_4}, m_4) \qquad (17)\\
&= \max_{I_{1f}, I_{2f}} \Big[K_{1f}(1-(K_{1A}+K_{1B})-\eta(K_{2A}+K_{2B})) - \mu_1 I_{1f} - \frac{1}{2}\gamma_{1f} I_{1f}^2 \\
&\quad +K_{2f}(1+\theta-\eta(K_{1A}+K_{1B})-(K_{2A}(t)+K_{2B})) - \mu_2 I_{2f} - \frac{1}{2}\gamma_{2f} I_{2f}^2 \\
&\quad +\frac{\partial V_f(\vec{K}^{m_4}, m_4)}{\partial K_{1f}}(I_{1f}-\delta_1 K_{1f}) + \frac{\partial V_f(\vec{K}^{m_4}, m_4)}{\partial K_{2f}}(I_{2f}-\delta_2 K_{2f}) \\
&\quad + \frac{\partial V_f(\vec{K}^{m_4}, m_4)}{\partial K_{1(-f)}}(\phi_{1(-f)}(\vec{K}^{m_4})-\delta_1 K_{1(-f)}) + \frac{\partial V_f(\vec{K}^{m_4}, m_4)}{\partial K_{2(-f)}}(\phi_{2(-f)}(\vec{K}^{m_4})-\delta_2 K_{2(-f)})\Big].
\end{aligned}
$$

By the strict concavity of the RHS in (I_{1f}, I_{2f}), the first order conditions are necessary and sufficient and again yield the expressions (9) for the investment functions.

Online Appendix

Appendix B: Details of the Numerical Procedure

In order to numerically determine a Markov Perfect Equilibrium strategy profile of the entire game, we first find an MPE in the final mode m_4 and calculate the corresponding value functions of both players. Due to the linear quadratic structure of the game in mode m_4 we assume value functions of the form

$$\begin{aligned} V_f(\vec{K}^{m_4}, m_4) = \kappa_0^f &+ \sum_{i=1,2} \sum_{j=A,B} \left(\kappa_{ij}^f K_{ij} + \frac{1}{2} \xi_{ij}^f K_{ij}^2 \right) \\ &+ \zeta_{1f2f}^f K_{1f} K_{2f} + \zeta_{1f1(-f)}^f K_{1f} K_{1(-f)} + \zeta_{1f2(-f)}^f K_{1f} K_{2(-f)} \\ &+ \zeta_{2f1(-f)}^f K_{2f} K_{1(-f)} + \zeta_{2f2(-f)}^f K_{2f} K_{2(-f)} + \zeta_{1(-f)2(-f)}^f K_{1(-f)} K_{2(-f)}. \end{aligned}$$

Using the first order conditions, the feedback investment function of firm $f = A, B$ can be expressed as

$$\phi_{if}^*(\vec{K}^{m_4}, m_4) = \frac{1}{\gamma_{if}} \left(\kappa_{if}^f - \mu_i + \xi_{if}^f K_{if} + \zeta_{ifi(-f)}^f K_{i(-f)} + \zeta_{if(-i)f}^f K_{(-i)f} + \zeta_{if(-i)(-f)}^f K_{(-i)(-f)} \right) \tag{18}$$

with $i \in \{1, 2\}$. Inserting these value and feedback functions into the HJB equations (17) and equating the coefficients of the different orders of K_{ij} on both sides of the resulting equation yields a system of 30 non-linear algebraic equations with 30 unknowns, namely the coefficient vectors κ^f, ξ^f and ζ^f for $f = A, B$. Using the symmetry between the value functions of the two firms this system reduces to 15 equations with 15 unknowns. For a given parameter set this system of algebraic equations can be solved by standard numerical methods (e.g. a Newton method). Typically, several solutions to this system of equations can be found. In order to identify possible candidates for Markov Perfect equilibria in this mode, we insert (18) into the state dynamics in m_4, using the calculated coefficient vector and check the stability of the (unique) steady state of the resulting dynamical system. In all numerical settings that we have considered, a single solution has been found fulfilling this stability property.[24] The stability of the steady state in mode m_4 induces that the corresponding transversality conditions for the dynamic optimization problems of both firms $\lim_{t \to +\infty} e^{-rt} V_A(\vec{K}^{m_4}, m_4) = 0$ and $\lim_{t \to +\infty} e^{-rt} V_B(\vec{K}^{m_4}, m_4) = 0$ are satisfied.

After the determination of the value functions of both firms in mode m_4, we solve the coupled HJB equations of both players in modes m_2 and m_3. We restrict our description here to mode m_2. The value and feedback functions in mode m_3 can be obtained from those in m_2 by swapping the roles of the firms A and B.

The general approach underlying our procedure in these modes, as well as in mode m_1, is to determine polynomial approximations of the different value functions (see Vedenov and Miranda (2001), Dawid et al. (2017)), with the property that, after inserting these approximate value functions and the corresponding feedback functions into the HJB equation of the corresponding mode, the (absolute) value of the difference between the left and the right hand side of the HJB equation is sufficiently small. To obtain such an

[24] This statement does not imply that the MPE which we found is necessarily unique. By assuming a quadratic value function we have restricted attention to MPE with linear feedback functions and there could be additional MPE exhibiting non-linear feedback functions.

approximation we define a set of basis functions consisting of multivariate polynomials of the four state variables in mode $m \in \{m_1, m_2, m_3\}$ and a set of nodes $\mathcal{N}^m$ in the state space of mode m. In the standard approach for collocation, the set of multivariate basis functions and of the collocation nodes are given by the tensor product of uni-variate Chebyshev polynomials and uni-dimensional Chebyshev nodes respectively. However, due to the dimension of the state space in our problem, this approach has proven ineffective with respect to computational effort and convergence properties of the collocation scheme. Therefore, we apply a Smolyak collocation, which is a sparse grid method suitable for generating approximate solutions to the Hamilton-Jacobi-Bellman (HJB) equations for large scale dynamic economic systems subject to the curse of dimensionality. The method consists of selecting collocation nodes and associated polynomials by a non-tensor product rule that guarantees accuracy in spite of considering only a small subset of nodes and basis functions resulting from the tensor product (see e.g. Maliar and Maliar (2014), Judd et al. (2014)). The cardinality of the Smolyak grid is a polynomial function of the number of nodes in each dimension, compared to the exponential function that results from the use of a tensor product.

The relevant state variables in mode m_2 are K_{1A}, K_{1B}, K_{2A} and K_{RB} with the state space $\mathcal{K}^{m_2} = [0, \bar{K}_1]^2 \times [0, \bar{K}_2] \times [0, \bar{K}_R]$. In order to determine the sparse grid of collocation nodes $\mathcal{N}^{m_2}$, we follow the approach originally introduced by Smolyak (1963) for representing smooth functions on multidimensional hypercubes (see Maliar and Maliar (2014)). We define for each variable K_{if} a set $\mathcal{N}_{if} = \left\{\hat{K}_{if}^j\right\}_{j=1}^{n_{if}}$ of Chebyshev nodes with

$$\hat{K}_{if}^j = \frac{\bar{K}_i}{2}\left(1 + \cos\left(\frac{(n_{if} - j + 0.5)\pi}{n_{if}}\right)\right), \; i = 1, 2, R, \; f = A, B.$$

We always choose an odd number of nodes n_{if} such that $\hat{K}_{if}^j$ for $j = \frac{n_{if}+1}{2}$ is positioned in the middle of the state interval and the other nodes are symmetric around that node. Based on this set we construct a sequence of nested subsets of $\mathcal{N}_{if}$ in a way that $\mathcal{N}_{if}^1 = \left\{\hat{K}_{if}^{\frac{n_{if}+1}{2}}\right\}$, $\mathcal{N}_{if}^2 = \mathcal{N}_{if}^1 \cup \{\hat{K}_{if}^1, \hat{K}_{if}^{n_{if}}\}$. Afterwards, $\mathcal{N}_{if}^{l+1}$ is obtained by adding to $\mathcal{N}_{if}^l$ for each interval between neighboring nodes in $\mathcal{N}_{if}^l$ the point in $\mathcal{N}_{if} \setminus \mathcal{N}_{if}^l$ which is closest to the center between these two nodes. Hence, the number of nodes in the set $\mathcal{N}_{if}^l$ is $2^{l-1}+1$. The number of sets in the sequence $\{\mathcal{N}_{if}^l\}_{l=1}^{\bar{l}_{if}}$ is therefore determined by $2^{\bar{l}_{if}-1} - 1 = n_{if}$. In particular, for $n_{if} = 5$ the number of sets in this sequence is $\bar{l}_{if} = 3$, whereas we have $\bar{l}_{if} = 4$ for $n_{if} = 9$. Intuitively, each set $\mathcal{N}_{if}^l$ in this sequence of sets (apart from the first) covers the entire considered interval for this state variable, where the coverage becomes finer the larger l is. In order to construct the set of nodes in the full four-dimensional state space we define levels of precision $\nu_i, i = 1, 2, R$ for each type of state variable and $\bar{\nu} = \max[\nu_1, \nu_2, \nu_R]$. We then obtain the set of nodes as

$$\mathcal{N}^{m_2} = \bigcup_{\substack{l_{if} \leq \nu_i \\ 4 \leq l_{1A}+l_{1B}+l_{2A}+l_{RB} \leq 4+\bar{\nu}}} \; \bigcup_{\hat{K}_{if} \in \mathcal{N}_{if}^{l_{if}}} \left\{(\hat{K}_{1A}, \hat{K}_{1B}, \hat{K}_{2A}, \hat{K}_{RB})\right\}.$$

This procedure of constructing the sparse grid ensures that a relatively fine coverage of the range of a certain state variables is included in the grid for 'important' values of the other state variables. This holds in particular for values of these other state variables in the center and close to the boundary of their range.

The other values of these state-variables are combined with a much coarser set of values of the considered state variable. For further reference we write $\mathcal{N}^{m_2} = \{\hat{K}_i^{m_2}\}_{i=1}^n$ such that we associate each node with an index $i = 1, .., n$.

Based on extensive numerical explorations, we have chosen a setting with $n_{1A} = n_{1B} = 5, n_{2A} = n_{2B} = n_{RA} = n_{RB} = 9$ as the number of grid points for the different dimensions as well as a degree of precision of 2 (for the variable K_{1f}) respectively 3 (for the variables K_{2f} and K_{Rf}) for the construction of the Smolyak grids and basis functions in the different modes. In each mode this setting generates a grid and a set of basis functions of cardinality 93. If simple tensor products were used, a cardinality of 2025 would arise. The number of the grid points and basis functions determines the number of nonlinear algebraic equations to be solved in each mode. Hence, the relatively low number induced by the Smolyak approach improves computational time compared to the use of tensor grids. This is essential for the feasibility of our analysis and robustness checks in light of the fact that numerical calculations have to be carried out sequentially in all the different modes. Furthermore, the relatively low dimension of the systems of equations also fosters convergence of the numerical procedures used to solve them.

The construction of the multivariate basis functions for the collocation is analogous to that of the construction of the grid. For each variable in the relevant state space we define a set of univariate basis functions $\mathcal{B}_{if} = \left\{b_{if}^j\right\}_{j=1}^{n_{if}}$ with

$$b_{if}^j(K_{if}) = T_{j-1}\left(K_{if}/\bar{K}_{if}\right),$$

where $T_j(x)$ is the Chebyshev polynomial of degree j defined on the interval $[0,1]$. We then define for each state variable a sequence of sets of basis functions such that the cardinality of each set coincides with that of the corresponding set of nodes: $\mathcal{B}_{if}^l = \{b_{if}^j\}_{j=1}^{k_{if}^l}$ with $k_{if}^l = |\mathcal{N}_{if}^l|$ for each $l \leq \bar{l}_{if}$. The set of multi-variate basis functions is then given by

$$\mathcal{B}^{m_2} = \bigcup_{\substack{l_{if} \leq \nu_i \\ 4 \leq l_{1A}+l_{1B}+l_{2A}+l_{RB} \leq 4+\bar{\nu}}} \quad \bigcup_{b_{if} \in \mathcal{B}_{if}^{l_{if}}} \left\{b_{1A}(K_{1A}) \cdot b_{1B}(K_{1B}) \cdot b_{2A}(K_{2A}) \cdot b_{RB}(K_{RB})\right\}.$$

By construction we have $|\mathcal{B}^{m_2}| = |\mathcal{N}^{m_2}| = n$. Writing $\mathcal{B}^{m_2} = \{b_j^{m_2}\}_{j=1}^n$ the approximate value function of firm $f = A, B$ in mode m_2 is then expressed as

$$\hat{V}_f(\vec{K}^{m_2}, m_2) = \sum_{j=1}^n c_j^{f,m_2} b_j^{m_2}(\vec{K}^{m_2}).$$

As a first step in the process of determining an appropriate value of the coefficient vector $\vec{c}^{f,m_2}$, we calculate for each node $\hat{K}_i^{m_2} = (K_{1A}^i, K_{1B}^i, K_{2A}^i, K_{RB}^i) \in \mathcal{N}^{m_2}$ the market profit for both firms denoted by

$$\begin{aligned}\pi_i^{A,m_2} &= (1-(K_{1A}^i+K_{1B}^i) - \eta K_{2A}^i)K_{1A}^i + (1+\theta-\eta(K_{1A}^i+K_{1B}^i) - K_{2A}^i)K_{2A}^i,\\ \pi_i^{B,m_2} &= (1-(K_{1A}^i+K_{1B}^i) - \eta K_{2A}^i)K_{1B}^i\end{aligned}$$

and the continuation value of the game if the mode would switch to m_4 at that point in the state space:

$v_i^{f,m_4} = V_f((K_{1A}^i, K_{1B}^i, K_{2A}^i, 0), m_4)$. Furthermore, we introduce the $n \times n$ matrices B and B^K with entries

$$B_{ij} = b_j^{m_2}(\hat{K}_i^{m_2}), \quad B_{ij}^K = \frac{\partial}{\partial K} b_j^{m_2}(\hat{K}_i^{m_2}), \quad i,j = 1,..,n, \; K \in \{K_{1A}, K_{1B}, K_{2A}, K_{RB}\},$$

giving the values of the basis functions and their state derivatives at all nodes in $\mathcal{N}^{m_2}$. Using this notation it follows from (9) and (12) that, for a given coefficient vector $\vec{c}^{f,m2}$ the optimal investments of the two firms at node $\hat{K}_i^{m_2} \in \mathcal{N}^{m_2}$ are given by

$$\begin{array}{rcl} I_{1A,i}^*(\vec{c}^{A,m_2}) &=& \frac{1}{\gamma_{1A}} \left(\sum_{j=1}^n c_j^{A,m2} B_{ij}^{K_{1A}} - \mu_1 \right), \\ I_{1B,i}^*(\vec{c}^{B,m_2}) &=& \frac{1}{\gamma_{1B}} \left(\sum_{j=1}^n c_j^{B,m2} B_{ij}^{K_{1B}} - \mu_1 \right), \\ I_{2A,i}^*(\vec{c}^{A,m_2}) &=& \frac{1}{\gamma_{2A}} \left(\sum_{j=1}^n c_j^{A,m2} B_{ij}^{K_{2A}} - \mu_2 \right), \\ I_{RB,i}^*(\vec{c}^{B,m_2}) &=& \frac{1}{\gamma_{RB}} \left(\sum_{j=1}^n c_j^{B,m2} B_{ij}^{K_{RB}} - \mu_R + \alpha \left(v_i^{B,m_4} - \sum_{j=1}^n c_j^{B,m2} B_{ij} \right) \right). \end{array} \tag{19}$$

The collocation problem then is to find coefficient vectors $\vec{c}^{A,m2}, \vec{c}^{B,m2}$ such that the following two equations, corresponding to (15) and (16), are satisfied for all $i = 1,..,n$:

$$\begin{array}{rcl} 0 &=& \pi_i^{A,m_2} - \mu_1 I_{1A,i}^*(\vec{c}^{A,m_2}) - \frac{\gamma_{1A}}{2} \left(I_{1A,i}^*(\vec{c}^{A,m_2}) \right)^2 - \mu_2 I_{2A,i}^*(\vec{c}^{A,m_2}) - \frac{\gamma_{2A}}{2} \left(I_{2A,i}^*(\vec{c}^{A,m_2}) \right)^2 \\ && + \sum_{j=1}^n c_j^{A,m2} B_{ij}^{K_{1A}} \left(I_{1A,i}^*(\vec{c}^{A,m_2}) - \delta_1 K_{1A}^i \right) + \sum_{j=1}^n c_j^{A,m2} B_{ij}^{K_{1B}} \left(I_{1B,i}^*(\vec{c}^{B,m_2}) - \delta_1 K_{1B}^i \right) \\ && + \sum_{j=1}^n c_j^{A,m2} B_{ij}^{K_{2A}} \left(I_{2A,i}^*(\vec{c}^{A,m_2}) - \delta_2 K_{2A}^i \right) + \sum_{j=1}^n c_j^{A,m2} B_{ij}^{K_{RB}} \left(I_{RB,i}^*(\vec{c}^{B,m_2}) - \delta_R K_{RB}^i \right) \\ && + \left(\alpha I_{RB,i}^*(\vec{c}^{B,m_2}) + \beta \left(K_{RB}^i \right)^\Psi \right) \left(v_i^{A,m_4} - \sum_{j=1}^n c_j^{A,m2} B_{ij} \right) - r \sum_{j=1}^n c_j^{A,m2} B_{ij}, \\ 0 &=& \pi_i^{B,m_2} - \mu_1 I_{1B,i}^*(\vec{c}^{B,m_2}) - \frac{\gamma_{1B}}{2} \left(I_{1B,i}^*(\vec{c}^{B,m_2}) \right)^2 - \mu_R I_{RB,i}^*(\vec{c}^{B,m_2}) - \frac{\gamma_{RB}}{2} \left(I_{RB,i}^*(\vec{c}^{B,m_2}) \right)^2 \\ && + \sum_{j=1}^n c_j^{B,m2} B_{ij}^{K_{1A}} \left(I_{1A,i}^*(\vec{c}^{A,m_2}) - \delta_1 K_{1A}^i \right) + \sum_{j=1}^n c_j^{B,m2} B_{ij}^{K_{1B}} \left(I_{1B,i}^*(\vec{c}^{B,m_2}) - \delta_1 K_{1B}^i \right) \\ && + \sum_{j=1}^n c_j^{B,m2} B_{ij}^{K_{2A}} \left(I_{2A,i}^*(\vec{c}^{A,m_2}) - \delta_2 K_{2A}^i \right) + \sum_{j=1}^n c_j^{B,m2} B_{ij}^{K_{RB}} \left(I_{RB,i}^*(\vec{c}^{B,m_2}) - \delta_R K_{RB}^i \right) \\ && + \left(\alpha I_{RB,i}^*(\vec{c}^{B,m_2}) + \beta \left(K_{RB}^i \right)^\Psi \right) \left(v_i^{B,m_4} - \sum_{j=1}^n c_j^{B,m2} B_{ij} \right) - r \sum_{j=1}^n c_j^{B,m2} B_{ij}. \end{array} \tag{20}$$

The solution to this system is found by defining a dynamical system on the space $\mathcal{R}^{2n}$ such that the fixed point of the system is a solution to (20) and each step involves the solution of a system of linear equations of dimension $2n$. More precisely, we determine a termination criterion $\bar{\xi} > 0$, and then, starting from an initial guess $(\vec{c}^{A,m_2,0}, \vec{c}^{B,m_2,0})$, in each iteration $\tau = 0, \ldots$ the following steps are carried out

1. Calculate $I_{z,i}^*(\vec{c}^{f,m_2,\tau}), \; i = 1,..,n, z \in \{1A, 1B, 2A, RB\}$ according to (19).
2. Calculate the $n \times n$ matrix $M = (m_{ij})$ with

$$\begin{array}{rcl} m_{ij} &=& B_{ij}^{K_{1A}} (I_{1A,i}^*(\vec{c}^{A,m_2,\tau}) - \delta_1 K_{1A}^i) + B_{ij}^{K_{1B}} (I_{1B,i}^*(\vec{c}^{B,m_2,\tau}) - \delta_1 K_{1B}^i) \\ && + B_{ij}^{K_{2A}} (I_{2A,i}^*(\vec{c}^{A,m_2,\tau}) - \delta_2 K_{1B}^i) + B_{ij}^{K_{RB}} (I_{RB,i}^*(\vec{c}^{B,m_2,\tau}) - \delta_R K_{RB}^i) \\ && - \left(\alpha I_{RB,i}^*(\vec{c}^{B,m_2,\tau}) + \beta \left(K_{RB}^i \right)^\Psi \right) B_{ij} - r B_{ij} \end{array}$$

and the vectors $\vec{y}^f = (y_i^f)_{i=1}^n,\ f = A, B$ as

$$y_i^f = \pi_i^{f,m_2} + \left(\alpha I^*_{RB,i}(\vec{c}^{B,m_2,\tau}) + \beta \left(K^i_{RB}\right)^{\Psi}\right) v_i^{f,m_4}.$$

3. Determine $(\vec{c}^{A,m_2,\tau+1}, \vec{c}^{B,m_2,\tau+1})$ as the solution of

$$M \cdot \vec{c}^{f,m_2,\tau+1} = \vec{y}^f,\ f = A, B.$$

4. Calculate the maximal (absolute) change of the value functions on the grid $\mathcal{N}^{m_2}$ as

$$\xi = \max_{i=1,..,n} \left[\max\left(\left|\left(B \cdot (\vec{c}^{A,m_2,\tau+1} - \vec{c}^{A,m_2,\tau})\right)_i\right|, \left|\left(B \cdot (\vec{c}^{B,m_2,\tau+1} - \vec{c}^{B,m_2,\tau})\right)_i\right|\right)\right]$$

and terminate if $\xi < \bar{\xi}$.

Once the algorithm has terminated, the maximal deviation between the left and the right hand side of the two HJB equations on the entire state space is checked. To illustrate, we denote by $\hat{V}_A(\vec{K}^{m_2}, m_2)$ the approximated value function of firm A in mode m_2 and by $\hat{\phi}_{iA}(\vec{K}^{m_2}, m_2), i = 1, 2$ the feedback function obtained by inserting this approximate value function (and those of the subsequent modes) into (9). The considered error is then given by

$$ERR^{A,m_2} = \max_{\vec{K}^{m_2} \in \mathcal{K}^{m_2}} \frac{|\Delta^{A,m_2}(\vec{K}^{m_2}, m_2)|}{\hat{V}_A(\vec{K}^{m_2})},$$

where $\mathcal{K}^{m_2} = [0,1] \times [0, \bar{K}_2] \times [0,1] \times [0, \bar{K}_R]$ is the relevant state space in mode m_2 and

$$\begin{aligned}
&\Delta^{A,m_2}(\vec{K}^{m_2}, m_2) \\
&= r\ \hat{V}_A(\vec{K}^{m_2}, m_2) - (1 - (K_{1A} + K_{1B}) - \eta K_{2A})K_{1A} - (1 - \eta(K_{1A} + K_{1B}) - K_{2A})K_{2A} \\
&\quad + \mu_1 \hat{\phi}_{1A}(\vec{K}^{m_2}, m_2) + \frac{\gamma_{1A}}{2}\hat{\phi}_{1A}(\vec{K}^{m_2}, m_2)^2 + \mu_2 \hat{\phi}_{2A}(\vec{K}^{m_2}, m_2) + \frac{\gamma_{2A}}{2}\hat{\phi}_{2A}(\vec{K}^{m_2}, m_2)^2 \\
&\quad - \frac{\partial \hat{V}_A(\vec{K}^{m_2}, m_2)}{\partial K_{1A}}(\hat{\phi}_{1A}(\vec{K}^{m_2}, m_2) - \delta_1 K_{1A}) - \frac{\partial \hat{V}_A(\vec{K}^{m_2}, m_2)}{\partial K_{2A}}(\hat{\phi}_{2A}(\vec{K}^{m_2}, m_2) - \delta_2 K_{2A}) \\
&\quad - \frac{\partial \hat{V}_A(\vec{K}^{m_2}, m_2)}{\partial K_{1B}}(\hat{\phi}_{1B}(\vec{K}^{m_2}, m_2) - \delta_1 K_{1B}) - \frac{\partial \hat{V}_A(\vec{K}^{m_2}, m_2)}{\partial K_{RB}}(\hat{\phi}_{RB}(\vec{K}^{m_2}, m_2) - \delta_R K_{RB}) \\
&\quad - (\alpha \hat{\phi}_{RB}(\vec{K}^{m_2}, m_2) + \beta K_{RB}^{\psi}) \left(\hat{V}_A(K_{1A}, K_{2A}, K_{1B}, 0, m_4) - \hat{V}_A(\vec{K}^{m_2}, m_2)\right) .
\end{aligned}$$

gives the difference between the left hand side and the right hand side of the HJB equation (15) of firm A. The error in the HJB equation of firm B is defined analogously. If the resulting error is too large, then the size of the grid n is adjusted through changes in the number of nodes n_i or levels of precision ν_i for at least one of the dimensions $i = 1, 2, R$. This procedure is repeated until a solution with a sufficiently small error in the HJB equations is obtained. In spite of the relatively sparse grid that has been used, for all numerical results reported in this paper the error of the HJB on the relevant state space satisfies $\max[ERR^{A,m}, ERR^{B,m}] < 5 * 10^{-4}$ for $m = m_1, m_2, m_3$.

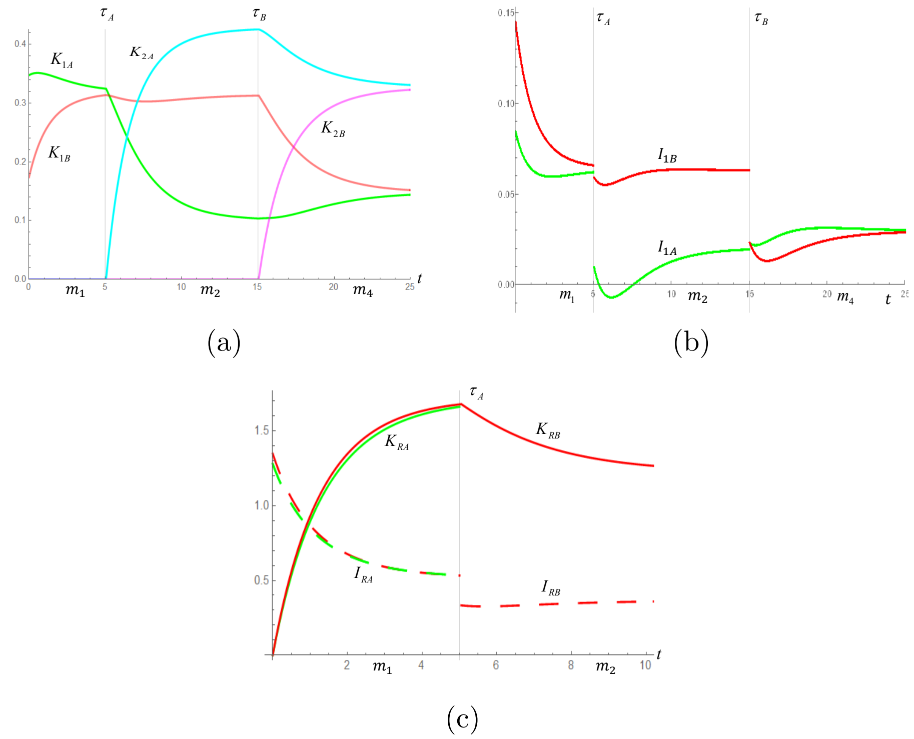

Figure 11: Equilibrium dynamics under the baseline parameter setting of (a) of production capacities, (b) investments in capacities for the established product, (c) knowledge stocks and R&D investments for asymmetric initial production capacities on the established market. It is assumed that the larger firm A (green lines) innovates earlier than firm B (red lines) and the innovation time of firm A is indicated as τ_A.

Initial guesses for the coefficient vectors $(\vec{c}^{A,m_2,0}, \vec{c}^{B,m_2,0})$ are typically obtained through a continuation method by using the obtained solution for a parameter setting in the close proximity of the parameter values under consideration.

Once the approximate value functions in modes m_2 and m_3 have been determined, an analogous procedure is applied to calculate the value and feedback functions in mode m_1, inserting the values in mode m_2, m_3 into the right hand side of the HJB equations in mode m_1.

Appendix C: Additional Robustness Checks

Dynamics if the larger firm innovates first

In our illustration of the dynamics of capital and knowledge stocks as well as the associated investments in Figure 2 we have assumed that firm B, which has smaller capacities on the established market, innovates first. Although, we have shown that this is the more likely scenario (see also Figure 2(d)) there is also a positive probability that the larger firm A innovates first. In Figure 11 below we show the analogous time series to panels (a) - (c) of Figure 2 for the case where firm A wins the innovation race and is the first innovator.

Figure 11 shows that all qualitative features of the dynamics, in particular the large downward jump of

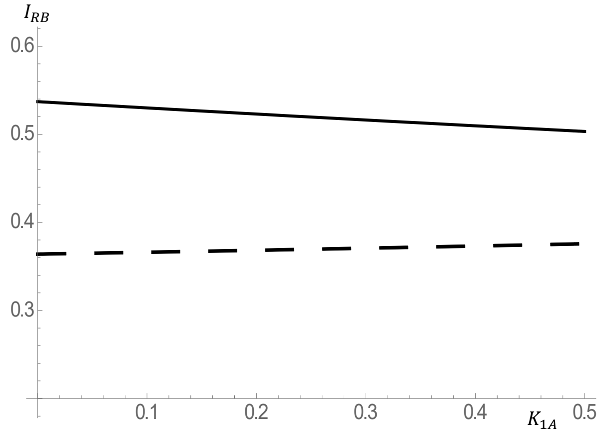

Figure 12: Investment in R&D knowledge of firm B depending on production capacity of firm A on the established market before (solid line) and after (dashed line) firm A has innovated. The figure is based on the baseline parameter setting with symmetric investment costs on the established market. The values of the variables apart from K_{1A} are given by their steady-state value in the respective mode.

the investments in established market capacities by the innovator and the reduction of R&D effort by the laggard after the competitor's innovation also hold if the larger firm innovates first. Similarly, Figure 12 shows that also the observation, that the slope of the laggard's R&D investment function with respect to the innovator's capacity on the established market is positive in the mode after the first innovation, while it is negative during the innovation race, stays intact if the larger firm A wins the innovation race. Figure 12 corresponds to Figure 9 in the main text and shows the optimal R&D investment of firm B for a variation of K_{1A} values in modes m_1 and m_2.

Robustness of qualitative insights under the simultaneous variation of several parameters

In this part of Appendix C we provide additional insights on the robustness of the qualitative properties (i)-(v) if more than one parameter is varied. In particular, we illustrate that the qualitative property (v) is less stable with respect to parameter variations than the other four properties in the sense that it might no longer hold if more than one parameter deviates from its baseline value. This issue arises for the case where the degree of vertical differentiation of the new product is with $\theta = 0.4$ substantially larger than in the baseline $\theta = 0.2$. Table 4 shows that under this parameter variation property (v) ceases to hold if one of the following three scenarios occur: (i) the innovation rate is sufficiently strongly moved upwards by an increase of the coefficient of the current R&D (α), of the exponent of the knowledge stock (ψ), or of the coefficient of the entire rate (ω); (ii) horizontal differentiation is sufficiently low, i.e. η is close to 1; (iii) capital investment is very costly ($\gamma_{if}, i = 1, 2$), or capital depreciates fast ($\delta_i, i = 1, 2$). Analysis of the laggard's equilibrium R&D strategy under these scenarios (not depicted here) shows that the R&D investment becomes completely flat with respect to changes of K_{1B}. In all three cases the intuition developed at the end of Section 7 provides a good explanation for the fact that the positive slope of this strategy disappears.

	Description	Baseline	Tested interval	Robust (i) - (iv)	Robust (v)
α	Effectiveness of current R&D	0.2	$[0.1, 0.3]$	✓	$[0.1, 0.23]$
β	Effectiveness of knowledge stock	0.2	$[0.1, 0.3]$	✓	✓
ψ	Exponent of know. stock in innov. rate	1	$[0.5, 2]$	✓	$[0.5, 1.4]$
ω	Coeff. of innov. rate in mode m_2/m_3	1	$[0.5, 1.5]$	✓	$[0.5, 1.1]$
η	Horizontal differentiation	0.65	$[0.3, 0.8]$	✓	$[0.3, 0.74]$
θ	Vertical differentiation	0.4	$[0, 0.4]$	✓	✓
μ_R	Unit costs of R&D investment	0.1	$[0.05, 0.3]$	✓	✓
γ_{1A}, γ_{1B}	Adjustment costs for product 1	3	$[1, 5]$	✓	$[1, 3.5]$
γ_{2A}, γ_{2B}	Adjustment costs for product 2	3	$[1, 5]$	✓	$[1, 3.5]$
γ_{RA}, γ_{RB}	Adjustment costs for knowledge stock	0.1	$[0.05, 0.5]$	✓	$[0.075, 0.5]$
δ_1	Depreciation rate for capacity K_1	0.2	$[0.1, 0.3]$	✓	$[0.1, 0.24]$
δ_2	Depreciation rate for capacity K_2	0.2	$[0.1, 0.3]$	✓	$[0.1, 0.24]$
δ_r	Depreciation rate of knowledge stock	0.3	$[0.1, 0.5]$	✓	✓
r	Discount rate	0.04	$[0.02, 0.06]$	✓	✓

Table 4: Range of parameter values for which the different qualitative properties listed in Table 2 are satisfied. The vertical differentiation parameter is set to $\theta = 0.4$ and all parameters apart from θ and the parameter under variation have the default values given in Table 1. A checkmark indicates that these properties were satisfied on the entire tested interval.

Appendix References

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