Capacity Investment under Uncertainty with Volume Flexibility — Preemption Analysis⋆

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Abstract

An investment decision involves different dimensions like, e.g., timing, size, and technology. Concerning the latter, this paper focuses on volume flexibility in the sense that the investing firm can choose between a dedicated technology, where the firm always has to produce up to capacity, and a volume flexible technology where the firm can also choose to produce below capacity. The present paper considers such investment decisions in a duopoly framework with demand uncertainty. Clearly, choosing for a flexible technology has the advantage that the firm can adjust its production amount to different demand realizations. On the other hand, choosing a dedicated technology implies that the firm is committed to produce a certain amount, which is advantageous from a strategic point of view. Our main results are threefold. First, the equilibrium is sequential where under limited demand uncertainty the first investor chooses for a dedicated technology, and the second investor takes the flexible technology. Second, if demand is more uncertain, the first investor goes for the flexible technology, where the second investor reacts by choosing the dedicated one. Third, in case the first investor chooses for a dedicated technology, we show that the optimal time and size of the investment is not influenced by the follower's choice regarding a dedicated or flexible technology.

Keywords: Investment under Uncertainty, Duopoly, Volume Flexibility, Preemption

1 Introduction

The advancement in production technology has made production firms more efficient in coping with market demand uncertainty. A significant advancement is the volume flexibility, i.e., the ability to operate profitably at different output levels (Sethi and Sethi, 1990). There are different concepts regarding the volume flexibility. In static models, Goyal and Netessine (2011) takes the volume flexibility as to increase or decrease production above and below installed capacity at a cost; Stigler (1939) considers from economics perspective that volume flexibility depends on the divisibility of the production plant, and a firm is less flexible if the average cost curve is steeper around the minimum because it is more costly to deviate from the corresponding output level. In a

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dynamic setting, the volume flexibility is more popular as the concept of adjusting output quantities within the constraint of installed maximal production capacity (Dangl, 1999; Hagspiel et al., 2016; Wen et al., 2017; Wen, 2017). It has also been established in literature that the volume flexibility is important and improves the firm's performance (Beach et al., 2000). In a market with two products, Goyal and Netessine (2011) show that the volume flexibility combats the aggregate demand uncertainty. Hagspiel et al. (2016) and Wen et al. (2017) conclude that the volume flexibility increases the value of the investment.

So far the discussion about adopting volume flexibility to combat the demand uncertainty restrains mainly to a monopoly firm. It makes sense that a monopoly firm chooses this production technology upon investment because it yields larger value. However, the strategic effect of volume flexibility is not very clear. Wen (2017) tries to gain insight about the influence of volume flexibility in a duopoly setting. He concludes that volume flexibility benefits not only the firm adopting it, but also the other firm without volume flexibility. The intuition is that volume flexibility has a buffer effect on the stochastic prices in the sense that the output is adjusted to a volatile market demand. So the the changes in the market price is less dramatic. Whereas this is based on the assumption that volume flexibility is assigned to the follower, i.e., the second investor. But it is not very clear whether the leader (first investor) being dedicated (without volume flexibility) and the follower being flexible (with volume flexibility) is an equilibrium outcome, if they have the choice to be volume flexible. A main reason the investigation is insufficient in the direction is stated by Huisman and Kort (2015) as,

"*We impose that the firm always produces up to capacity. Relaxing this constraint is doable in a monopoly framework (Dangl, 1999), but complicates the analysis considerably in the model with two firms.*"

Within this research work, we take on the challenge and answer the question that, if the volume flexibility is an endogenous decision, i.e., firms can choose whether to be volume flexible or not upon their investments, what is the equilibrium outcome under demand uncertainty in the dynamic setting. In particular, we consider the firm having irreversible capital investment projects to obtain a production plant, and the market demand for the potential product is stochastic. The firm has to decide when to invest, and in case it does invest, whether to be volume flexible and its maximal production capacity. A larger capacity is associated with larger sunk investment costs. This is a real option problem because the firm constantly forecasts the future demand and compares the decisions of investing now and delaying investment. So this paper has connection to the following streams of literature.

Dynamic investment under demand uncertainty. The traditional real option models considers investment timing as in Dixit and Pindyck (1994). Later on the capacity choices are incorporated into firm's investment decisions, which can be found in the research by Dixit (1993), Bar-Ilan and Strange (1999) and Décamps et al. (2006). The general conclusion is that market uncertainty induces the firm to invest later and with a larger investment capacity. Huisman and Kort (2015) extend the firm's investment decision of timing and capacity to a duopoly setting, which has motivated flourishing literature studying firm's strategy interactions. These interactions focus mainly on investment decisions to deter or accommodate the competitor's market entry, especially under some specific market conditions such as capacity expansion (Huberts et al., 2019), and the potential competition from a third firm (Lavrutich et al., 2016). An overview on this stream of literature has been conducted by Huberts et al. (2015), Trigeorgis et al. (1996) and Trigeorgis and Tsekrekos (2018). The underlying assumption of these literature is that the firm utilizes all the invested capacity during its production, i.e., without volume flexibility. Whereas the main contribution of this paper is to consider also the choice of volume flexibility apart from the investment timing and capacity. Besides, given the complexity of the analysis, the interaction between the duopolistic firms focuses mainly on the preemption/deterrence investment decisions.

The value of commitment in competition. Commitment has been considered valuable because the inability to back down poses a credible threat during competition and confrontation (Cong and Zhou, 2019; Schelling, 1980; Fudenberg and Tirole, 1991). For instance, the incumbent with excessive capacity can commit to an expanded output so as to deter the entry of its competitor (Spence, 1977). So far the literature about commitment finds itself mainly in the setting without uncertainty. When there is market uncertainty, the investigation about commitment have been conducted by (Anand and Girotra, 2007) and (Anupindi and Jiang, 2008), and they support that as long as there is competition the commitment is valuable. Their analysis bases mainly on a static setting. A common approach is to carry out analysis for the stages both before and after the uncertainty is resolved. Cong and Zhou (2019) consider duopoly competition on a Hotelling line with uncertainty about the customer distribution. Before the uncertainty realization, both firms decide on their rigidity or flexibility, simultaneously or sequentially. After the uncertainty about customer distribution is realized, the flexible firm can reposition itself on the Hotelling line and the rigid firm cannot, and the two firms compete on prices. In their model, both firms choosing flexibility can arise in the equilibrium if it yields larger payoff than rigidity for both firms. They find that under larger uncertainty, rigidity softens competition and generates commitment value, and flexibility generates option value. Both values can spill over to competitors. There are several differences between the present paper and (Cong and Zhou, 2019). The present paper studies a dynamic and continuous time model, which allows richer observations about the interactions between two firms, i.e., the timing decisions and preemption analysis. We show that both firms choosing flexibility is not an equilibrium in dynamic setting.

		Follower	
		Dedicated	Flexible
Leader	Dedicated	Huisman and Kort (2015)	Wen (2017)
	Flexible	This paper	This paper

Table 1: Extension to previous research work.

The main contribution for this paper is to analyze duopoly firms' volume flexibility decision in dynamic investment under uncertainty. So it naturally extends the research work featuring dynamic investment without volume flexibility by Huisman and Kort (2015). Specifically, in addition to their model where both the leader and the follower are dedicated (symmetric firm rols), we conduct analysis for other different firm role combinations, see Table 1. [1] Another difference is that to answer our research question it is sufficient to anlyze only the non-simultaneous investment, i.e., the deterrence strategy as in Huisman and Kort (2015). In particular, we derive the investment decisions of both the leader and the follower for the corresponding exogenous firm roles. The dedicated and flexible firms's value are then compared between different settings. This allows us to rule out both firms choosing volume flexibility as an equilibrium output. The preemption analysis is conducted for different demand uncertainty levels between a flexible and a dedicated firm. Our result shows that when the demand uncertainty is low, the firm choosing dedicated production invests first and the second investor chooses volume flexibility. When the demand uncertainty is high, it is the other way around. Both the dedicated and the flexible firms need balance the effects of investing earlier or later than its rival. Investing earlier brings monopoly profits, which is good. On the other hand, when investing earlier than the flexible rival the dedicated firm has to invest a relatively small capacity, which constrains its market share and benefits its rival. When investing later than its flexible rival the dedicated firm can

[1] Part of the analysis for dedicated leader and flexible follower can also be found in Wen (2017).

benefit from the buffer effect of its rival's volume flexibility.

The structure of the paper is as follows: Section 2 introduces the model. Section 3 derived the investment decisions under three different exogenous firm roles. Section 4 applies numerical examples and shows that when uncertainty is low the equilibrium outcome is dedicated leader and flexible follower, and vice versa when uncertainty is large. Section 5 concludes.

2 Model Setup

Two firms need to make investment decisions to enter a market with volatile market demand. The investment decisions not only include the timing and the size of the investment, but also the volume flexibility, i.e., the capability to produce below its investment capacity after investment in case the realized market demand is low. The volume flexibility decision at the moment of investment is irreversible, and once the firm chooses to be non-flexible (dedicated), the firm utilizes all its production capacity and produces always a constant output. Denote by $K_L \geq 0$ and $K_F \geq 0$ the capacity of the first investor (leader) and the second investor (follower) respectively. For both firms, the unit cost for capacity investment is $\delta > 0$ and the unit cost for production is $c > 0$. The price at time $t \geq 0$ is $p(t)$, and in an inverse demand structure when both firms are active the price equals to

$$p(t) = X(t)\Big(1 - \eta\left(Q_L(t) + Q_F(t)\right)\Big),$$

where $\eta > 0$ is a constant, $Q_s(t) \leq K_s$ denotes the production output for firm $s \in \{L, F\}$ at time t if firm s is flexible, $Q_s(t) = K_s$ if firm s is dedicated. The stochastic process $\{X(t)|t \geq 0\}$ follows a geometric Brownian Motion (GBM), i.e.,

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}W_t,$$

in which $X(0) > 0$, μ is the trend parameter, $\sigma > 0$ is the volatility parameter, and $\mathrm{d}W_t$ is the increment of a Wiener process. This inverse linear demand function has among others been adopted by Pindyck (1988) and Huisman and Kort (2015). Both firms are risk neutral and discount against rate r that is assumed to be larger than μ. This is to prevent that it is optimal for the firms to always delay the investment (see Dixit and Pindyck, 1994). From now on the argument of time is dropped whenever there can be no misunderstanding.

3 Investment Decisions under Exogenous Firm Roles

This section analyzes three models where the volume flexibility is designated exogenously to the leader or the follower or both. In particular, they are an extension to the model proposed in Huisman and Kort (2015), where both firms are dedicated. In the following analysis, we use superscript "fd" to denote the model of a flexible leader and a dedicated follower, and "df" the other way around. The subscript of "f" and "d" then represent the flexible and the dedicated firm in these two models. Furthermore, we use superscript "ff" to denote the model of a flexible leader and a flexible follower, and subscript "L" and "F" to represent the corresponding leader and the follower. For each model, we analyze the firms' optimal investment decisions, i.e., the investment capacity K_s^{ij} and the investment threshold X_s^{ij} with $i, j \in \{d, f\}$ and $s \in \{d, f, L, F\}$. Note that the investment happens when $X(t)$ reaches $X_s^{i,j}$ for the first time from below, and we assume $X(t = 0) < X_s^{i,j}$ holds, i.e., neither firm invests at time $t = 0$.

3.1 Dedicated Leader and Flexible Follower

In this model, the leader always produces up to capacity after investment, i.e., $Q_d^{df} = K_d^{df}$, and the follower can produce below capacity, i.e., $Q_f^{df} \leq K_f^{df}$.

3.1.1 Flexible Follower's Investment Decision

Given that $X(t) = X$ and the leader has installed a capacity size K_d^{df}, denote $\pi_f^{df}(K_d^{df}, X, K_f^{df})$ as the profit for the flexible follower after investing a capacity K_f^{df}. The follower's output maximizes its profit flow that is equal to

$$\pi_f^{df}(K_d^{df}, X, K_f^{df}) = \max_{0 \leq Q_f^{df} \leq K_f^{df}} \Big(X \big(1 - \eta \left(K_d^{df} + Q_f^{df} \right) \big) - c \Big) Q_f^{df}.$$

Because $0 \leq K_d^{df} < 1/\eta$, the optimal output level for the follower is

$$Q_f^{df*}(K_d^{df}, X, K_f^{df}) = \begin{cases} 0 & 0 < X < X_{f1}^{df}, \\ \frac{X-c}{2\eta X} - \frac{K_d^{df}}{2} & X_{f1}^{df} \leq X < X_{f2}^{df}, \\ K_f^{df} & X \geq X_{f2}^{df}, \end{cases} \tag{1}$$

where the two boundaries are [2]

$$X_{f1}^{df} = \frac{c}{1 - \eta K_d^{df}} \quad \text{and} \quad X_{f2}^{df} = \frac{c}{1 - \eta K_d^{df} - 2\eta K_f^{df}}.$$

The follower's corresponding profit flow is given by

$$\pi_f^{df*}(K_d^{df}, X, K_f^{df}) = \begin{cases} 0 & 0 < X < X_{f1}^{df}, \\ \frac{\left(X - c - \eta X K_d^{df}\right)^2}{4\eta X} & X_{f1}^{df} \leq X < X_{f2}^{df}, \\ X \left(1 - \eta K_d^{df} - \eta K_f^{df} \right) K_f^{df} - c K_f^{df} & X \geq X_{f2}^{df}. \end{cases} \tag{2}$$

The flexible follower's investment decision is solved as an optimal stopping problem and can be formalized as

$$\sup_{T \geq 0, K_f^{df} \geq 0} E \left[\int_T^\infty \pi_f(K_d^{df}, X(t), K_f^{df}) \exp(-rt) \mathrm{d}t - \delta K_f^{df} \exp(-rT) \middle| X(0) \right], \tag{3}$$

conditional on the available information at time 0, and T is the time of the investment, i.e, the first time that $x(t)$ reaches its investment threshold, and K_f^{df} is the acquired capacity at time T. Denote by $V_f(X, K_d^{df}, K_f^{df})$ the value of the flexible follower, and it satisfies the Bellman equation

$$rV_f^{df} = \pi_f^{df} + \frac{1}{\mathrm{d}t} \mathbb{E}[\mathrm{d}V_f^{df}]. \tag{4}$$

Applying Ito's Lemma, substituting and rewriting lead to the following differential equation (see also, e.g., Dixit and Pindyck (1994))

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_f^{df}(K_d^{df}, X, K_f^{df})}{\partial X^2} + \mu X \frac{\partial V_f^{df}(K_d^{df}, X, K_f^{df})}{\partial X} - rV_f^{df}(K_d^{df}, X, K_f^{df}) + \pi_f^{df}(K_d^{df}, X, K_f^{df}) = 0. \tag{5}$$

Substituting (2) into (5) and employing the value matching and smooth pasting conditions at X_{f1}^{df} and X_{f2}^{df} yield the follower's value after investment as given by

$$V_f^{df}(K_d^{df}, X, K_f^{df})$$
$$= \begin{cases} L^{df}(K_d^{df}, K_f^{df}) X^{\beta_1} & 0 < X < X_{f1}^{df}, \\ M_1^{df}(K_d^{df}, K_f^{df}) X^{\beta_1} + M_2^{df}(K_d^{df}) X^{\beta_2} + \frac{\left(1-\eta K_d^{df}\right)^2 X}{4\eta(r-\mu)} - \frac{c\left(1-\eta K_d^{df}\right)}{2\eta r} + \frac{c^2}{4\eta X(r+\mu-\sigma^2)} & X_{f1}^{df} \leq X < X_{f2}^{df}, \\ N^{df}(K_d^{df}, K_f^{df}) X^{\beta_2} - \frac{c K_f^{df}}{r} + \frac{X K_f^{df}(1-\eta K_d^{df} - \eta K_f^{df})}{r-\mu} & X \geq X_{f2}^{df}, \end{cases} \tag{6}$$

[2] X_{f1}^{df} is a function of K_d^{df} and X_{f2}^{df} is a function of K_d^{df} and K_f^{df}. We drop the arguments for the boundaries when there can be no mistandanding.

in which β_1 and β_2 are the positive and negative root for the quadratic equation $\beta(\beta-1)\sigma^2/2+\mu\beta-r=0$, and the expressions of $L(K_d^{df}, K_f^{df})$, $M_1(K_d^{df}, K_f^{df})$, $M_2(K_d^{df})$, $N(K_d^{df}, K_f^{df})$ can be found in the Appendix A. If $K_d=0$, then the model reduces to a monopolist with volume flexibility as in Wen et al. (2017) .

The follower does not produce right after the investment if $X < X_{f1}^{df}$. Thus, $L(K_d^{df}, K_f^{df})X^{\beta_1}$ is positive and represents the option value to start producing in the future as soon as $X(t)$ reaches X_{f1}^{df}. $M_1(K_d^{df}, K_f^{df})X^{\beta_1}$ is negative and corrects for the fact that if $X(t)$ reaches X_{f2}^{df}, the follower's output will be constrained by the installed capacity level. $M_2(K_d^{df})X^{\beta_2}$ has both a negative and a positive effect. The negative effect corrects for the positive quadratic form of cash flows even when $X(t)$ drops below X_{f1}^{df} in (2). The positive effect comes from the option that the follower would temporarily suspend production for a too small market demand. When $\sigma^2 < r+\mu$, the negative effect dominates the positive effect, and if $\sigma^2 > r+\mu$ the positive effect dominates[3]. $N(K_d^{df}, K_f^{df})X^{\beta_2}$ is positive and describes the option value that if the demand decreases, i.e., $X(t)$ drops below X_{f2}^{df}, the flexible follower produces below full capacity. The optimal investment decision is found in two steps. First, given K_d^{df} and the level of $X(t)=X$, the optimal value of K_f is found by maximizing $V_f^{df}(X, K_d^{df}, K_f^{df}) - \delta K_f$, which yields $K_f^{df}(K_d^{df}, X)$. Second, for a given capacity size K, the optimal investment threshold $X_f^{df}(K_d^{df}, K)$ for the follower can be derived. Combining $K_f^{df}(K_d^{df}, X)$ and $X_f^{df}(K_d^{df}, K)$ yields the optimal investment decision that is summarized in the following proposition.

Proposition 1 *Let*

$$\bar{\sigma}^2 = \frac{-2\left(\Lambda-\mu^2\right)(2r-\mu) - 4\sqrt{r\Lambda\left(\Lambda-\mu^2\right)(r-\mu)}}{\Lambda-(2r-\mu)^2} \quad \text{with} \quad \Lambda = \left(\frac{2\delta r(r-\mu)-\mu c}{c}\right)^2, \tag{7}$$

and given that the dedicated firm has already invested capacity $K_d \in [0, 1/\eta)$, there are two possibilities for the follower's investment decisions:

i. Suppose $\mu > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \mu \leq \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$, then the follower produces below capacity right after investment. For any $X \geq X_{f1}^{df}$, the optimal capacity $K_f^{df}(X, K_d^{df})$ that maximizes $V_f^{df}(X, K_d^{df}, K_f^{df}) - \delta K_f^{df}$ is given by

$$K_f^{df}(K_d^{df}, X) = \max\left\{0, \frac{1}{2\eta}\left(1-\eta K_d^{df} - \frac{c}{X}\left[\frac{2\delta\left(\beta_1-\beta_2\right)}{c\left(1+\beta_1\right)F\left(\beta_2\right)}\right]^{\frac{1}{\beta_1}}\right)\right\}, \tag{8}$$

and the optimal investment threshold $X_f^{df*}(K_d^{df})$ satisfies

$$\begin{aligned} &\frac{c\left(1-\eta K_d^{df}\right)F(\beta_1)}{4\eta\beta_1}\left(\frac{X\left(1-\eta K_d^{df}\right)}{c}\right)^{\beta_2} - \delta K_f^{df}(K_d^{df}, X) \\ &+\frac{1}{4\eta}\left[\frac{\beta_1-1}{\beta_1}\frac{X\left(1-\eta K_d^{df}\right)^2}{r-\mu} - \frac{2c\left(1-\eta K_d^{df}\right)}{r} + \frac{\beta_1+1}{\beta_1}\frac{c^2}{X\left(r+\mu-\sigma^2\right)}\right] = 0, \end{aligned} \tag{9}$$

where

$$F(\beta) = \frac{2\beta}{r} - \frac{\beta-1}{r-\mu} - \frac{\beta+1}{r+\mu-\sigma^2}. \tag{10}$$

[3] Compared to Hagspiel et al. (2016), the dominance of positive and negative effect can be determined in this paper. This is probably due to the fact that I adopt a multiplicative inverse demand structure, and they study an additive inverse demand function.

ii. Suppose $\mu \leq r - c/\delta$, or both $r - c/\delta < \mu \leq \delta r^2/(c+\delta r)$ and $\sigma \leq \bar{\sigma}$, then the follower produces up to capacity right after investment. For any $X \geq X_{f1}^{df}$, the optimal capacity $K_f^{df}(K_d^{df}, X) = \max\{0, k_f\}$ with k_f satisfies

$$\frac{c\,(1+\beta_2)\,F\,(\beta_1)}{2\,(\beta_1-\beta_2)}\left(\frac{X\left(1-2\eta k_f-\eta K_d^{df}\right)}{c}\right)^{\beta_2}+\frac{X\left(1-2\eta k_f-\eta K_d^{df}\right)}{r-\mu}-\frac{c}{r}-\delta=0, \tag{11}$$

and the optimal investment threshold $X_f^{df*}(K_d^{df})$ satisfies

$$\begin{aligned}&\frac{cF(\beta_1)}{4\eta\beta_1}\left(\frac{X}{c}\right)^{\beta_2}\left(\left(1-\eta K_d^{df}\right)^{1+\beta_2}-\left(1-2\eta K_f^{df}(K_d^{df},X)-\eta K_d^{df}\right)^{1+\beta_2}\right)\\&+\frac{(\beta_1-1)X}{\beta_1}\times\frac{K_f^{df}(K_d^{df},X)\left(1-\eta K_d^{df}-\eta K_f^{df}(K_d^{df},X)\right)}{r-\mu}-\left(\frac{c}{r}+\delta\right)K_f^{df}(K_d^{df},X)=0.\end{aligned} \tag{12}$$

3.1.2 Dedicated Leader's Investment Decision

The leader takes the follower's decisions into consideration when deciding on the market entry, and the leader's maximization problem is given by

$$\begin{aligned}\sup_{\tau\geq 0, K_d^{df}\geq 0} E\Bigg[&\int_{\tau}^{T}\left(K_d^{df}\left(1-\eta K_d^{df}\right)X(t)-cK_d^{df}\right)\exp\left(-rt\right)\mathrm{d}t-\delta K_d^{df}\exp\left(-r\tau\right)\\&+\int_{T}^{\infty}\left(K_d^{df}\left(1-\eta K_d^{df}-\eta Q_f^{df}(K_d^{df},X_f^{df},K_f^{df})\right)X(t)-cK_d^{df}\right)\exp\left(-rt\right)\mathrm{d}t\,\Bigg|\,X(0)=X\Bigg],\end{aligned}$$

where τ is the leader's investment timing, and T is the moment that the flexible follower invests. Note that $T > \tau$ for the non-simultaneous investment between the leader and the follower.

The leader's investment value is generated by the leader's profit flow. Before the follower's entry, the leader is a monopolist in the market. After the follower's entry, both firms are active in the market, putting an end to the leader's monopoly privilege. The follower might not produce, produce below, and produces up to capacity after its investment. Thus there are three cases for the leader's profit flow. For the given GBM level X and the leader's capacity size K_d^{df}, the leader's profit flow $\pi_d^{df}(X, K_d^{df})$ is given by

$$\pi_d^{df}(X,K_d^{df})=\begin{cases}K_d^{df}\left(1-\eta K_d^{df}\right)X-cK_d^{df} & 0<X<X_{f1}^{df},\\ \frac{K_d^{df}}{2}\left(X-\eta XK_d^{df}-c\right) & X_{f1}^{df}\leq X<X_{f2}^{df},\\ XK_d^{df}\left(1-\eta\left(K_d^{df}+K_f^{df*}(K_d^{df})\right)\right)-cK_d^{df} & X\geq X_{f2}^{df}.\end{cases}$$

Then the value function of the leader after the follower's investment can be derived as being equal to

$$V_d^{df}(X,K_d^{df})=\begin{cases}\mathcal{L}^{df}(K_d^{df})X^{\beta_1}+\frac{K_d^{df}\left(1-\eta K_d^{df}\right)}{r-\mu}X-\frac{cK_d^{df}}{r} & 0<X<X_{f1}^{df},\\ \mathcal{M}_1^{df}(K_d^{df})X^{\beta_1}+\mathcal{M}_2^{df}(K_d^{df})X^{\beta_2}+\frac{XK_d^{df}\left(1-\eta K_d^{df}\right)}{2(r-\mu)}-\frac{cK_d^{df}}{2r} & X_{f1}^{df}\leq X<X_{f2}^{df},\\ \mathcal{N}^{df}(K_d^{df})X^{\beta_2}+\frac{XK_d^{df}\left(1-\eta K_d^{df}-\eta K_f^{df*}(K_d^{df})\right)}{r-\mu}-\frac{cK_d^{df}}{r} & X\geq X_{f2}^{df}.\end{cases} \tag{13}$$

The expressions of $\mathcal{L}^{df}(K_d^{df})$, $\mathcal{M}_1^{df}(K_d^{df})$, $\mathcal{M}_2^{df}(K_d^{df})$, $\mathcal{N}^{df}(K_d^{df})$, and their signs can be found in Appendix A. For $X < X_{f1}^{df}$, the demand is so low that the follower's production is temporarily suspended. However, the dedicated leader still produces at full capacity. In the leader's value function, $\mathcal{L}^{df}(K_d^{df})X^{\beta_1}$ corrects for the decrease in the leader's value when the follower resumes production in the future. This happens as soon as $X(t)$ becomes larger than X_{f1}^{df}. For $X_{f1}^{df} \leq X < X_{f2}^{df}$, the follower produces below capacity right after investment. $\mathcal{M}_1^{df}(K_d^{df})X^{\beta_1}$ corrects for the fact that if $X(t)$ reaches $X_{f2}^{df}(K_d^{df}, K_f^{df^*}(K_d^{df}))$, then the production of the follower is constrained by its installed capacity, hence the value of the leader increases. The term $\mathcal{M}_2^{df}(K_d^{df})X^{\beta_2}$ corrects for fact that when $X(t)$ falls below X_{f1}^{df}, a negative Q_f^{df*} enlarges the leader's profit. Whereas this cannot happen in reality, which requires a negative $\mathcal{M}_2^{df}(K_d^{df})$ to correct this. For $X \geq X_{f2}^{df}$, the follower produces up to capacity right after investment. The term $\mathcal{N}^{df}(K_d^{df})X^{\beta_2}$ corrects for the fact that when $X(t)$ drops below X_{f2}^{df}, the follower produces below capacity, and the value of the leader would increase.

Before the follower invests, the leader's value function consists of two parts: One part represents the net present value of the monopolistic profit flow, and the other part corrects for the decrease in leader's value when the follower invests and ends its monopoly privilege. Assume the leader invests at X, let the leader's value before the follower's entry be

$$V_d^{df}(X, K_d^{df}) = \mathcal{B}^{df}(K_d^{df})X^{\beta_1} + \frac{K_d^{df}\left(1 - \eta K_d^{df}\right)}{r - \mu}X - \frac{cK_d^{df}}{r},$$

where $\mathcal{B}^{df}(K_d^{df})$ has different expressions for the two cases, i.e., the follower produces below and up to capacity right after investment. [4]

In case that $\mu > \delta r^2/(c + \delta r)$, or both $r - c/\delta < \mu \leq \delta r^2/(c + \delta r)$ and $\sigma > \bar{\sigma}$, the flexible follower produces below capacity right after investment. The value function of the leader before and after the follower's entry equals to

$$V_d^{df}(X, K_d^{df}) = \begin{cases} \mathcal{B}_1^{df}(K_d^{df})X^{\beta_1} + \frac{K_d^{df}\left(1-\eta K_d^{df}\right)}{r-\mu}X - \frac{c}{r}K_d^{df} & X < X_f^{df^*}(K_d^{df}), \\ \mathcal{M}_1^{df}(K_d^{df})X^{\beta_1} + \mathcal{M}_2^{df}(K_d^{df})X^{\beta_2} + \frac{K_d^{df}(1-\eta K_d)}{2(r-\mu)}X - \frac{cK_d^{df}}{2r} & X \geq X_f^{df^*}(K_d^{df}), \end{cases} \tag{14}$$

with

$$\begin{aligned} \mathcal{B}_1^{df}(K_d^{df}) = \mathcal{M}_1(K_d^{df}) + \mathcal{M}_2(K_d^{df})\left(X_f^{df^*}(K_d)\right)^{\beta_2-\beta_1} + \frac{cK_d^{df}}{2r}\left(X_f^{df^*}(K_d^{df})\right)^{-\beta_1} \\ - \frac{K_d^{df}\left(1 - \eta K_d^{df}\right)}{2(r-\mu)}\left(X_f^{df^*}(K_d^{df})\right)^{1-\beta_1}, \end{aligned} \tag{15}$$

according to value matching condition at $X_f^{df^*}(K_d^{df})$ that satisfies (9).

In case that $\mu \leq r - c/\delta$, or both $r - c/\delta < \mu \leq \delta r^2/(c + \delta r)$ and $\sigma \leq \bar{\sigma}$, the flexible follower produces up to capacity right after the investment. Given that the leader invests at X, the leader's value function before and after the follower's entry can be written as

$$V_d^{df}(X, K_d^{df}) = \begin{cases} \mathcal{B}_2^{df}(K_d^{df})X^{\beta_1} + \frac{K_d^{df}\left(1-\eta K_d^{df}\right)}{r-\mu}X - \frac{c}{r}K_d^{df} & X < X_f^{df^*}(K_d^{df}), \\ \mathcal{N}^{df}(K_d^{df})X^{\beta_2} + \frac{K_d^{df}\left(1-\eta K_d^{df} - \eta K_f^{df^*}(K_d^{df})\right)}{r-\mu}X - \frac{c}{r}K_d^{df} & X \geq X_f^{df^*}(K_d^{df}), \end{cases} \tag{16}$$

[4] $\mathcal{B}(K_d^{df})$ and $\mathcal{L}(K_d^{df})$ are different. According to Dixit and Pindyck (1994), the fundamental component in the leader's value function, i.e., $\left(K_d^{df}\left(1 - \eta K_d^{df}\right)X - K_d^{df}\right)/r$, is generated by the profit flows. $\mathcal{L}(K_d^{df})X^{\beta_1}$ describes the deviation of $V_d^{df}(X, K_d^{df})$ from the fundamental component due to the possibility that X will move across the boundary $X_{f1}^{df}(K_d^{df})$. $\mathcal{B}(K_d^{df})X^{\beta_1}$ describes the deviation of $V_d^{df}(X, K_d^{df})$ from the fundamental component due to the possibility that X will move across the follower's optimal investment threshold $X_f^{df^*}$.

with

$$\mathcal{B}_2^{df}(K_d^{df}) = \mathcal{N}(K_d^{df}) X_f^{df*\beta_2-\beta_1}(K_d^{df}) - \frac{\eta K_d^{df} K_f^{df*}(K_d^{df})}{r-\mu} X_f^{df*1-\beta_1}(K_d^{df}), \tag{17}$$

according to the value matching condition at the flexible follower's investment threshold $X_f^{df*}(K_d^{df})$ that satisfies equation (12). The leader's investment decisions are described in the following proposition (see Appendix A for the proof).

Proposition 2 *The dedicated leader's optimal investment threshold X_d^{df*} and investment capacity K_d^{df*} are*

$$\begin{aligned} X_d^{df*} &= \frac{(\beta_1+1)(r-\mu)}{\beta_1-1}\left(\frac{c}{r}+\delta\right), \\ K_d^{df*} &= \frac{1}{(\beta_1+1)\eta}. \end{aligned}$$

When compared to the leader's entry deterrence strategy by Huisman and Kort (2015), Proposition 2 suggests that the follower's volume flexibility does not influence the leader's investment decisions. The intuition is as follows. The capacity decision is from a long-run perspective. Because the leader commits to a certain output, the flexible follower has to adapt to this fixed output level. In this sense, the long-run perspective is the same for the dedicated leader regardless of the follower's flexibility. The timing decision is from a short-run perspective, i.e., to find a sufficiently large enough market demand for a given investment size. In the non-simultaneous investment, the dedicated leader finds the same demand level for the same size of investment regardless of the follower's volume flexibility.

3.2 Flexible Leader and Dedicated Follower

This section analyzes the model where the leader can produce below capacity right after investment, $Q_f^{fd} \leq K_f^{fd}$ and the follower produces up to capacity after investment, $Q_d^{fd} = K_d^{fd}$.

3.2.1 Dedicated Follower's Investment Decision

Given that the leader is already in the market and producing Q_f^{fd} when the follower enters the market at X, the follower's instantaneous profit equals to

$$\pi_d^{fd}(Q_f^{fd}, K_d^{fd}, X) = \left(X\left(1-\eta(Q_f^{fd}+K_d^{fd})\right)-c\right)K_d^{fd}, \quad 0 \leq Q_f^{fd} \leq K_f^{fd}$$

In fact, the leader adjusts its output immediately from the moment of the follower's investment on to maximize its instantaneous profit such that

$$Q_f^{fd}(K_d^{fd}, X) = \frac{X\left(1-\eta K_d^{fd}\right)-c}{2\eta X}.$$

There are three cases/regions for the leader's output: no production ($Q_f^{fd} = 0$), producing below capacity ($0 < Q_f^{fd} < K_f^{fd}$), and producing up to capacity ($Q_f^{fd} = K_f^{fd}$). These three regions are characterized by the GBM level X, and the follower's instantaneous profit in each region is given by

$$\pi_d^{fd}(K_f^{fd}, X, K_d^{fd}) = \begin{cases} K_d^{fd}\left(X\left(1-\eta K_d^{fd}\right)-c\right) & X \leq X_{d1}^{fd}, \\ \frac{K_d^{fd}}{2}\left(X\left(1-\eta K_d^{fd}\right)-c\right) & X_{d1}^{fd} < X \leq X_{d2}^{fd}, \\ K_d^{fd}\left(X\left(1-\eta K_f^{fd}-\eta K_d^{fd}\right)-c\right) & X > X_{d2}^{fd}, \end{cases}$$

where

$$X_{d1}^{fd} = \frac{c}{1-\eta K_d^{fd}} \quad \text{and} \quad X_{d2}^{fd} = \frac{c}{1-\eta K_d^{fd} - 2\eta K_f^{fd}}$$

are the boundaries for the three regions.[5] By comparing the dedicated follower in this model with the dedicated leader in subsection 3.1, the instantaneous profit functions are the same. This is because once both firms are active the market, the economic condition becomes similar for both models in the sense that, the dedicated firm produces a constant output and the flexible firm adjusts its output according to the demand fluctuations. As shown in the follower's profit function, the dedicated follower could influence the boundaries of regions, but not in a direct way as the dedicated leader influencing the boundaries of the flexible follower in subsection 3.1. When the flexible leader makes investment decisions, the leader knows that the dedicated firm will enter the market later. The more the follower invests, the more installed capacity of the leader would remain idle once the realized market demand is small. So the follower can influence the leader's capacity choice and thus the boundaries of the regions, implying the follower's value function is differentiable at X_{d1}^{fd} and X_{d2}^{fd}, and takes the form as

$$V_d^{fd}(K_f^{fd}, X, K_d^{fd}) = \begin{cases} \mathcal{L}^{fd}(K_f^{fd}, K_d^{fd})X^{\beta_1} + K_d^{fd}\left(\frac{X\left(1-\eta K_d^{fd}\right)}{r-\mu} - \frac{c}{r}\right) & X \le X_{d1}^{fd}, \\ \mathcal{M}_1^{fd}(K_f^{fd}, K_d^{fd})X^{\beta_1} + \mathcal{M}_2^{fd}(K_d^{fd})X^{\beta_2} + K_d^{fd}\left(\frac{X\left(1-\eta K_d^{fd}\right)}{2(r-\mu)} - \frac{c}{2r}\right) & X_{d1}^{fd} < X \le X_{d2}^{fd}, \\ \mathcal{N}^{fd}(K_f^{fd}, K_d^{fd})X^{\beta_2} + \frac{\left(1-\eta K_f^{fd} - \eta K_d^{fd}\right)XK_d^{fd}}{r-\mu} - \frac{c}{r}K_d^{fd} & X > X_{d2}^{fd}, \end{cases}$$

where $\mathcal{L}^{fd}(\cdot) = \mathcal{L}^{df}(\cdot)$, $\mathcal{M}_1^{fd}(\cdot) = \mathcal{M}_1^{df}(\cdot)$, $\mathcal{M}_2^{fd}(\cdot) = \mathcal{M}_2^{df}(\cdot)$ and $\mathcal{N}^{fd}(\cdot) = \mathcal{N}^{df}(\cdot)$. This is because these coefficients correct for the changes in the dedicated firm's value function that are caused by the flexible firm adjusting its output. They are identical regardless of whether the dedicated firm is the leader or the follower. The dedicated follower's investment decisions are presented in the following proposition.

Proposition 3 *Given that the flexible firm has already invested with a capacity size K_f^{fd}, there are two possibilities for the dedicated follower's investment decisions:*

i. The flexible leader produces below capacity right after the follower's investment, i.e., $X_{d1}^{fd}(K_d^{fd}(K_f^{fd}, X)) < X \le X_{d2}^{fd}(K_f^{fd}, K_d^{fd}(K_f^{fd}, X))$, where $K_d^{fd}(K_f^{fd}, X)$ is the follower's investment capacity for a given X, and equals to

$$K_d^{fd}(K_f^{fd}, X) = \begin{cases} \frac{X-c}{\eta X} & \text{if } k_d^{fd}(K_f^{fd}, X) \ge \frac{X-c}{\eta X}, \\ k_d^{fd}(K_f^{fd}, X) & \text{if } \frac{X(1-2\eta K_f^{fd})-c}{\eta X} \le k_d^{fd}(K_f^{fd}, X) < \frac{X-c}{\eta X}, \\ \frac{X(1-2\eta K_f^{fd})-c}{\eta X} & \text{otherwise}, \end{cases} \tag{18}$$

and $k_d^{fd}(K_f^{fd}, X)$ satisfies the implicit equation that

$$\frac{1-2\eta K_f^{fd} - (\beta_1+1)\eta k}{k\left(1-2\eta K_f^{fd} - \eta k\right)}\mathcal{M}_1^{fd}(K_f^{fd}, k)X^{\beta_1} + \frac{1-(\beta_2+1)\eta k}{k(1-\eta k)}\mathcal{M}_2^{fd}(k)X^{\beta_2} + \frac{X(1-2\eta k)}{2(r-\mu)} - \frac{c}{2r} - \delta = 0 .$$

For a given K, the investment threshold $X_d^{fd}(K_f^{fd}, K)$ makes it hold that

$$2(\beta_1-\beta_2)\mathcal{M}_2^{fd}(K)X^{\beta_2} + \frac{XK(\beta_1-1)(1-\eta K)}{r-\mu} - \frac{c\beta_1 K}{r} - 2\beta_1\delta K = 0 . \tag{19}$$

[5] The boundary X_{d1}^{fd} is a function of K_d^{fd}, and X_{d2}^{fd} is a function of K_f^{fd} and K_d^{fd}. We drop the argument for the boundaries when there can be no misunderstanding.

ii. The flexible leader produces up to capacity right after the follower's investment, i.e., $X > X_{d2}^{fd}(K_f^{fd}, K_d^{fd}(K_f^{fd}, X))$, *where* $K_d^{fd}(K_f^{fd}, X)$ *is the follower's investment capacity for a given* X*, and equals to*

$$K_d^{fd}(K_f^{fd}, X) = \begin{cases} \frac{X\left(1-2\eta K_f^{fd}\right)-c}{\eta X} & \text{if } k_d^{fd}(K_f^{fd}, X) \geq \frac{X\left(1-2\eta K_f^{fd}\right)-c}{\eta X} , \\ k_d^{fd}(K_f^{fd}, X) & \text{if } 0 < k_d^{fd}(K_f^{fd}, X) < \frac{X\left(1-2\eta K_f^{fd}\right)-c}{\eta X} , \\ 0 & \text{otherwise} , \end{cases} \tag{20}$$

and $k_d^{fd}(K_f^{fd}, X)$ *satisfies the implicit equation that*

$$\frac{\partial \mathcal{N}^{fd}(K_f^{fd}, k)}{\partial k} X^{\beta_2} + \frac{X(1-\eta K_f^{fd} - 2\eta k)}{r-\mu} - \frac{c}{r} - \delta = 0 .$$

For a given K*, the dedicated follower invests at a threshold level* $X_d^{fd}(K_f^{fd}, K)$ *that satisfies*

$$(\beta_1 - \beta_2)\mathcal{N}^{fd}(K_f^{fd}, K)X^{\beta_2} + \frac{(\beta_1 - 1)XK\left(1 - \eta K_f^{fd} - \eta K\right)}{r-\mu} - \frac{\beta_1 K(c + r\delta)}{r} = 0 . \tag{21}$$

Combining $K_d^{fd}(K_f^{fd}, X)$ *and* $X_d^{fd}(K_f^{fd}, K)$ *yields the optimal investment decision* $K_d^{fd^*}(K_f^{fd})$ *and* $X_d^{fd^*}(K_f^{fd})$ *for the dedicated follower.*

Note that even though the dedicated follower's value function takes similar expressions as the dedicated leader's value function (13) in subsection 3.1, it generates different investment decisions for the dedicated follower. This is because in subsection 3.1 the dedicated leader's investment capacity K_d^{df} influences the flexible follower's decision K_f^{df} and X_f^{df}, which has to be taken into account by the leader. However, the analysis of the dedicated follower here takes the flexible leader's capacity K_f^{fd} as given.

3.2.2 Flexible Leader's Investment Decision

As a designated leader, the flexible firm invests before the dedicated firm. After its investment, the flexible leader becomes a monopolist. This monopoly period ends at the dedicated follower's time of investment, assumed to be τ_d. The follower's investment naturally decreases the leader's profit flow. If the leader were dedicated, the decrease would be due to the shrink of market share. However, for a flexible leader, the decrease could also be because of its output adjustment.

There are in total three possibilities for the leader's instant profit change at time τ_d. Denote by τ_d^- the moment right before the follower invests and τ_d^+ right after the follower invests. Then the possibilities are demonstrated in Figure 1. For the flexible leader, given that it produces below capacity ("LB") at τ_d^+, the leader might have been producing below ("LB") or up to capacity ("LU") at τ_d^-. Given that the leader produces up to capacity ("LU") at time τ_d^+, the leader must also be producing up to capacity ("LU") at τ_d^-. This is because $X(\tau_d^-) = X(\tau_d^+) = X(\tau_d)$, if the leader produces up to capacity right after the follower's investment, the market demand must be sufficiently high such that it also produces up to capacity before the follower's investment.

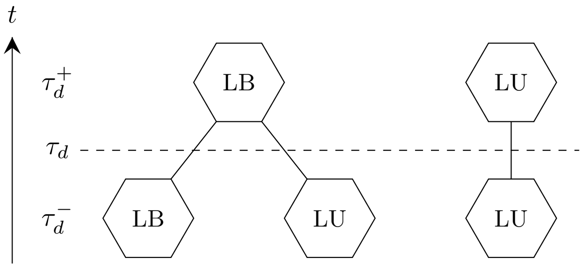


Figure 1: Possibilities for the leader's instant profit changes at the follower's investment timing τ_d.

So we can derived the leader's output Q_f^{fd} as given by

$$\tau_d^+ :\ Q_f^{fd} = \begin{cases} \frac{\left(1-\eta K_d^{fd*}(K_f^{fd})\right)X(\tau_d^+)-c}{2\eta X(\tau_d^+)} & \text{LB},\\ K_f^{fd} & \text{LU}, \end{cases} \quad \text{and} \quad \tau_d^- :\ Q_f^{fd} = \begin{cases} \frac{X(\tau_d^-)-c}{2\eta X(\tau_d^-)} & \text{LB},\\ K_f^{fd} & \text{LU}. \end{cases} \tag{22}$$

The corresponding leader's profits after and before the follower's investment are equal to

$$\tau_d^+ :\ \pi_f^{fd}(Q_f^{fd}, X(\tau_d^+), K_d^{fd*}(K_f^{fd})) = \begin{cases} \frac{\left(\left(1-\eta K_d^{fd*}(K_f^{fd})\right)X(\tau_d^+)-c\right)^2}{4\eta X(\tau_d^+)} & \text{LB},\\ K_f^{fd}\left(\left(1-\eta K_f^{fd}-\eta K_d^{fd*}(K_f^{fd})\right)X(\tau_d^+)-c\right) & \text{LU}, \end{cases} \tag{23}$$

and

$$\tau_d^- :\ \pi_f^{fd}(Q_f^{fd}, X(\tau_d^-)) = \begin{cases} \frac{\left(X(\tau_d^-)-c\right)^2}{4\eta X(\tau_d^-)} & \text{LB},\\ K_f^{fd}\left(\left(1-\eta K_f^{fd}\right)-c\right) & \text{LU}. \end{cases} \tag{24}$$

To derive the flexible leader's value function, we first need to calculate the Expected Change in the leader's Profit flow (ECP) caused by the follower's market entry. For a given GBM level X, ECP is denoted by

$$\mathrm{ECP}^{fd}(X, K_f^{fd}) = \left(\frac{X}{X_d^{fd*}(K_f^{fd})}\right)^{\beta_1} \mathbb{E}_{\tau_d}\left[\int_0^\infty \left(\pi_f^{fd}\big(Q_f^{fd}, X(t)\big) - \pi_f^{fd}\big(Q_f^{fd}, X(t), K_d^{fd*}(K_f^{fd})\big)\right)\exp(-rt)\mathrm{d}t\right], \tag{25}$$

where $\mathbb{E}_{\tau_d}$ calculates the expected changes in the flexible leader's profit flow from time τ_d on.[6] In particular, τ_d is the dedicated follower's investment moment, i.e., the first time that $X(t)$ reaches $X_d^{fd*}(K_f^{fd})$. The stochastic discount factor $\big(X/X_d^{fd*}(K_f^{fd})\big)^{\beta_1}$ discounts this expected change back to a point in time after the flexible leader's investment and is characterized by X.[7] In $\mathrm{ECP}^{fd}(X, K_f^{fd})$, it holds that

$$\begin{aligned} &\mathbb{E}_{\tau_d}\left[\int_0^\infty \left(\pi_f^{fd}\big(Q_f^{fd}, X(t)\big) - \pi_f^{fd}\big(Q_f^{fd}, X(t), K_d^{fd*}(K_f^{fd})\big)\right)\exp(-rt)\mathrm{d}t\right] \\ &= \begin{cases} \frac{K_d^{fd*}(K_f^{fd})}{4}\left(\frac{X_d^{fd*}(K_f^{fd})}{r-\mu}\left(2-\eta K_d^{fd*}(K_f^{fd})\right)-\frac{2c}{r}\right) & \text{if } \tau_d^- :\ \text{LB};\ \tau_d^+ :\ \text{LB},\\ \frac{K_f^{fd}\left(1-\eta K_f^{fd}\right)}{r-\mu}X_d^{fd*}(K_f^{fd})-\frac{cK_f^{fd}}{r} & \\ -\frac{1}{4\eta}\left(\frac{\left(1-\eta K_d^{fd*}(K_f^{fd})\right)^2 X_d^{fd*}(K_f^{fd})}{r-\mu}+\frac{c^2}{(r+\mu-\sigma^2)X_d^{fd*}(K_f^{fd})}-\frac{2c\left(1-\eta K_d^{fd*}(K_f^{fd})\right)}{r}\right) & \text{if } \tau_d^- :\ \text{LU};\ \tau_d^+ :\ \text{LB},\\ \frac{\eta X_d^{fd*}(K_f^{fd})}{r-\mu}K_f^{fd}K_d^{fd*}(K_f^{fd}) & \text{if } \tau_d^- :\ \text{LU};\ \tau_d^+ :\ \text{LU}. \end{cases} \end{aligned}$$

We use the following lemma to summarize how the expected profit changes depend on the flexible leader's investment size K_f^{fd}.

[6] The calculator $\mathbb{E}_t$ denotes the expectation operator conditional on the available information at time t.
[7] Please find detailed explanation by Huisman and Kort (2015).

Lemma 1 *At the dedicated follower's investment threshold $X_d^{fd*}(K_f^{fd})$, it holds that for the leader's expected change of profits, i.e.,* $\mathrm{ECP}^{fd}(X, K_f^{fd})$*, such that*

- *If the leader produces below capacity both at τ_d^- and at τ_d^+, then*

$$\frac{\mathrm{dECP}^{fd}(X, K_f^{fd}|\tau_d^- : \mathrm{LB}; \tau_d^+ : \mathrm{LB})}{\mathrm{d}K_f^{fd}}$$
$$= \left(\frac{X}{X_d^{fd*}(K_f^{fd})}\right)^{\beta_1} \times \left[\left(\frac{\left(1 - \eta K_d^{fd*}(K_f^{fd})\right) X_d^{fd*}(K_f^{fd})}{2(r-\mu)} - \frac{c}{2r}\right) \times \frac{\mathrm{d}K_d^{fd*}(K_f^{fd})}{\mathrm{d}K_f^{fd}}\right.$$
$$\left. - \frac{K_d^{fd*}(K_f^{fd})}{X_d^{fd*}(K_f^{fd})}\left(\frac{(\beta_1 - 1)\left(2 - \eta K_d^{fd*}(K_f^{fd})\right) X_d^{fd*}(K_f^{fd})}{4(r-\mu)} - \frac{c\beta_1}{2r}\right) \times \frac{\mathrm{d}X_d^{fd*}(K_f^{fd})}{\mathrm{d}K_f^{fd}}\right].$$

- *If the leader produces below capacity at τ_d^- and up to capacity at τ_d^+, then*

$$\frac{\mathrm{dECP}^{fd}(X, K_f^{fd}|\tau_d^- : \mathrm{LU}; \tau_d^+ : \mathrm{LB})}{\mathrm{d}K_f^{fd}} = \left(\frac{X}{X_d^{fd*}(K_f^{fd})}\right)^{\beta_1} \times \left[-\frac{c}{2r} \times \left(2 + \frac{\mathrm{d}K_d^{fd*}(K_f^{fd})}{\mathrm{d}K_f^{fd}}\right)\right.$$
$$+ \frac{X_d^{fd*}(K_f^{fd})}{2(r-\mu)} \times \left(2 - 4\eta K_f^{fd} + \left(1 - \eta K_d^{fd*}(K_f^{fd})\right)\frac{\mathrm{d}K_d^{fd*}(K_f^{fd})}{\mathrm{d}K_f^{fd}}\right)$$
$$+ \frac{\mathrm{d}X_d^{fd*}(K_f^{fd})}{\mathrm{d}K_f^{fd}} \times \frac{\beta_1 - 1}{4\eta(r-\mu)} \times \left(\left(1 - 2\eta K_f^{fd}\right)^2 - \eta\left(2 - \eta K_d^{fd*}(K_f^{fd})\right) K_d^{fd*}(K_f^{fd})\right)$$
$$\left. + \frac{\mathrm{d}X_d^{fd*}(K_f^{fd})}{\mathrm{d}K_f^{fd}} \times \frac{1}{4\eta} \times \left(\frac{c^2(\beta_1 + 1)}{(r + \mu - \sigma^2)X_d^{fd*2}(K_f^{fd})} - \frac{2c\beta_1}{rX_d^{fd*}(K_f^{fd})}\left(1 - 2\eta K_f^{fd} - \eta K_d^{fd*}(K_f^{fd})\right)\right)\right].$$

- *If the leader produces up to capacity both at τ_d^- and at τ_d^+, then*

$$\frac{\mathrm{dECP}^{fd}(X, K_f^{fd}|\tau_d^- : \mathrm{LU}; \tau_d^+ : \mathrm{LU})}{\mathrm{d}K_f^{fd}} = \left(\frac{X}{X_d^{fd*}(K_f^{fd})}\right)^{\beta_1} \times \frac{\eta K_f^{fd} X_d^{fd*}(K_f^{fd})}{r-\mu} \times \left[\frac{\mathrm{d}K_d^{fd*}(K_f^{fd})}{\mathrm{d}K_f^{fd}}\right.$$
$$\left. + \frac{K_d^{fd*}(K_f^{fd})}{K_f^{fd}} - (\beta_1 - 1) \times \frac{K_d^{fd*}(K_f^{fd})}{X_d^{fd*}(K_f^{fd})} \times \frac{\mathrm{d}X_d^{fd*}(K_f^{fd})}{\mathrm{d}K_f^{fd}}\right].$$

Right after investment, the leader adjusts the output according to the market demand. There are three regions characterizing the leader's output levels, i.e., no production, producing below, and up to capacity. We denote the boundaries for the these three regions as $X_1^D = c$ and $X_2^D = c/\left(1 - 2\eta K_f^{fd}\right)$. For the non-simultaneous investment, the boundaries are the same as that of a monopolistic flexible firm in Wen et al. (2017). This is because the flexible leader remains a monopolist until the follower's entry. In this sense, the flexible leader's value function is to some extent also similar to that in Wen et al. (2017). The difference is due to the entry of a dedicated follower that decreases the leader's value at the moment of the follower's investment.

If the leader invests at a GBM level X and $X \leq X_1^D$, the leader does not produce right after investment. In the monopoly model by Wen et al. (2017), the flexible firm's value function at the moment of investment consist of two terms: a positive option value correting for that the flexible firm resumes production once $X(t)$ reaches X_1^D from below, and a negative term that represents the investment cost. According to Wen et al. (2017), the flexible firm already does not invest in this region. In our model, there is an additional negative third term that corrects for the decrease in the flexible leader's value due to the follower's entry. So the flexible firm does not invest in this region in our model either.

If the flexible leader invests at a GBM level X and $X_1^D < X \leq X_2^D$, the leader produces below capacity right after its own investment. Then there are two possibilities for the leader's output at the follower's investment threshold X_d^{fd}, i.e., at time τ_d^+: if $X_{d1}^{fd} < X_d^{fd} \leq X_{d2}^{fd}$, then the leader produces below capacity at τ_d^+; if $X_d^{fd} > X_{d2}^{fd}$, then the leader produces up to capacity at τ_d^+. Note that the dedicated follower does not invest when the leader suspends production, i.e., $X < X_d^{fd} \leq X_{d1}^{fd}$, as shown in the proof of Proposition 3. In the following analysis we leave out this possibility.

If the flexible leader invests at a GBM level X and $X > X_2^D$, the leader produces up to capacity right after its own investment. Then the two possibilities for the leader's output at time τ_d are the same as above.

We analyze the leader's investment decisions based on cases of whether the leader produces below or up to capacity right after its investment. Within each case, we distinguish right after the follower enters the market, whether the leader produces below or up to capacity.

Case 1: flexible leader produces below capacity right after investment, i.e., $X_1^D < X \leq X_2^D$

The flexible leader's value right after its own investment is

$$V_f^{fd}(X, K_f^{fd}) = M_1^{fd}(K_f^{fd})X^{\beta_1} + M_2^{fd}X^{\beta_2} + \frac{1}{4\eta}\left(\frac{X}{r-\mu} - \frac{2c}{r} + \frac{c^2}{X(r+\mu-\sigma^2)}\right) - \mathrm{ECP}^{fd}(X, K_f^{fd}), \quad (26)$$

where the first three terms represent the value for the flexible firm producing below capacity right after investment according to Wen et al. (2017), and $M_1^{fd}(K_f^{fd})$ and M_2^{fd} have the expressions as M_1^{df} and M_2^{df} in the analysis of the dedicated follower in Appendix A with X_1^D and X_2^D substituting X_{f1}^{df} and X_{f2}^{df}. The last term in the value function denotes the decrease in the leader's value function due to the follower investment at $X_d^{fd^*}(K_f^{fd})$.

Given the flexible leader's profit change at $X_d^{fd^*}(K_f^{fd})$, we can derive the leader's investment decisions as in Proposition 4. The expression of the flexible leader's value functions and the proof of the proposition can be found in the appendix.

Proposition 4 *When the flexible leader produces below capacity right after investment, there are two possibilities depending on whether the leader will be producing below or up to capacity right after the follower invests.*

i. The leader will be producing below capacity right after the follower invests. For a given $X > c$, the flexible leader's corresponding investment capacity $K_f^{fd}(X)$ is such that $K_f^{fd}(X) = \max\left\{k_f^{fd}(X),\ \frac{X-c}{2\eta X}\right\}$, where $k_f^{fd}(X)$ is such that:

- *If the leader produces below capacity right before the follower invests, $k_f^{fd}(X)$ satisfies*

$$\frac{c(\beta_1+1)F(\beta_2)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\eta k)}{c}\right)^{\beta_1} - \delta - \frac{\mathrm{dECP}^{fd}(X, k|\tau_d^- : \mathrm{LB}; \tau_d^+ : \mathrm{LB})}{\mathrm{d}k} = 0\ , \quad (27)$$

- *If the leader produces up to capacity right before the follower invests, $k_f^{fd}(X)$ satisfies*

$$\frac{c(\beta_1+1)F(\beta_2)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\eta k)}{c}\right)^{\beta_1} - \delta - \frac{\mathrm{dECP}^{fd}(X, k|\tau_d^- : \mathrm{LU}; \tau_d^+ : \mathrm{LB})}{\mathrm{d}k} = 0. \quad (28)$$

For a given capacity size K, the corresponding investment threshold $X_f^{fd}(K)$ satisfies the equation of

$$\left(\frac{X}{c}\right)^{\beta_2} cF(\beta_1) + \frac{X(\beta_1-1)}{r-\mu} - \frac{2c\beta_1}{r} + \frac{c^2(\beta_1+1)}{X(r+\mu-\sigma^2)} - 4\delta\beta_1\eta K = 0\ . \quad (29)$$

ii. The leader will be producing up to capacity right after the follower invests. For a given $X > c$, the flexible leader's corresponding investment capacity $K_f^{fd}(X)$ euqals to $K_f^{fd}(X) = \max\left\{k_f^{fd}(X),\ \frac{X-c}{2\eta X}\right\}$, where $k_f^{fd}(X)$ satisfies the implicit equation as

$$\frac{c(1+\beta_1)F(\beta_2)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\eta k)}{c}\right)^{\beta_1} - \delta - -\frac{\mathrm{dECP}^{fd}(X,k|\tau_d^- : \mathrm{LU}; \tau_d^+ : \mathrm{LU})}{\mathrm{d}k} = 0\ . \tag{30}$$

For a given K, the flexible leader's corresponding investment threshold $X_f^{fd}(K)$ also satisfies equation (29).

Combining $K_f^{fd}(X)$ and $X_f^{fd}(K)$ yields the flexible leader's optimal investment decision $K_f^{fd^*}$ and $X_f^{fd^*}$.

In Proposition 4, the flexible firm's investment threshold $X_f^{fd}(K)$ for a given K, i.e., equation (29), is the same as that in the monopolistic model with volume flexibility in Wen et al. (2017). This implies that if the flexible leader invests with the same capacity, i.e., $\mathrm{dECP}^{fd}(\cdot)/\mathrm{d}k = 0$, then the investment timing is the same regardless of a follower or not. The intuition is that for the given capacity size, the leader's investment timing has no effect on the optimal reaction by the dedicated follower (see Huisman and Kort (2015)), and it depends only on the leader's investment capacity. In other words, timing decision is from the short-run perspective, and the negative correction in the leader's value due to the follower's entry does not influence the flexible leader's timing decision.However, because the follower's market entry decreases the flexible leader's expected profit flow, i.e., $\mathrm{ECP}^{fd} > 0$, the duopoly flexible leader invests with a capacity that is smaller than the monopolist.

Case 2: flexible leader produces up to capacity right after investment, i.e., $X > X_2^D$

The value of the flexible firm right after investment equals to

$$V_f^{fd}(X, K_f^{fd}) = N^{fd}(K_f^{fd})X^{\beta_2} + \frac{X\left(1-\eta K_f^{fd}\right)K_f^{fd}}{r-\mu} - \frac{cK_f^{fd}}{r} - \mathrm{ECP}^{fd}(X, K_f^{fd}),$$

where the expression for $N^{fd}(K_f^{fd})$ is similar to N^{df} as in the analysis for the flexible follower in Appendix A with X_1^D and X_2^D substituting X_{f1}^{df} and X_{f2}^{df}. Similar as in Case 1, the first three terms represent the leader's value if there would be no potential follower, and the last term corrects for the fact that, the flexible leader's value decreases when the follower invests at threshold $X_d^{fd^*}(K_f^{fd})$. The flexible leader's optimal investment decision can be found in the following proposition.

Proposition 5 *When the flexible leader produces up to capacity right after investment, there are two possibilities depending on whether the leader will be producing below or up to capacity right after the follower invests.*

i. The leader will be producing below capacity right after the dedicated follower invests. For a given $X > c$, the flexible leader's corresponding investment capacity $K_f^{fd}(X)$ equals to $\min\left\{k_f^{fd}(X), \frac{X-c}{2\eta X}\right\}$ where $k_f^{fd}(X)$ is such that

- *If the leader produces below capacity right before the follower invests, $k_f^{fd}(X)$ satisfies that*

$$\frac{c(1+\beta_2)F(\beta_1)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\eta k)}{c}\right)^{\beta_2} + \frac{X(1-2\eta k)}{r-\mu} - \frac{c}{r} - \delta - \frac{\mathrm{dECP}^{fd}(X,k|\tau_d^- : \mathrm{LB}; \tau_d^+ : \mathrm{LB})}{\mathrm{d}k} = 0. \tag{31}$$

- *If the leader produces up to capacity right before the follower invests, $k_f^{fd}(X)$ satisfies that*

$$\frac{c(1+\beta_2)F(\beta_1)}{2(\beta_1-\beta_2)}\left(\frac{X\,(1-2\eta k)}{c}\right)^{\beta_2}+\frac{X\,(1-2\eta k)}{r-\mu}-\frac{c}{r}-\delta-\frac{\mathrm{dECP}^{fd}(X,k|\tau_d^-:\mathrm{LU};\tau_d^+:\mathrm{LB})}{\mathrm{d}k}=0. \quad (32)$$

For a given K, the corresponding investment threshold $X_f^{fd}(K)$ makes it hold that

$$\frac{c^2F(\beta_1)}{4\eta X}\left(\left(\frac{X}{c}\right)^{\beta_2+1}-\left(\frac{X\,(1-2\eta K)}{c}\right)^{\beta_2+1}\right)+\frac{(\beta_1-1)(1-\eta K)XK}{r-\mu}-\frac{\beta_1(c+r\delta)K}{r}=0. \quad (33)$$

ii. *The leader will be producing up to capacity right after the dedicated follower invests. For a given X, the flexible leader's corresponding investment capacity $K_f^{fd}(X)$ equals to $\min\left\{k_f^{fd}(X),\frac{X-c}{2\eta X}\right\}$ where $k_f^{fd}(X)$ makes it hold that*

$$\frac{c(\beta_2+1)F(\beta_1)}{2(\beta_1-\beta_2)}\left(\frac{X(1-2\eta k)}{c}\right)^{\beta_2}+\frac{X(1-2\eta k)}{r-\mu}-\frac{c}{r}-\delta-\frac{\mathrm{dECP}^{fd}(X,k|\tau_d^-:\mathrm{LU};\tau_d^+:\mathrm{LU})}{\mathrm{d}k}=0. \quad (34)$$

For a given K, the flexible leader's corresponding investment threshold $X_f^{fd}(K)$ also makes equation (33) *hold.*

Combining $K_f^{fd}(X)$ and $X_f^{fd}(K)$ yields the flexible leader's optimal investment decision $K_f^{fd^*}$ and $X_f^{fd^*}$.

Similar as in Proposition 4, for a given capacity size K, the flexible firm's investment threshold is the same as that of a flexible monopolist.

3.3 Flexible Leader and Flexible follower

In this subsection we consider both the follower and the leader can adjust their output according to the market demand. Then there are three regions for each firm concerning their output right after investment, i.e., production suspension, below-capacity production, and up-to-capacity production. These three regions are characterized by two boundaries for each firm given the current market demand. Because of the symmetric unit production cost, these four boundaries are reduced to three. In the following analysis we first analyze the flexible follower and then the flexible leader. Because there are multiple combination possibilities for the two firms' output, especially the leader's output right before and after the follower invests, we would like to only specify firms' value functions. Interested readers can refer to the Appendix C for the derivation of the firms' optimal investment decisions.

3.3.1 Flexible follower

Suppose the flexible follower invests at time τ_F. From time τ_F on, both firms are active in the market and can adjust their output within the constraint of installed capacity sizes. A smaller capacity implies it is relatively easy to reach the constraint, and vice versa. There are in total two cases depending on the comparison between the leader and the follower's investment sizes.

Case 1: $K_F^{ff}\geq K_L^{ff}$

This case is when the follower's invests a capacity that is no smaller than the leader's capacity. Denote the three boundaries in this case as

$$X_{F1}^{ff}=c,\quad X_{F2}^{ff}=\frac{c}{1-3\eta K_L^{ff}},\ \text{ and }\ X_{F3}^{ff}=\frac{c}{1-\eta K_L^{ff}-2\eta K_F^{ff}}.$$

It holds that when $X < X_{F1}^{ff}$, both firms suspend their production. When $X \in \left[X_{F1}^{ff}, X_{F2}^{ff}\right)$, both firms produce below capacity. When $X \in \left[X_{F2}^{ff}, X_{F3}^{ff}\right)$, the leader produces up to capacity but the follower produces below capacity. When $X \geq X_{F3}^{ff}$, both firms produce at full capacity. According to the analysis in the Appendix C, the flexible follower's value function after investment for a given GBM level X and the leader's capacity size K_L^{ff} is equal to

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} L1_F(K_L^{ff}, K_F^{ff})X^{\beta_1} & X < X_{F1}^{ff}, \\ M1_{F1}(K_L^{ff}, K_F^{ff})X^{\beta_1} + M1_{F2}X^{\beta_2} + \frac{1}{9\eta}\left(\frac{X}{r-\mu} + \frac{c^2}{X(r+\mu-\sigma^2)} - \frac{2c}{r}\right) & X_{F1}^{ff} \leq X < X_{F2}^{ff}, \\ M1_{F1}(K_L^{ff}, K_F^{ff})X^{\beta_1} + M1_{F2}X^{\beta_2} + \frac{1}{4\eta}\left(\frac{X\left(1-\eta K_L^{ff}\right)^2}{r-\mu} + \frac{c^2}{X(r+\mu-\sigma^2)} - \frac{2c\left(1-\eta K_L^{ff}\right)}{r}\right) & X_{F2}^{ff} \leq X < X_{F3}^{ff}, \\ N1_F(K_L^{ff}, K_F^{ff})X^{\beta_2} + \frac{XK_F^{ff}\left(1-\eta K_L^{ff}-\eta K_F^{ff}\right)}{r-\mu} - \frac{cK_F^{ff}}{r} & X \geq X_{F3}^{ff}. \end{cases}$$

The expressions of the coefficients $L1_F(K_L^{ff}, K_F^{ff})$, $M1_{F1}(K_L^{ff}, K_F^{ff})$, $M1_{F2}$, and $N1_F(K_L^{ff}, K_F^{ff})$ can also be found in Appendix C. The option values in each region corrects for changes in the value, i.e., when $X(t)$ hits the boundaries the follower's production suspends or becomes constrained by capacity. Note that at the boundary of X_{F2}^{ff}, the leader produces at full capacity, which does not generate extra option value for the follower, and the follower's value function is continuous at the boundary but not differentiable.

Case 2: $K_L^{ff} > K_F^{ff}$

In this case the leader installs a larger capacity than the follower. The corresponding three boundaries characterizing the follower's producing below or up to capacity after its investment time τ_F are

$$X_{L1}^{ff} = c, \quad X_{L2}^{ff} = \frac{c}{1-3\eta K_F^{ff}}, \quad \text{and} \quad X_{L3}^{ff} = \frac{c}{1-\eta K_F^{ff} - 2\eta K_L^{ff}}.$$

If $X < X_{L1}^{ff}$, both firms suspend their production. If $X \in \left[X_{L1}^{ff}, X_{L2}^{ff}\right)$, both produce below capacity. For $X \in \left[X_{L2}^{ff}, X_{L3}^{ff}\right)$, the follower produces up to capacity while the leader produces below capacity. If $X \geq X_{L3}^{ff}$, both produce up to capacity. The follower's instantaneous profit lead to the follower's value function in the stopping region as given by

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} L2_F(K_F^{ff})X^{\beta_1} & X < X_{L1}^{ff}, \\ M2_{F1}(K_F^{ff})X^{\beta_1} + M2_{F2}X^{\beta_2} + \frac{1}{9\eta}\left(\frac{X}{r-\mu} - \frac{2c}{r} + \frac{c^2}{X(r+\mu-\sigma^2)}\right) & X_{L1}^{ff} \leq X < X_{L2}^{ff}, \\ N2_F(K_F^{ff})X^{\beta_2} + \frac{K_F^{ff}}{2}\left(\frac{X(1-\eta K_F^{ff})}{r-\mu} - \frac{c}{r}\right) & X_{L2}^{ff} \leq X < X_{L3}^{ff}, \\ N2_F(K_F^{ff})X^{\beta_2} + \frac{XK_F^{ff}\left(1-\eta K_L^{ff}-\eta K_F^{ff}\right)}{r-\mu} - \frac{cK_F^{ff}}{r} & X \geq X_{L3}^{ff}. \end{cases}$$

The expressions for coefficients $L_{4F}(K_F^{ff})$, $M2_{F1}(K_F^{ff})$, $M2_{F2}$ and $N2_F(K_F^{ff})$ are given in the Appendix C. Different from Case 1, the coefficients for the option values here are independent of the leader's capacity size K_L^{ff}. This is due to the fact that the leader has a larger capacity than the follower and thus requires a larger market demand to produce at full capacity, i.e., the follower will be already producing at full capacity then. From the follower's value function, we could derive the follower's investment decisions, which are summarized in the corresponding proposition in Appendix C.

3.3.2 Flexible Leader

The flexible leader has monopoly profits before the follower enters the market. Its instantaneous profits right after investment are not affected by the potential market entry of the follower. Same as in subsection

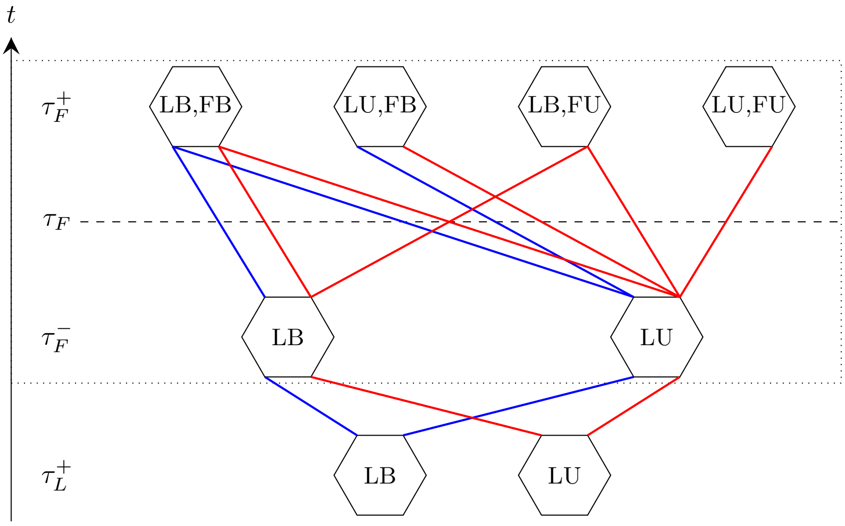


Figure 2: Possibilities for the leader's instant profit changes at the flexible follower's investment timing τ_F.

3.2, the flexible leader's value takes a similar functional expression as that of a monopolistic flexible firm in Wen et al. (2017). The follower's entry only generates a negative correction term in the flexible leader's value function. In particular, there are in total 9 possibilities for the leader's expected change of profit flows (ECP) at the flexible follower's time of investment, denoted by τ_F. These possibilities are illustrated in Figure 2, where τ_i^+ with $i \in \{L, F\}$ denotes the point in time right after the leader's (L) or the follower's (F) investment, and τ_F^- denotes the point in time right before the follower's investment. "LB" and "LU" imply the flexible leader produces below capacity and up to capacity. "FB" and "FU" denote that the flexible follower produces below and up to capacity. The possibilities of the value functions are indicated by three time points $\tau_L^+ \to \tau_F^- \to \tau_F^+$. The blue (red) lines connect the combination that the leader produces below (up to) capacity right after its own investment at time τ_L^+. For instance, "τ_L^+ : LB; τ_F^- : LU, τ_F^+ : LB, FB" is a possibility that, the leader produces below capacity right after its own investment, and produces up to capacity right before the follower's investment. Right after the follower's investment both the leader and the follower produce below capacity.

To derive the leader's value function, we first need to calculate the Expected Change in the flexible leader's Profit flow (ECP) due to the follower's market entry after the leader's investment. Similar as in subsection 3.2, denote this ECP for a given X as

$$\mathrm{ECP}^{ff}(X, K_L^{ff}) = \left(\frac{X}{X_F^{ff*}(K_L^{ff})}\right)^{\beta_1} \mathbb{E}_{\tau_F}\left[\int_0^\infty \left(\pi_L^{ff}(Q_L^{ff}, X(t)) - \pi_L^{ff}(Q_L^{ff}, X(t), K_F^{ff*}(K_L^{ff}))\right)\exp(-rt)\mathrm{d}t\right], \tag{35}$$

where $\pi_L^{ff}(Q_L^{ff}, X(t))$ represents the instant profit at time τ_F^-, and $\pi_L^{ff}(Q_L^{ff}, X(t), K_F^{ff*}(K_L^{ff}))$ represents the instant profit at time τ_F^+. The calculated expression for the expected term in (35) can be found in Appendix C. In order to navigate these 9 value functions for the flexible leader, we group them based on whether the leader produces below or up to capacity right after its own investment. So we distinguish the following two groups.

- The leader produces below capacity right after its own investment, i.e., τ_L^+ : LB, then the leader's value is given by

$$V_L^{ff}(X, K_L^{ff}) = M_1^{ff}(K_L^{ff})X^{\beta_1} + M_2^{ff}X^{\beta_2} + \frac{1}{4\eta}\left(\frac{X}{r-\mu} - \frac{2c}{r} + \frac{c^2}{X(r+\mu-\sigma^2)}\right) - \delta K_L^{ff} - \mathrm{ECP}^{ff}(X, K_L^{ff}).$$

In the value function, $M_1^{ff}(K_L^{ff})$ and M_2^{ff} have similar expressions as M_1^{df} and M_2^{df} in Appendix A, with X_1^D and X_2^D replacing X_{f1}^{df} and X_{f2}^{df}. The expression of $\mathrm{ECP}^{ff}(X, K_L^{ff})$ is conditional upon the leader and the follower's output at time τ_F^- and τ_F^+. The conditions are listed in the following Table 2.

Table 2: Output possibilities for the leader and follower at time τ_F^- and τ_F^+

τ_L^+ : LB		
τ_F^- : LB; τ_F^+ : LB, FB	τ_F^- : LU; τ_F^+ : LB, FB	τ_F^- : LU; τ_F^+ : LU, FB

According to Proposition 6 in Appendix C, τ_F^+ : LB, FU and τ_F^+ : LU, FU are not possible.

- The leader produces up to capacity right after its own investment, i.e., τ_L^+ : LU, then the leader's value function equals to

$$V_L^{ff}(X, K_L^{ff}) = N(K_L^{ff})X^{\beta_2} + \frac{X\left(1-\eta K_L^{ff}\right)K_L^{ff}}{r-\mu} - \frac{cK_L^{ff}}{r} - \delta K_L^{ff} - \mathrm{ECP}^{ff}(X, K_L^{ff}),$$

where $N(K_L^{ff})$ has the similar expression as N^{df} in Appendix A with X_1^D and X_2^D substituting X_{f1}^{df} and X_{f2}^{df}. There are six possibilities for the expression of $\mathrm{ECP}^{ff}(X, K_L^{ff})$, conditional on the leader and the follower's output at time τ_F^- and τ_F^+. These conditions are summarized in Table 3.

Table 3: Output possibilities for the leader and follower at time τ_F^- and τ_F^+

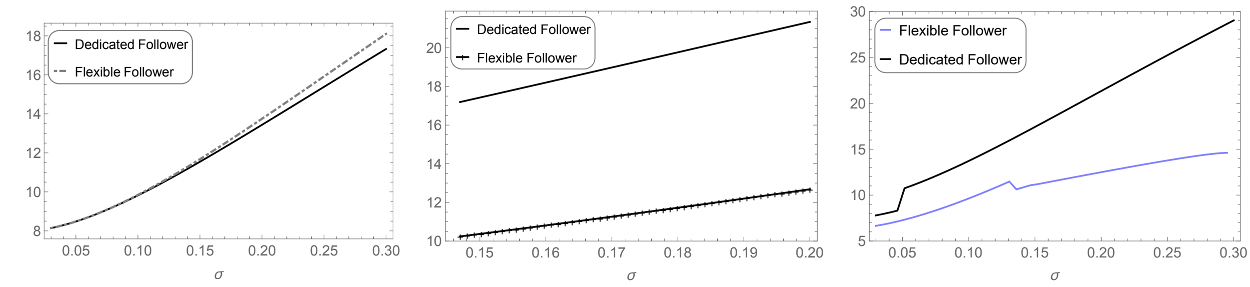
τ_L^+ : LU		
τ_F^- : LB; τ_F^+ : LB, FB	τ_F^- : LU; τ_F^+ : LB, FB	τ_F^- : LU; τ_F^+ : LU, FB
τ_F^- : LB; τ_F^+ : LB, FU	τ_F^- : LU; τ_F^+ : LB, FU	τ_F^- : LU; τ_F^+ : LU, FU

4 Equilibria under Endogenous Firm Roles

In this section, we analyze the equilibrium outcome when two firms in the duopoly setting can choose their volume flexibility at the moment of investment. Given the complexity of the analysis, we have to resort to numerical examples.

4.1 Asymmetric production technologies

We explore for a given exogenous leader, either flexible or dedicated, which production technology the corresponding follower chooses, i.e., which technology yields a larger value for the follower. Because our ultimate purpose is to analyze the preemption game in the duopoly setting, we consider only the non-simultaneous investment between a leader and a follower in this subsection. For a given leader's production technology, we observe the following figure.



(a) Flexible follower dominates dedicated follower's value when the leader is dedicated

(b) Dedicated follower dominates flexible follower when the leader is flexible and $K_F^{ff} \geq K_L^{ff}$.

(c) Dedicated follower dominates flexible follower when the leader is flexible and $K_F^{ff} < K_L^{ff}$.

Figure 3: Parameter values are $r = 0.1$, $\mu = 0.03$, $\eta = 0.05$, $c = 2$, $\delta = 10$ and $X_0 = 3$.

The analysis of a dedicated leader and a dedicated follower is carried out by Huisman and Kort (2015), and Wen (2017). The investment decisions for the dedicated leader and the flexible follower can be found in subsection 3.1. We compare the two different followers' values in the Figure 3a, which shows that given a dedicated leader, the flexible follower's value is larger than the dedicated follower's value, i.e., it is better for the follower to choose the volume flexibility if the leader chooses to be dedicated.

In order to conduct the analysis of a dedicated follower dominating a flexible follower, we can compare the value of a dedicated and a flexible follower at the moment of their corresponding investment for a given K_L, i.e., assume the same size of investment by the leader regardless of whether the follower is flexible. However, the follower's production technology inevitably affects the leader's capacity choice. So it is difficult to assume a representative size of capacity for the leader in both models. In our analysis we compute also the flexible leader's investment decision. In particular, we consider that the leader's optimal investment should be such that the corresponding output generates the largest net present value. Then we compare the dedicated and flexible follower's investment value under their corresponding leaders' optimal investment decisions. If the follower's value in model "flexible leader and dedicated follower" (FD) is larger than that in the model "flexible leader and flexible follower" (FF), then we can conclude that for a given flexible leader, a dedicated follower dominates a flexible follower. For the FD model, both the leader and the follower investment decisions can be found in Appendix B. For the FF model, the firms' investment decision can be found in Appendix C.

We distinguish two cases in FF model, based on the difference between K_F^{ff} and K_L^{ff}. Figure 3b depicts for the case of $K_F^{ff} \geq K_L^{ff}$, and shows the dominance of a dedicated follower for a given exogenous flexible leader [8]. The dedicated follower corresponds to the flexible leader in FD model and the leader produces up to capacity right after its own investment. The flexible follower corresponds to the flexible leader in the FF model and the leader produces up to capacity before the follower's investment.

Subfigure 3c compares for the case of $K_F^{ff} < K_L^{ff}$ in FF model. It is shown that the dedicated follower dominates the flexible follower when the leader is flexible. Note that there are jumps in the follower's values. This is because for each σ, the leader compares which scenario (up-to or below-capacity productions at τ_f^+, τ_d^- and τ_d^+, see the appendix D.1) generates the largest value. When the flexible leader's production changes at $\sigma = 0.049$ from τ_d^+ : LB to τ_d^+ : LU, the dedicated follower's value jumps upwards, which seems counter-intuitive. Apparently, the flexible leader has different investment capacities between these two scenarios.

[8] Note that 3b is under the condition that the flexible follower produces below capacity right after investment in FF model, i.e., τ_F^+ : FB, because the flexible follower does not produce up to capacity right after investment, as proved in Appendix C.

Between these two scenarios, the flexible leader invests less when it produces up to capacity both before and after the follower's investment. From the dedicated follower's perspective, this is better because the follower only needs the leader to provide the buffer effect.

4.2 Preemption analysis between a dedicated and a flexible firm

In this subsection, we analyze the preemption game between a flexible and a dedicated firm. For the flexible firm, the calculation of the preemption point is where the firm is indifferent from being a leader and being a follower. When the flexible firm is the follower, the parameter values define whether it produces below or up to capacity right after its investment. However, when the flexible firm is the leader, we have no knowledge if the parameter values still define its output quantity right after investment, and the leader's investment depends on which case yields larger value. So we first conduct the preemption analysis for the scenario of a consistent flexible firm, that is if the flexible firm as a follower produces below capacity right after investment, as a leader it also produces below capacity right after investment. Then we analyze for the scenario where the flexible firm is inconsistent, i.e., as a follower it produces below capacity right after investment, but as a leader it produces up to capacity right after investment.

According to the analysis in subsection 3.2, there are three cases for the flexible leader's output right before and after the follower's investment, i.e. at time τ_d^- and τ_d^+. "LB" indicates the flexible leader produces below capacity and "LU" indicates up to capacity. These three cases are listed in the following Table 4.

Table 4: Flexible leader's output possibilities at the dedicated follower's investment time τ_d

τ_d^- : LB; τ_d^+ : LB	τ_d^- : LU; τ_d^+ : LB	τ_d^- : LU; τ_d^+ : LU

When we take into account the flexible leader can produce below and up to capacity right after its own investment, we have to distinguish 6 cases in order to calculate the consistent flexible firm's preemption points, and 6 cases to calculate the inconsistent flexible firm's preemption points. In all the cases, the flexible firm's preemption point X_f^P makes it hold that

$$V_f^{fd}(X_f^P, K_f^{fd}(X_f^P)) = \left(X_f^P / X_f^{df}(K_d^{df}(X_f^P))\right)^{\beta_1} V_f^{df}(K_d^{df}(X_f^P), X_f^{df}(K_d^{df}(X_f^P)), K_f^{df}(K_d^{df}(X_f^P))) \ ,$$

and the dedicated firm's preemption point X_d^P satisfies the equation that

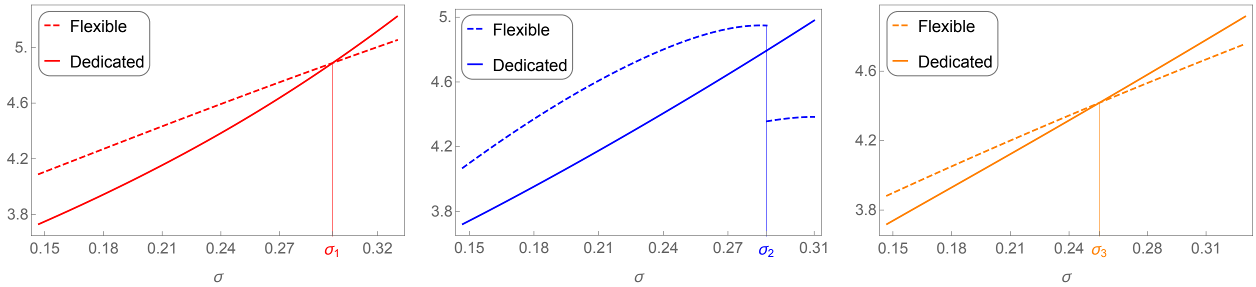
$$V_d^{df}(X_d^P, K_d^{df}(X_d^P)) = \left(X_d^P / X_d^{fd}(K_f^{fd}(X_d^P))\right)^{\beta_1} V_d^{fd}(K_f^{fd}(X_d^P), X_d^{fd}(K_f^{fd}(X_d^P)), K_d^{fd}(K_f^{fd}(X_d^P))).$$

The value functions V_f^{fd}, V_f^{df}, V_d^{df}, and V_d^{fd} can be found in the subsections 3.1 and 3.2.

4.2.1 Consistent Flexible Firm

The consistent flexible firm produces below capacity right after its own investment.

The firms' preemption points are illustrated in Figure 4. The three subfigures correspond to the three cases in Table 4. As shown in the subfigures, the dedicated firm has smaller preemption points when the market uncertainty is relatively low, i.e., $\sigma < \sigma_{j\in\{1,2,3\}}$. For relatively larger σ, i.e., $\sigma > \sigma_{j\in\{1,2,3\}}$, the flexible firm has smaller preemption points. The jump in the flexible firm's preemption points at σ_2 in Subfigure 4b is due to that the boundary solutions are encountered for the dedicated follower's investment. Given that the firms are asymmetric in our model, the preemption, especially by the dedicated firm is more about the strategic interaction and taking advantage of the other firm's volume flexibility. Our intuition is that the dedicated firm has to balance two effects: If it invests earlier than the flexible firm, it can benefit from a monopoly profit until the flexible firm invests, but it has to invest a smaller size that limits its market share in the future; If it invests later than the flexible firm, it can invest with a larger capacity and the flexible firm's volume flexibility provides a "buffer effect" against the demand fluctuations. The flexible firm also needs to balance the trade-off effects between investing earlier and later than the dedicated firm. If it invests earlier, the flexible firm can have some monopoly profits. If the firm invests later than the dedicated firm, then it can invest a larger size, which is good for the flexible firm given that the dedicated firm has to invest earlier and less to become a leader.



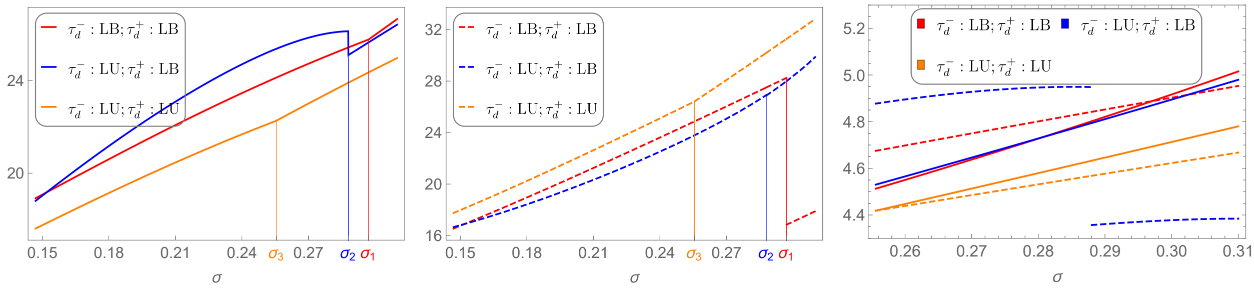
(a) τ_d^- : LB; τ_d^+ : LB (b) τ_d^- : LU; τ_d^+ : LB (c) τ_d^- : LU; τ_d^+ : LU

Figure 4: The preemption points for the flexible and the dedicated firms when the flexible firm produces below capacity right after its own investment. Parameter values are $r = 0.1$, $\mu = 0.03$, $\sigma = 0.1$, $\eta = 0.05$, $c = 2$, and $\delta = 10$.

With Figure 5, we show the dedicated and flexible firms' value as functions of σ in the preemption games for the three cases described in Table4, where the leader invests at the follower's preemption points. [9] Subfigure 5a shows that the dedicated firm prefers to be a leader if $\sigma < \sigma_1$. However, by comparing σ_1, σ_2 and σ_3, it is obvious that when $\sigma > \sigma_3$, it is possible for the flexible firm to preempt the dedicated firm. For flexible firm depicted in subfigure 5b, being in the case of τ_d^- : LU; τ_d^+ : LU always generates larger values, and it prefers to be a follower when $\sigma < \sigma_3$, and to be a leader when $\sigma > \sigma_3$. Subfigure 5c compares the preemption points for both firms given that $\sigma > \sigma_3$. It shows that the flexible firm has smaller preemption points. Overall, if the consistent flexible firm produces below capacity right after investment, an equilibrium outcome where firms choose their production technology upon investment is: When $0.147 < \sigma < \sigma_3$, the firm choosing dedicated production becomes the leader;[10] When $\sigma > \sigma_3$, the firm choosing flexible production becomes the leader, and the flexible leader produces up to capacity both before and after the dedicated follower's entry.

[9] Note the jumps in the flexible firm's value functions at σ_1 and σ_2 are because of the boundary solutions when calculating the optimal investment decisions. Especially when the boundary solutions are encountered in the calculation of one firm's preemption, but not in that of its opponent's preemption, then the equations in the analysis of the firm being the leader and being the followers are different.

[10] Please check the appendix for the analysis of how the dedicated leader switches among different preemption points in the three cases.



(a) Dedicated firm's value

(b) Flexible firm's value

(c) Preemption points comparison for $\sigma \geq \sigma_3$

Figure 5: The dedicated and flexible firm's values in the preemption games for cases in Table 4. Parameter values are $r = 0.1$, $\mu = 0.03$, $\sigma = 0.1$, $\eta = 0.05$, $c = 2$, $\delta = 10$ and $X_0 = 3$.

The consistent flexible firm produces up to capacity right after its own investment.

The flexible firm producing up to capacity right after investment implies that, the market uncertainty is small such that the firm can utilize all its production capacity then. Recall from the previous case, i.e., the flexible firm produces below capacity right after investment, that the dedicated firm preempts the flexible firm if the market uncertainty is small. This holds for all the three cases listed in Table 4, as shown in Figure 6. So when the firms choose the production technology upon investment for $\sigma < 0.147$, we conclude that the firm choosing dedicated production becomes the leader, and the other firm chooses volume flexibility and becomes the follower.

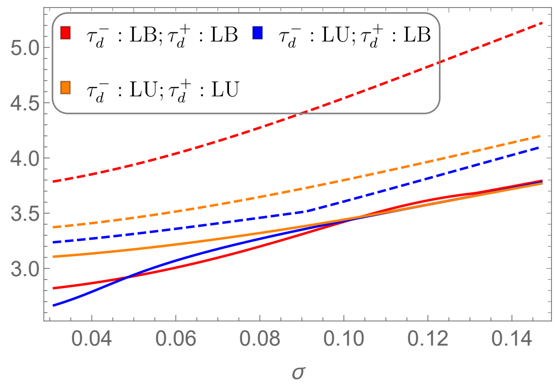
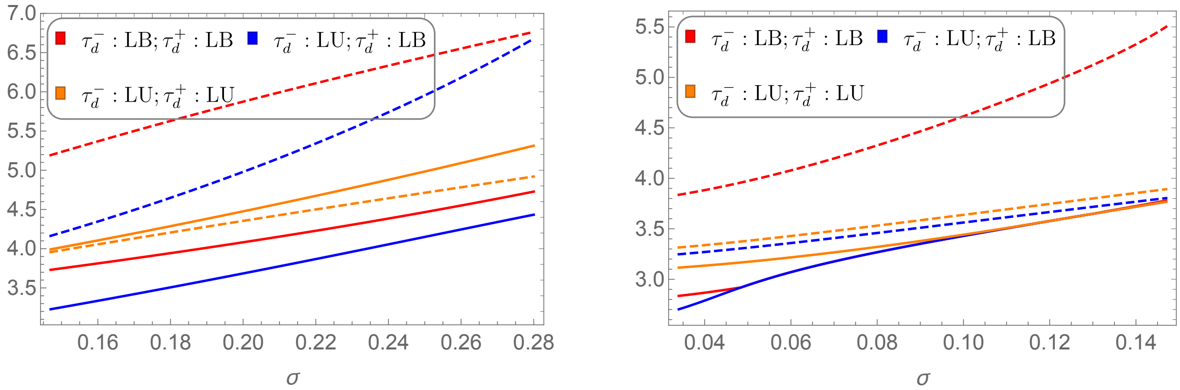


Figure 6: Preemption points comparison for cases in Table 4: flexible firm (dashed line), dedicated firm (real line). Parameter values are $r = 0.1$, $\mu = 0.03$, $\sigma = 0.1$, $\eta = 0.05$, $c = 2$, and $\delta = 10$.

4.2.2 Inconsistent Flexible Firm

In this subsection we consider that the flexible firm is not consistent with its production right after investment, i.e., if as a follower it produces below capacity right after investment, then as a leader it produces up to capacity right after investment, and vice versa. With Figure 7, we show that the the dedicated firm always preempts the inconsistent flexible firm in the equilibrium, i.e., it has a smaller preemption points for a given σ. This is different than that for a consistent flexible firm, where the flexible firm preempts the dedicated firm under larger market uncertainty. The reason is that the inconsistent flexible firm has larger preemption points than a consistent flexible firm, especially when there is more volatility in the market. Note that when X_t is below the preemption point, firms prefer to be the follower. Otherwise, they prefer to be leader. When the flexible firm is inconsistent, i.e., as follower it produces below capacity right after investment, but as leader it produces up to capacity right after investment, the inconsistent flexible firm needs a larger market demand to become the leader because it produces up to capacity right after investment in a more volatile market.

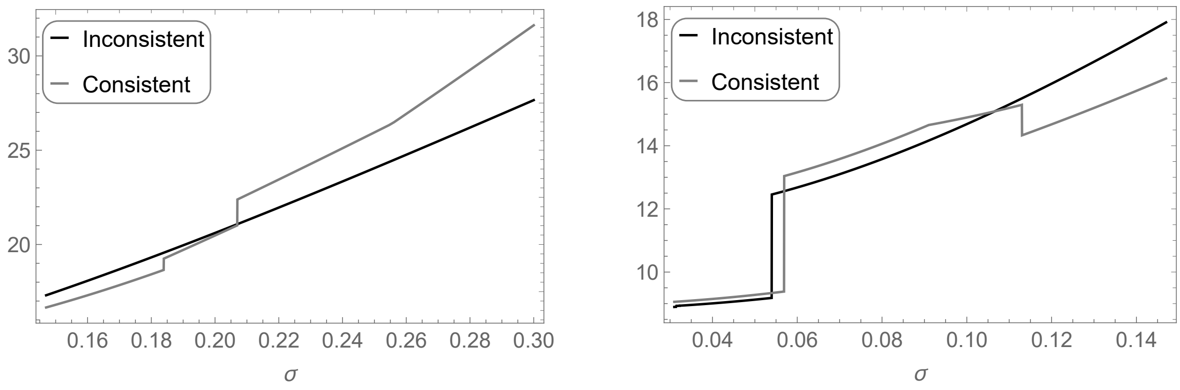


(a) Flexible follower below capacity right after investment

(b) Flexible follower up to capacity right after investment

Figure 7: Comparison of the flexible and dedicated firm's preemption points for an inconsistent flexible firm: flexible firm (dashed line), dedicated firm (real line). Parameter values are $r = 0.1$, $\mu = 0.03$, $\sigma = 0.1$, $\eta = 0.05$, $c = 2$, and $\delta = 10$.

In Figure 8 the value of the flexible firm is compared for being consistent and inconsistent. The jumps in the flexible firm's value functions are due to the dedicated firm's investment as a leader switches among different preemption points. This switch is further explained in the appendix. For a given σ, if being consistent generates a larger value for the flexible firm, then the flexible firm chooses to be consistent. Otherwise, the flexible firm chooses to be inconsistent. When the flexible firm as a follower produces below capacity, as depicted in subfigure 8b, the flexible firm switches between consistent and inconsistent for different σs, but the dedicated firm always preempts the flexible firm. When the flexible firm as a follower produces up to capacity, as displayed in subfigure 8a, the flexible firm chooses to be consistent when $\sigma > 0.207$. Note that the consistent flexible firm preempts the dedicated firm when $\sigma > 0.2556$. So the choice for the flexible firm to be consistent does not influence our preemption conclusion, i.e., when the market uncertainty is small, a dedicated firm preempts a flexible firm; when the market uncertainty is large, a flexible firm preempts a dedicated firm.



(a) Flexible follower below capacity right after investment

(b) Flexible follower up to capacity right after investment

Figure 8: Value comparison between an inconsistent flexible firm and a consistent flexible firm. Parameter values are $r = 0.1$, $\mu = 0.03$, $\sigma = 0.1$, $\eta = 0.05$, $c = 2$, $\delta = 10$ and $X(0) = 3$.

5 Conclusion

Volume flexibility is a technological advancement that allows firms to adjust output levels optimally according to the market demand change. This research considers firms' investment decisions under demand uncertainty consist not only the timing and capacity, as suggested by (Huisman and Kort, 2015), but also whether to be a flexible firm with volume flexibility, or a dedicated firm without volume flexibility.

This paper analyzes investment decisions for duopoly firms in exogenous firm roles where the volume flexibility is assigned upon investment, i.e., dedicated leader and flexible follower, flexible leader and dedicated follower, and flexible leader and flexible follower. For a flexible firm, the analysis distinguishes whether the firm produces below or up to capacity right after its own investment. In particular, if the flexible firm is the first investor in the market, the analysis takes into account different situations of the leader's output based on whether it produces below or up to capacity right before and after the corresponding follower's market entry.

The result of the analysis supports that both firms being flexible is not an equilibrium outcome. This is because given that the leader is flexible, a dedicated follower dominates a flexible follower. The intuition is that one firm being flexible is enough in generating the buffer effect for the firms in the market. Given a flexible firm and a dedicated firm, preemption analysis is carried out and this research concludes that when the market uncertainty is low, in the market equilibrium the leader is dedicated and the follower is flexible, and vice versa when the market uncertainty is large. This outcome is due to the buffer effect generated by the flexible firm. When uncertainty is low, the value of the commitment by the dedicated firm outweighs the buffer effect on market price under uncertainty. When uncertainty is large, the buffer effect become more attractive for the dedicated firm.

One limitation of this work is the assumption of the symmetric unit cost for investment. One would expect volume flexibility is more expensive to adopt. However, by assuming the symmetric cost allows us a clear picture about the influence of volume flexibility. Besides, our final conclusion is still robust given that the volume flexibility costs more, i.e., the dedicated firm would still preempt the flexible firm under low uncertainty, and probably a larger uncertainty is necessary for the flexible firm to preempt the dedicated firm.

Another limitation is that firms only invest once. In fact, multiple investment options or capacity expansions can also be considered as an operational flexibility. It would be interesting to explore how the operations flexibility influences the preemption between firms under demand uncertainty.

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Appendix

A Dedicated leader and flexible follower

Expressions of L^{df}, M_1^{df}, M_2^{df} and N^{df}: The coefficients for the option values in the value function $V_f^{df}(K_d^{df}, X, K_f^{df})$ are equal to

$$
\begin{aligned}
L^{df}(K_d^{df}, K_f^{df}) &= \frac{c^2 F(\beta_2)}{4\eta\,(\beta_1 - \beta_2)} \left(\left(X_{f1}^{df}(K_d^{df}) \right)^{-\beta_1 - 1} - \left(X_{f2}^{df}(K_d^{df}, K_f^{df}) \right)^{-\beta_1 - 1} \right), \\
N^{df}(K_d^{df}, K_f^{df}) &= \frac{c^2 F(\beta_1)}{4\eta\,(\beta_1 - \beta_2)} \left(\left(X_{f1}^{df}(K_d^{df}) \right)^{-\beta_2 - 1} - \left(X_{f2}^{df}(K_d^{df}, K_f^{df}) \right)^{-\beta_2 - 1} \right). \\
M_1^{df}(K_d^{df}, K_f^{df}) &= -\frac{c^2 F(\beta_2)}{4\eta\,(\beta_1 - \beta_2)} \left(X_{f2}^{df}(K_d^{df}, K_f^{df}) \right)^{-\beta_1 - 1}, \\
M_2^{df}(K_d^{df}) &= \frac{c^2 F(\beta_1)}{4\eta\,(\beta_1 - \beta_2)} \left(X_{f1}^{df}(K_d^{df}) \right)^{-\beta_2 - 1}.
\end{aligned}
$$

Proof of Proposition 1: The optimal investment capacity of the follower for a given X maximizes the value at the moment of investment. Taking the first order partial derivative of $V_f^{df}(K_d^{df}, X, K_f^{df})$ with respect to K_f^{df} yields the equations of (8) and (11) for the follower produces below and up to capacity right after investment respectively. Assuming the value before investment has the expression of $AX_f^{df\,\beta_1}(K_d^{df}, K)$, the optimal investment threshold $X_f^{df}(K_d^{df}, K)$ for a given capacity size K and the leader's investment capacity K_d^{df} satisfy the following value matching and smooth pasting conditions:

$$
\begin{cases}
AX^{\beta_1} & = V_f^{df}(K_d^{df}, X, K), \\
\beta_1 A X^{\beta_1 - 1} & = \partial V_f^{df}(K_d^{df}, X, K_f^{df}) \Big/ \partial X.
\end{cases}
$$

Thus, $X_f^{df}(K_d^{df}, K)$ satisfies the following implicit equation

$$
V_f^{df}(K_d^{df}, X, K) = \frac{X}{\beta_1} \partial V_f^{df}(K_d^{df}, X, K_f^{df}) \Big/ \partial X. \tag{36}
$$

The implicit equation (36) leads to equation (9) and (12). For the region where $0 < X < c/(1 - \eta K_D)$, equation (36) implies that $\delta K_f^{df} = 0$, so the flexible follower does not invest in this region.

Note that in the monopoly case by Wen et al. (2017), whether the flexible firm producing up to capacity depends on the parameter values. Similarly as for the follower in the duopoly situation, if the firm produces below capacity right after investment, then $Q_f^{df*}(K_d^{fd}, X) < K_f^{df*}(K_d^{fd}, X)$, i.e.,

$$
\frac{1}{2\eta} \left(1 - \eta K_d^{fd} - \frac{c}{X} \left(\frac{2\delta\,(\beta_1 - \beta_2)}{cF\,(\beta_2)\,(1 + \beta_1)} \right)^{\frac{1}{\beta_1}} \right) > \frac{X(1 - \eta K_d^{fd}) - c}{2\eta X}.
$$

It is equivalent to

$$
2\delta(\beta_1 - \beta_2) < cF(\beta_2)(1 + \beta_1), \tag{37}
$$

which is the same as in the monopoly case. Furthermore, it can be deduced that

$$
2\delta(\beta_1 - \beta_2) \geq cF(\beta_2)(1 + \beta_1) \tag{38}
$$

defines Region 3, where the firm produces up to capacity right after investment. The definitions of Region 2 where $X \geq \frac{c}{1 - \eta K_d^{df}}$ and $K_f^{df} > \frac{X - c}{2\eta X} - \frac{K_d^{df}}{2}$, equation (37), and Region 3 where $X \geq \frac{c}{1 - \eta K_d^{df}}$ and

$K_f^{df} \leq \frac{X-c}{2\eta X} - \frac{K_d^{df}}{2}$, equation (38), for the flexible follower firm are the same as that for the flexible monopoly firm in Wen et al. (2017).

Expressions of $\mathcal{L}^{df}(K_d^{df})$, $\mathcal{M}_1^{df}(K_d^{df})$, $\mathcal{M}_2^{df}(K_d^{df})$, $\mathcal{N}^{df}(K_d^{df})$: Employing value matching and smooth pasting at $X = c/(1-\eta K_d^{df})$ and $X = c/\left(1-\eta K_d^{df} - 2\eta K_f^{df*}(K_d^{df})\right)$, it can be derived for a given $K_d^{df} \in [0, 1/\eta)$ that

$$\begin{aligned}
\mathcal{M}_2^{df}(K_d^{df}) &= \frac{cK_d^{df}}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\mu} - \frac{\beta_1}{r}\right)\left(\frac{c}{1-\eta K_d^{df}}\right)^{-\beta_2}, \\
\mathcal{M}_1^{df}(K_d^{df}) &= -\frac{cK_d^{df}}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu} - \frac{\beta_2}{r}\right)\left(\frac{c}{1-\eta K_d^{df} - 2\eta K_f^{df*}(K_d^{df})}\right)^{-\beta_1}, \\
\mathcal{L}^{df}(K_d^{df}) &= \frac{cK_d^{df}}{2(\beta_1-\beta_2)}\left(\frac{\beta_2-1}{r-\mu} - \frac{\beta_2}{r}\right)\left(\left(\frac{c}{1-\eta K_d^{df}}\right)^{-\beta_1} - \left(\frac{c}{1-\eta K_d^{df} - 2\eta K_f^{df*}(K_d^{df})}\right)^{-\beta_1}\right), \\
\mathcal{N}^{df}(K_d^{df}) &= \frac{cK_d^{df}}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\mu} - \frac{\beta_1}{r}\right)\left(\left(\frac{c}{1-\eta K_d^{df}}\right)^{-\beta_2} - \left(\frac{c}{1-\eta K_d^{df} - 2\eta K_f^{df*}(K_d^{df})}\right)^{-\beta_2}\right).
\end{aligned}$$

In order to check the signs for $\mathcal{L}^{df}(K_d^{df})$, $\mathcal{M}_1^{df}(K_d^{df})$, $\mathcal{M}_2^{df}(K_d^{df})$, and $\mathcal{N}^{df}(K_d^{df})$, first analyze the signs of $(\beta-1)/(r-\mu) - \beta/r = \frac{\mu\beta - r}{r(r-\mu)}$ for $\beta = \beta_1$ and $\beta = \beta_2$.

If $\mu \geq 0$, then $\mu\beta_2 - r < 0$ because $\beta_2 < 0$. If $\mu < 0$, then $\mu\beta_2 - r = \mu\left(\frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{r}{\mu} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}\right)$, with $\frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{r}{\mu} > 0$. From $\left(\frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{r}{\mu}\right)^2 - \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 - \frac{2r}{\sigma^2} = -\frac{r}{\mu} + \frac{r^2}{\mu^2} > 0$, we get $\mu\beta_2 - r < 0$. So, $\frac{\beta_2-1}{r-\mu} - \frac{\beta_2}{r} < 0$.

If $\mu \leq 0$, then $\mu\beta_1 - r < 0$. If $\mu > 0$, then $\mu\beta_1 - r = \mu\left(\frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{r}{\mu} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}\right)$, with $\frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{r}{\mu} < 0$, because $r > \mu$. From $\left(\frac{r}{\mu} + \frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 - \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 - \frac{2r}{\sigma^2} = \frac{r^2}{\mu^2} - \frac{r}{\mu} > 0$, it holds that $\mu\beta_1 - r < 0$. So, $\frac{\beta_1-1}{r-\mu} - \frac{\beta_1}{r} < 0$.

Thus, it can be concluded that when $0 \leq K_d^{df} < 1/\eta$, then $\mathcal{L}^{df}(K_d^{df}) < 0$, $\mathcal{M}_1^{df}(K_d^{df}) > 0$, $\mathcal{M}_2^{df}(K_d^{df}) < 0$, $\mathcal{N}^{df}(K_d^{df}) > 0$.

To derive the optimal investment decision, we first note that

$$\frac{\mathrm{d}\mathcal{B}_1(K_d^{df})}{\mathrm{d}K_d^{df}} = \frac{1-\eta K_d^{df} - \beta_1\eta K_d^{df}}{K_d^{df}\left(1-\eta K_d^{df}\right)}\mathcal{B}_1(K_d^{df}).$$

Next, we analyze the entry deterrence (non-simultaneous) and accommodation (simultaneous) strategies for the dedicated leader, which include the optimal investment capacities and optimal investment thresholds.

Proof of Proposition 2: We derive the leader's investment decisions based on whether the flexible follower produces below or up to capacity right after its investment.

Case1: The flexible follower produces below capacity right after investment, i.e., $\mu > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \mu \leq \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$.

The investment capacity $K_d^{df}(X)$ for a given level of X maximizes $V_d^{df}(X, K_d^{df})$ and satisfies the following equation (39).

$$\frac{1-\eta K_d^{df} - \beta_1\eta K_d^{df}}{K_d^{df}(1-\eta K_d^{df})}\mathcal{B}_1(K_d^{df})X^{\beta_1} + \frac{1-2\eta K_d^{df}}{r-\mu}X - \frac{c}{r} - \delta = 0. \tag{39}$$

Suppose the investment threshold of the dedicated leader is $X_d^{df^*}$. The leader's value function before and after the investment is as follows

$$V_d^{df}(X, K_d^{df}) = \begin{cases} \mathcal{A}(K_d^{df})X^{\beta_1} & X < X_d^{df^*}, \\ \mathcal{B}_1(K_d^{df})X^{\beta_1} + \frac{K_d^{df}(1-\eta K_d^{df})}{r-\mu}X - \frac{cK_d^{df}}{r} & X_d^{df^*} \leq X < X_f^{df^*}(K_d^{df}), \\ \mathcal{M}_1^{df}(K_d^{df})X^{\beta_1} + \mathcal{M}_2^{df}(K_d^{df})X^{\beta_2} & \\ +\frac{K_d^{df}(1-\eta K_d^{df})}{2(r-\mu)}X - \frac{cK_d^{df}}{2r} - \delta K_d^{df} & X \geq X_f^{df^*}(K_d^{df}). \end{cases} \tag{40}$$

The value matching and smooth pasting conditions at the investment threshold $X_d^{df}(K)$ for a given capacity size K lead to

$$X_d^{df}(K) = \frac{\beta_1}{\beta_1 - 1} \times \frac{r-\mu}{1-\eta K}\left(\frac{c}{r} + \delta\right). \tag{41}$$

Substituting $X_d^{df}(K)$ into (39), the optimal investment capacity $K_d^{df^*}$ and investment threshold $X_d^{df^*}$ can be derived as

$$K_d^{df^*} = \frac{1}{(\beta_1+1)\eta}, \quad X_d^{df^*} = \frac{(\beta_1+1)(r-\mu)}{\beta_1 - 1}\left(\frac{c}{r} + \delta\right).$$

Case 2:The flexible follower produces up to capacity right after the investment, i.e., $\mu \leq r - c/\delta$, or both $r - c/\delta < \mu \leq \delta r^2/(c + \delta r)$ and $\sigma \leq \bar{\sigma}$.

To derive the optimal investment for the dedicated leader in this case, we first derive that

$$\frac{\mathrm{d}\mathcal{B}_2(K_d^{df})}{\mathrm{d}K_d^{df}} = \frac{1 - \eta K_d^{df} - \beta_1\eta K_d^{df}}{K_d^{df}\left(1 - \eta K_d^{df}\right)}\mathcal{B}_2(K_d^{df}).$$

If the leader applies the entry deterrence strategy and invests at $X_d^{df^*}$, then the value function before and after investment is

$$V_d^{df}(X, K_d^{df}) = \begin{cases} \mathcal{A}(K_d^{df})X^{\beta_1} & X < X_d^{df^*}, \\ \mathcal{B}_2(K_d^{df})X^{\beta_1} + \frac{K_d^{df}(1-\eta K_d^{df})}{r-\mu}X - \frac{cK_d^{df}}{r} - \delta K_d^{df} & X_d^{df^*} \leq X < X_f^{df^*}(K_d^{df}), \\ \mathcal{N}^{df}(K_d^{df})X^{\beta_2} + \frac{K_d^{df}(1-\eta K_d^{df} - \eta K_f^{df^*}(K_d^{df}))}{r-\mu}X - \frac{cK_d^{df}}{r} & X \geq X_f^{df^*}(K_d^{df}). \end{cases} \tag{42}$$

The optimal capacity by the dedicated leader for a given X, $K_d^{df}(X)$, can be derived by taking the first order condition with respect to K_d^{df}, which satisfies equation

$$\frac{\left(1 - \eta K_d^{df} - \beta_1\eta K_d^{df}\right)X^{\beta_1}}{K_d^{df}\left(1 - \eta K_d^{df}\right)}\mathcal{B}_2(K_d^{df}) + \frac{1 - 2\eta K_d^{df}}{r-\mu}X - \frac{c}{r} - \delta = 0. \tag{43}$$

For a given capacity level K, from value matching and smooth pasting at the investment threshold $X_d^{df}(K)$, which reads

$$X_d^{df}(K) = \frac{\beta_1(r-\mu)}{(\beta_1-1)(1-\eta K)}\left(\frac{c}{r} + \delta\right). \tag{44}$$

Combing (43) and (44) yields the optimal investment decision under the non-simultaneous investment as

$$K_d^{df^*} = \frac{1}{(\beta_1+1)\eta}, \quad X_d^{df^*} = \frac{(\beta_1+1)(r-\mu)}{\beta_1 - 1}\left(\frac{c}{r} + \delta\right).$$

B Flexible leader and dedicated follower

Proof of Proposition 3: In order to analyze the dedicate follower's investment decision for a given flexible leader's investment capacity K_f^{fd}, we assume the follower's value before investment is denoted as $A_d^{fd}X^{\beta_1}$. Then for the three regions, i.e., the leader is not producing, producing below capacity and producing up to capacity when the follower enters the market, we can derive the follower's optimal investment decision: the capacity decision $K_d^{fd^*}(K_f^{fd})$ and the threshold decision $X_d^{fd^*}(K_f^{fd})$ for each region separately.

Region 1: $K_d^{fd} \geq \frac{X-c}{\eta X}$. Given a capacity size for the dedicated follower K_d^{fd}, the value matching and smooth pasting conditions imply an equation that $\beta_1 K_d^{fd}\left(\frac{c}{r}+\delta\right)=0$, i.e., $K_d^{fd}=0$. So the dedicated follower does not investment in this case.

Region 2: $\frac{X(1-2\eta K_f^{fd})-c}{\eta X} \leq K_d^{fd} < \frac{X-c}{\eta X}$. Note that

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{M}_1^{fd}}{\mathrm{d}K_d^{fd}} &= \frac{1-2\eta K_f^{fd}-(\beta_1+1)\eta K_d^{fd}}{K_d^{fd}\left(1-2\eta K_f^{fd}-\eta K_d^{fd}\right)}\mathcal{M}_1^{fd}\,,\\
\frac{\mathrm{d}\mathcal{M}_2^{fd}}{\mathrm{d}K_d^{fd}} &= \frac{1-(\beta_2+1)\eta K_d^{fd}}{K_d^{fd}\left(1-\eta K_d^{fd}\right)}\mathcal{M}_2^{fd}\,.
\end{aligned}
$$

Maximizing $V_d^{fd}(K_f^{fd},X,K_d^{fd})$ with respect to K_d^{fd} by taking in the boundary solutions yield the investment capacity $K_d^{fd}(K_f^{fd},X)$ for a given X as in equation (18). For a given K, the investment threshold $X_d^{fd}(K_f^{fd},K)$ as in equation (19) can be derived.

Region 3: $K_d^{fd} < \frac{X(1-2\eta K_f^{fd})-c}{\eta X}$. In this region it holds that

$$
\begin{aligned}
\frac{\partial\mathcal{N}^{fd}}{\partial K_d^{fd}} = \frac{1}{2(\beta_1-\beta_2)}\left(\frac{\beta_1-1}{r-\mu}-\frac{\beta_1}{r}\right)&\left(\left(1-(\beta_2+1)\eta K_d^{fd}\right)\left(\frac{c}{1-\eta K_d^{fd}}\right)^{1-\beta_2}\right.\\
&\left.-\left(1-2\eta K_f^{fd}-(\beta_2+1)\eta K_d^{fd}\right)\left(\frac{c}{1-\eta K_d^{fd}-2\eta K_f^{fd}}\right)^{1-\beta_2}\right)\,.
\end{aligned}
$$

Similar as in *Region 2*, the follower's capacity $K_d^{fd}(K_f^{fd},X)$, i.e., equation (20), for a given X is calculated by maximizing $V_d^{fd}(K_f^{fd},X,K_d^{fd})$ with respect to K_d^{fd} wile taking into account of the boundary. The investment threshold $X_d^{fd}(K_f^{fd},K)$ as in equation (21) is from the value matching and smoothing pasting conditions.

Proof of Lemma 1: The influence of the flexible leader's investment capacity K_f^{fd} on the expected change in profit $\mathrm{ECP}^{fd}(K_f^{fd})$ can be derived by taking the derivative of $\mathrm{ECP}^{fd}(K_f^{fd})$ with respect to K_f^{fd}, where the follower's investment decision $X_d^{fd^*}(K_f^{fd})$ and $K_d^{fd^*}(K_f^{fd})$ are as in Proposition 3.

Proof of Proposition 4: Assume the flexible leader's value before investment is given by $A_f X^{\beta_1}$. Given that the flexible leader produces below capacity right after its own investment, its value function at the moment of investment is equal to

$$
V_f^{fd}(X,K_f^{fd}) = M_1^{fd}(K_f^{fd})X^{\beta_1}+M_2^{fd}X^{\beta_2}+\frac{1}{4\eta}\left(\frac{X}{r-\mu}-\frac{2c}{r}+\frac{c^2}{X(r+\mu-\sigma^2)}\right)-\delta K_f^{fd}-\mathrm{ECP}^{fd}(X,K_f^{fd}).
$$

This value depends on whether it will be producing up to or below capacity right before and right after the dedicated follower enters the market, i.e., at time τ_d^- and τ_d^-.

i. When the flexible leader produces below capacity at at time τ_d^+, i.e., τ_d^+ : LB, $\mathrm{ECP}^{fd}(X, K_f^{fd}) = \mathrm{ECP}^{fd}(X, K_f^{fd} | \tau_d^- : \mathrm{LB}; \tau_d^+ : \mathrm{LB})$ if it also produces below capacity at time τ_d^-; and $\mathrm{ECP}^{fd}(X, K_f^{fd}) = \mathrm{ECP}^{fd}(X, K_f^{fd} | \tau_d^- : \mathrm{LU}; \tau_d^+ : \mathrm{LB})$ if it also produces up to capacity at time τ_d^-.

ii. When the flexible leader will be producing up to capacity at at time τ_d^+, i.e., τ_d^+ : LU, then in the leader's value function $\mathrm{ECP}^{fd}(X, K_f^{fd}) = \mathrm{ECP}^{fd}(X, K_f^{fd} | \tau_d^- : \mathrm{LU}; \tau_d^+ : \mathrm{LU})$.

In the value functions, the coefficients for the option values are

$$M_1^{fd}(K_f^{fd}) = -\frac{c^2 F(\beta_2)}{4\eta(\beta_1 - \beta_2)} \left(\frac{c}{1 - 2\eta K_f^{fd}} \right)^{-1-\beta_1} \quad \text{and} \quad M_2^{fd} = \frac{c^{1-\beta_2} F(\beta_1)}{4\eta(\beta_1 - \beta_2)} .$$

For a given X, the flexible leader's investment capacity $K_f^{fd}(X)$ maximizes the value at the moment of investment. Taking the first order condition of $V_f^{fd}(X, K_f^{fd})$ with respect to K_f^{fd} yields the equations of (27), (28) and (30). For a given capacity size K, the corresponding investment threshold $X_f^{fd}(K)$ can be derived by the value matching and smooth pasting conditions and $X_f^{fd}(K)$ satisfies (29) regardless of whether the flexible leader will be producing below or up to capacity when the dedicated follower enters the market.

Proof of Proposition 5: Suppose the flexible leader's value before investment is $A_f X^{\beta_1}$. Given that the flexible leader produces below capacity right after its own investment, its value function at the moment of investment is equal to

$$V_f^{fd}(X, K_f^{fd}) = N(K_f^{fd}) X^{\beta_2} + \frac{X\left(1 - \eta K_f^{fd}\right) K_f^{fd}}{r - \mu} - \frac{c K_f^{fd}}{r} - \delta K_f^{fd} - \mathrm{ECP}^{fd}(X, K_f^{fd}).$$

Same as in the proof of Proposition 4. In the leader's value function $\mathrm{ECP}^{fd}(X, K_f^{fd})$ also depends on its output at time τ_d^- and at time τ_d^+, implying three possibilities the same as in Proposition 4. In the leader's value function, the coefficient for the option values is

$$N(K_f^{fd}) = \frac{c^2 F(\beta_1)}{4\eta(\beta_1 - \beta_2)} \left(c^{-1-\beta_2} - \left(\frac{c}{1 - 2\eta K_f^{fd}} \right)^{-1-\beta_2} \right) .$$

For a given X, the flexible leader's investment capacity $K_f^{fd}(X)$ maximizes the value at the moment of investment. Taking the first order condition of $V_f^{fd}(X, K_f^{fd})$ with respect to K_f^{fd} yields the equations of (31), (32) and (34). For a given capacity size K, the corresponding investment threshold $X_f^{fd}(K)$ can be derived by the value matching and smooth pasting conditions and $X_f^{fd}(K)$ satisfies (33) regardless of whether the flexible leader will be producing below or up to capacity when the dedicated follower enters the market.

C Flexible leader and flexible follower

C.1 Flexible follower

Case 1: The follower invests a larger capacity that the leader, i.e., $K_F^{ff} \geq K_L^{ff}$.

Right after the follower's investment at time τ_F, the flexible follower's profit for a given X and K_L^{ff} is equal to

$$\pi_F^{ff}(K_L^{ff},X,Q_F^{ff})=\begin{cases}0 & \text{if } X<X_{F1}^{ff},\\ \big(X(1-\eta Q_L^{ff}-\eta Q_F^{ff})-c\big)Q_F^{ff} & \text{if } X_{F1}^{ff}\le X<X_{F2}^{ff},\\ \big(X(1-\eta K_L^{ff}-\eta Q_F^{ff})-c\big)Q_F^{ff} & \text{if } X_{F2}^{ff}\le X<X_{F3}^{ff},\\ \big(X(1-\eta K_L^{ff}-\eta K_F^{ff})-c\big)K_F^{ff} & \text{if } X\ge X_{F3}^{ff}.\end{cases}$$

From the follower's profit we can calculate the follower's optimal output quantities in each case, and they are given by

$$Q_F^{ff}(K_L^{ff},X)=\begin{cases}0 & \text{if } X<X_{F1}^{ff},\\ \frac{X-c}{3\eta X} & \text{if } X_{F1}^{ff}\le X<X_{F2}^{ff},\\ \frac{X-X\eta K_L^{ff}-c}{2\eta X} & \text{if } X_{F2}^{ff}\le X<X_{F3}^{ff},\\ K_F^{ff} & \text{if } X\ge X_{F3}^{ff}.\end{cases}$$

From the follower's optimal output, we can characterize the boundaries for the follower as

$$X_{F1}^{ff}=c \quad\text{and}\quad X_{F3}^{ff}=\frac{c}{1-\eta K_L^{ff}-2\eta K_F^{ff}}.$$

In order to characterize the other two boundaries, we also need to get the expressions of the leader's optimal output quantity and it is equal to

$$Q_L^{ff}(X,K_F^{ff})=\begin{cases}0 & \text{if } X<X_{F1}^{ff},\\ \frac{X-c}{3\eta X} & \text{if } X_{F1}^{ff}\le X<X_{F2}^{ff},\\ K_L^{ff} & \text{if } X\ge X_{F2}^{ff}.\end{cases}$$

Thus, it can be derived that

$$X_{F2}^{ff}=\frac{c}{1-3\eta K_L^{ff}}.$$

Expressions of $L1_F$, $M1_{F1}$, $M1_{F1}$, and $N1_F$: To derive the coefficients of the option values, we apply the value matching and smooth pasting conditions at the boundary X_{F1}^{ff} for regions $X<X_{F1}^{ff}$ and $X_{F1}^{ff}\le X<X_{F2}^{ff}$, and at the boundary X_{F3}^{ff} for regions $X_{F2}^{ff}\le X<X_{F3}^{ff}$ and $X\ge X_{F3}^{ff}$. Then we can derive the following expressions.

$$\begin{aligned}
M1_{F2} &= \frac{c^2F(\beta_1)}{9(\beta_1-\beta_2)\eta}\left(X_{F1}^{ff}\right)^{-\beta_1-1},\\
M1_{F1}(K_L^{ff},K_F^{ff}) &= -\frac{c^2F(\beta_2)}{4(\beta_1-\beta_2)\eta}\left(X_{F3}^{ff}\right)^{-\beta_1-1},\\
N1_F(K_L^{ff},K_F^{ff}) &= \frac{c^2F(\beta_1)}{36(\beta_1-\beta_2)\eta}\left(4\left(X_{F1}^{ff}\right)^{-\beta_2-1}-9\left(X_{F3}^{ff}\right)^{-\beta_2-1}\right),\\
L1_F(K_L^{ff},K_F^{ff}) &= \frac{c^2F(\beta_2)}{36(\beta_1-\beta_2)\eta}\left(4\left(X_{F1}^{ff}\right)^{-\beta_1-1}-9\left(X_{F3}^{ff}\right)^{-\beta_1-1}\right).
\end{aligned}$$

For the calculated coefficients the option values, we can further derive the follower's investment decisions based on both firm's output quantities right after the follower's investment.

Proposition 6 *The flexible follower enters the market and produces below capacity right after investment when $\mu > \delta r^2/(c+\delta r)$, or both $r - c/\delta < \mu \le \delta r^2/(c+\delta r)$ and $\sigma > \bar{\sigma}$. Given that the flexible leader is already active in the market and has invested a capacity size K_L^{ff}, the flexible follower's investment decisions are as follows.*

i. The leader is producing below capacity when the follower invests: For a given GBM level X, the follower's investment capacity $K_F^{ff}(K_L^{ff}, X)$ is given by

$$\max\left\{K_L^{ff}, \frac{1}{2\eta}\left(1-\eta K_L^{ff} - \frac{c}{X}\left(\frac{(\beta_1+1)c}{2(\beta_1-\beta_2)\delta}\left(\frac{2\beta_2}{r} - \frac{\beta_2-1}{r-\mu} - \frac{\beta_2+1}{r+\mu-\sigma^2}\right)\right)^{-1/\beta_1}\right)\right\} . \tag{45}$$

For a given $K \ge K_L^{ff}$, the follower's investment threshold is $X_F^{ff}(K_L^{ff}, K)$, which satisfies equation

$$(\beta_1-\beta_2)M1_{F2}X^{\beta_2} - \frac{1}{9\eta}\left(\frac{2\beta_1 c}{r} - \frac{(\beta_1-1)X}{r-\mu} - \frac{(\beta_1+1)c^2}{X(r+\mu-\sigma^2)}\right) - \beta_1\delta K = 0 . \tag{46}$$

ii. The leader is producing up to capacity when the follower invests: For a given GBM level X, the follower's investment capacity $K_F^{ff}(K_L^{ff}, X)$ maximizes the value at the moment of investment and has the same expression as equation (45). For a given capacity size $K \ge K_L^{ff}$, the follower's investment threshold $X_F^{ff}(K_L^{ff}, K)$, according to the value matching and smooth pasting conditions, satisfies the implicit equation,

$$(\beta_1-\beta_2)M1_{F2}X^{\beta_2} - \frac{1}{4\eta}\left(\frac{2\beta_1 c\left(1-\eta K_L^{ff}\right)}{r} - \frac{(\beta_1-1)X\left(1-\eta K_L^{ff}\right)^2}{r-\mu} - \frac{(\beta_1+1)c^2}{X(r+\mu-\sigma^2)}\right) - \beta_1\delta K = 0 . \tag{47}$$

Combining $K_F^{ff}(K_L^{ff}, X)$ and $X_F^{ff}(K_L^{ff}, K)$ yield the follower's optimal investment decision $K_F^{ff^*}(K_L^{ff})$ and $X_F^{ff^*}(K_L^{ff})$.

Note that the follower's investment decision should make it hold that $X_F^{ff^*}(K_L^{ff}) \in \left[X_{F1}^{ff}, < X_{F2}^{ff}\right)$ when the leader is producing below capacity and $X_F^{ff^*}(K_L^{ff}) \in \left(X_{F2}^{ff}, X_{F3}^{ff}\right)$ when the leader is producing up to capacity. If the derived $X_F^{ff^*}(K_L^{ff})$ is larger than the upper bound of the corresponding interval, then the follower does not invest in the corresponding scenario. Furthermore, Proposition 6 shows that for a given GBM level X, the flexible follower's investment capacity is the same. The intuition is that when the flexible follower firm produces below capacity right after investment, the firm's instantaneous profit is only influenced by the instantaneous demand, rather than the firm's capacity size. This implies that the decision about the capacity is from the long-term perspective, which is the same regardless of whether the leader is producing below or up to capacity when the follower invests.

Proof of Proposition 6:

Flexible follower does not produce right after investment: $X < X_{F1}^{ff}$

For a given K_L^{ff}, the follower's value function before and after investment is given by

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} AX^{\beta_1} & \text{if } X < X_F^{ff^*}(K_L^{ff}) , \\ L1_F(K_L^{ff}, K_F^{ff})X^{\beta_1} - \delta K_F^{ff} & \text{if } X \ge X_F^{ff^*}(K_L^{ff}) . \end{cases}$$

The follower does not invest in this region because the value matching and smooth pasting conditions do not hold unless $K_F^{ff} = 0$.

Flexible follower producing below capacity right after investment: $X_{F1}^{ff} \leq X < X_{F2}^{ff}$

For a given $K_L^{ff} > \frac{X-c}{3\eta X}$, suppose the follower's value before and after investment takes the following form

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} AX^{\beta_1} & \text{if } X < X_F^{ff^*}(K_L^{ff}) \ , \\ M1_{F1}(K_L^{ff}, K_F^{ff})X^{\beta_1} + M1_{F2}X^{\beta_2} & \\ +\frac{1}{9\eta}\left(\frac{X}{r-\mu} + \frac{c^2}{X(r+\mu-\sigma^2)} - \frac{2c}{r}\right) - \delta K_F^{ff} & \text{if } X \geq X_F^{ff^*}(K_L^{ff}) \ . \end{cases}$$

For a given X, taking the first order condition of $V_F^{ff}(K_L^{ff}, X, K_F^{ff})$ with respect to K_F^{ff} and the boundary condition that $K_F^{ff} \geq K_L^{ff}$ yield equation (45). For a given $K \geq K_L^{ff}$, the follower's investment threshold $X_F^{ff}(K)$ is derived by the value matching and smooth pasting conditions at the threshold, which leads to equation (46). Moreover, becaus the leader is producing below capacity when the follower invests, when $K_F^{ff}(K_L^{ff}) > K_L^{ff}$ it holds that

$$K_L^{ff} > \frac{X-c}{3\eta X} \Longrightarrow K_F^{ff}(X) < \frac{1}{6\eta X}\left(2X + c - 3c\left(\frac{(\beta_1+1)c}{2(\beta_1-\beta_2)\delta}\left(\frac{2\beta_2}{r} - \frac{\beta_2-1}{r-\mu} - \frac{\beta_2+1}{r+\mu-\sigma^2}\right)\right)^{-1/\beta_1}\right) .$$

Because $K_F^{ff}(K_L^{ff}, X) > \frac{X-c}{3\eta X}$, it can be derive that

$$\begin{aligned} &\frac{1}{6\eta X}\left(2X + c - 3c\left(\frac{(\beta_1+1)c}{2(\beta_1-\beta_2)\delta}\left(\frac{2\beta_2}{r} - \frac{\beta_2-1}{r-\mu} - \frac{\beta_2+1}{r+\mu-\sigma^2}\right)\right)^{-1/\beta_1}\right) > \frac{X-c}{3\eta X} \\ \Longrightarrow \quad &\frac{(\beta_1+1)c}{2(\beta_1-\beta_2)\delta}\left(\frac{2\beta_2}{r} - \frac{\beta_2-1}{r-\mu} - \frac{\beta_2+1}{r+\mu-\sigma^2}\right) > 1 \ . \end{aligned}$$

This condition is the same as the definition for which the flexible firm produces below capacity right after investment as in the monopoly setting.

Flexible follower producing below capacity right after investment and flexible leader is producing up to capacity: $X_{F2}^{ff} \leq X < c/X_{F3}^{ff}$.

For a given capacity size by the leader K_L^{ff}, the follower's value before and after investment is assumed to take the functional form as

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} AX^{\beta_1} & \text{if } X < X_F^{ff^*}(K_L^{ff}) \ , \\ M1_{F1}(K_L^{ff}, K_F^{ff})X^{\beta_1} + M1_{F2}X^{\beta_2} & \\ +\frac{1}{4\eta}\left(\frac{X\left(1-\eta K_L^{ff}\right)^2}{r-\mu} + \frac{c^2}{X(r+\mu-\sigma^2)} - \frac{2c\left(1-\eta K_L^{ff}\right)}{r}\right) - \delta K_F^{ff} & \text{if } X \geq X_F^{ff^*}(K_L^{ff}) \ . \end{cases}$$

At a given GBM level X, the follower's investment capacity $K_F^{ff}(K_L^{ff}, X)$ maximizes the value at the moment of investment. Taking the first order condition of $V_F^{ff}(K_L^{ff}, X, K_F^{ff})$ with respect to $K^{ff})K$ and the boundary condition that $K_F^{ff} \geq K_L^{ff}$ lead to equation (45). For a given capacity size $K \geq K_L^{ff}$, the follower's investment threshold $X_F^{ff}(K_L^{ff}, K)$, according to the value matching and smooth pasting conditions, satisfies the following implicit equation (47). Note that when the leader' is producing up to capacity and the follower produces below capacity right after investment, given that $K_F^{ff} > K_L^{ff}$, we have the following inequality,

$$K_F^{ff}(K_L^{ff}, X) > \frac{X(1-\eta K_L^{ff})-c}{2\eta X} \Longrightarrow \frac{(\beta_1+1)c}{2(\beta_1-\beta_2)\delta}\left(\frac{2\beta_2}{r}-\frac{\beta_2-1}{r-\mu}-\frac{\beta_2+1}{r+\mu-\sigma^2}\right) > 1 \ .$$

This is the same definition for the region that the flexible firm produces below capacity right after investment in the monopoly setting.

Flexible follower producing up to capacity right after investment and the flexible leader is also producing up to capacity: $X \geq X_{F3}^{ff}$

For a given K_L^{ff}, the follower's value function before and after investment is supposed to be equal to

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} AX^{\beta_1} & \text{if } X < X_F^{ff^*}(K_L^{ff}) \ , \\ N1_F(K_L^{ff}, K_F^{ff})X^{\beta_2} + \frac{XK_F^{ff}(1-\eta K_L^{ff}-\eta K_F^{ff})}{r-\mu} - \frac{cK_F^{ff}}{r} - \delta K_F^{ff} & \text{if } X \geq X_F^{ff^*}(K_L^{ff}) \ . \end{cases}$$

The second order derivative of V_F^{ff} with respect to K_F^{ff} in this region is

$$\frac{\partial^2 V_F^{ff}}{\partial {K_F^{ff}}^2} \quad = \quad -\frac{\beta_2(\beta_2+1)\eta}{\beta_1-\beta_2}\left(\frac{c}{1-\eta K_L^{ff}-2\eta K_F^{ff}}\right)^{1-\beta_2} F(\beta_1)\beta_2(\beta_2-1)X^{\beta_2-2} > 0 \ .$$

Given that in this region $K_L^{ff} < K_F^{ff} < \left(1-\eta K_L^{ff}\right)/(2\eta)$, the flexible follower would invest for a given X with capacity size $\left(1-\eta K_L^{ff}\right)/(2\eta)$, because this capacity generates the highest value of $+\infty$. But the corresponding investment would be delayed infinitely. So the follower does not invest in this case.

Case 2: The follower invests larger capacity that the leader, i.e., $K_F^{ff} < K_L^{ff}$.
Right after the follower's investment at time τ_F, the flexible follower's profit for a given X and K_L^{ff} is equal to

$$\pi_F^{ff}(K_L^{ff}, X, Q_F^{ff}) = \begin{cases} 0 & \text{if } X < X_{L1}^{ff} \ , \\ \left(X\left(1-\eta Q_L^{ff}-\eta Q_F^{ff}\right)-c\right)Q_F^{ff} & \text{if } X_{L1}^{ff} \leq X < X_{L2}^{ff} \ , \\ \left(X\left(1-\eta Q_L^{ff}-\eta K_F^{ff}\right)-c\right)K_F^{ff} & \text{if } X_{L2}^{ff} \leq X < X_{L3}^{ff} \ , \\ \left(X\left(1-\eta K_L^{ff}-\eta K_F^{ff}\right)-c\right)K_F^{ff} & \text{if } X \geq X_{L3}^{ff} \ . \end{cases}$$

The corresponding optimal output maximized the follower's profits in each region and is as follows:

$$Q_F^{ff^*}(K_L^{ff}, X) = \begin{cases} 0 & \text{if } X < X_{L1}^{ff} \ , \\ \frac{X-c}{3\eta X} & \text{if } X_{L1}^{ff} \leq X < X_{L2}^{ff} \ , \\ K_F^{ff} & \text{if } X \geq X_{L2}^{ff} \ . \end{cases}$$

So we can get the boundaries for the follower to suspend production, producing below capacity and to produce up to capacity, and they equal to

$$X_{L1}^{ff} = c \ \text{ and } \ X_{L2}^{ff} = \frac{c}{1-3\eta K_F^{ff}} \ .$$

The leader's profit in different regions is given by

$$\pi_L^{ff}(X, K_L^{ff}) = \begin{cases} 0 & \text{if } X < X_{L1}^{ff} \ , \\ \left(X\left(1-\eta Q_L^{ff}-\eta Q_F^{ff}\right)-c\right)Q_L^{ff} & \text{if } X_{L1}^{ff} \leq X < X_{L2}^{ff} \ , \\ \left(X\left(1-\eta Q_L^{ff}-\eta K_F^{ff}\right)-c\right)Q_L^{ff} & \text{if } X_{L2}^{ff} \leq X < X_{L3}^{ff} \ , \\ \left(X\left(1-\eta K_L^{ff}-\eta K_F^{ff}\right)-c\right)K_L^{ff} & \text{if } X \geq X_{L3}^{ff} \ . \end{cases}$$

The leader's optimal output is denoted as

$$Q_L^{ff*}(X) = \begin{cases} 0 & \text{if } X < X_{L1}^{ff}, \\ \frac{X-c}{3\eta X} & \text{if } X_{L1}^{ff} \leq X < X_{L2}^{ff}, \\ \frac{X-X\eta K_F^{ff}-c}{2\eta X} & \text{if } X_{L2}^{ff} \leq X < X_{L3}^{ff}, \\ K_L^{ff} & \text{if } X \geq X_{L3}^{ff}. \end{cases}$$

This implies that

$$X_{L3}^{ff} = \frac{c}{1-\eta K_F^{ff}-2\eta K_L^{ff}} \ .$$

Expressions of $L2_F$, $M2_{F1}$, $M2_{F2}$ and $N2_F$: In this case by the value matching and smooth pasting conditions at the follower's boundaries X_{L1}^{ff} and X_{L2}^{ff}, we can get the expressions of the coefficients for the option values.

$$\begin{aligned} M2_{F2} &= \frac{c^2F(\beta_1)}{9(\beta_1-\beta_2)\eta}\left(X_{L1}^{ff}\right)^{-1-\beta_2} \ , \\ M2_{F1}(K_F^{ff}) &= -\frac{c^2F(\beta_2)}{9\eta(\beta_1-\beta_2)}\left(X_{L2}^{ff}\right)^{-1-\beta_1} - \frac{cK_F^{ff}}{6(\beta_1-\beta_2)}\left(\frac{\beta_2}{r}-\frac{\beta_2-1}{r-\mu}\right)\left(X_{L2}^{ff}\right)^{-\beta_1} \ , \\ N2_F(K_F^{ff}) &= \frac{c^2F(\beta_1)}{9\eta(\beta_1-\beta_2)}\left(\left(X_{L1}^{ff}\right)^{-1-\beta_2}-\left(X_{L2}^{ff}\right)^{-1-\beta_2}\right) - \frac{cK_F^{ff}}{6(\beta_1-\beta_2)}\left(\frac{\beta_1}{r}-\frac{\beta_1-1}{r-\mu}\right)\left(X_{L2}^{ff}\right)^{-\beta_2} \ , \\ L2_F(K_F^{ff}) &= M2_{F1} + \frac{c^2F(\beta_2)}{9\eta(\beta_1-\beta_2)}\left(X_{F1}^{ff}\right)^{-1-\beta_1} \ . \end{aligned}$$

For the calculated coefficients the option values, we can further derive the follower's investment decisions based on both firm's output quantities right after the follower's investment.

Proposition 7 *The follower's investment decision depends on the leader's output quantity and can be characterized by the following situations.*

i. Both the leader and the follower produce below capacity right after the follower's investment. For a given GBM level X, the follower firm's corresponding investment capacity maximizes the value at the moment of investment and equals to

$$K_F^{ff}(X) = \begin{cases} K_L^{ff} & \text{if } k_1(X) > K_L^{ff}, \\ k_1(X) & \text{if } 0 < k_1(X) \leq K_L^{ff}, \\ 0 & \text{otherwise,} \end{cases} \tag{48}$$

where $k_1(X)$ satisfies the following equation

$$-\frac{c}{6(\beta_1-\beta_2)}\frac{1-(\beta_1+1)3\eta k_1}{1-3\eta k_1}\left(\frac{c}{X(1-3\eta k_1)}\right)^{-\beta_1}\left(\frac{\beta_2}{r}-\frac{\beta_2-1}{r-\mu}\right)$$
$$\frac{(\beta_1+1)F(\beta_2)}{3(\beta_1-\beta_2)}\left(\frac{c}{X(1-3\eta k_1)}\right)^{-\beta_1} - \delta = 0 \ .$$

Given a capacity level K, the follower's investment threshold $X_F^{ff}(K)$ solves the following equation

$$M2_{F2} \times (\beta_1 - \beta_2) X^{\beta_2} + \frac{1}{9\eta}\left(\frac{(\beta_1 - 1)X}{r-\mu} + \frac{(\beta_1+1)c^2}{X(r+\mu-\sigma^2)} - \frac{2\beta_1 c}{r}\right) - \beta_1 \delta K = 0 \ . \tag{49}$$

ii. The leader produces below capacity and the follower produces up to capacity right after the follower's investment. For a given X, the follower's corresponding investment capacity is given by

$$K_F^{ff}(X) = \begin{cases} K_L^{ff} & \textit{if } k_2(X) > K_L^{ff}, \\ k_2(X) & \textit{if } 0 < k_2(X) \leq K_L^{ff}, \\ 0 & \textit{otherwise,} \end{cases} \tag{50}$$

where $k_2(X)$ satisfies the equation

$$\begin{aligned} &\frac{(\beta_2+1)cF(\beta_1)}{3(\beta_1-\beta_2)}\left(\frac{c}{X(1-3\eta k_2)}\right)^{-\beta_2} + \frac{1}{2}\left(\frac{X(1-2\eta k_2)}{r-\mu} - \frac{c}{r}\right) - \delta \\ &-\frac{c}{6(\beta_1-\beta_2)}\frac{1-(\beta_2+1)3\eta k_2}{1-3\eta k_2}\left(\frac{c}{X(1-3\eta k_2)}\right)^{-\beta_2}\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{r-\mu}\right) = 0 \ . \end{aligned}$$

Given a capacity size K, the corresponding investment threshold $X_F^{ff}(K)$ satisfies the following implicit equation

$$N2_F(K_F^{ff}) \times (\beta_1 - \beta_2) X^{\beta_2} - \left(\frac{\beta_1(c+2r\delta)K}{2r} - \frac{(\beta_1-1)X(1-\eta K)K}{2(r-\mu)}\right) = 0 \ . \tag{51}$$

iii. Both the leader and the follower produce up to capacity right after the follower's investment. Given a GBM level X, the follower's investment capacity is equal to

$$K_F^{ff}(K_L^{ff}, X) = \begin{cases} K_L^{ff} & \textit{if } k_3(K_L^{ff}, X) > K_L^{ff}, \\ k_3(K_L^{ff}, X) & \textit{if } 0 < k_3(K_L^{ff}, X) \leq K_L^{ff}, \\ 0 & \textit{otherwise,} \end{cases} \tag{52}$$

where $k_3(K_L^{ff}, X)$ satisfies the equation of

$$\begin{aligned} &\frac{(\beta_2+1)cF(\beta_1)}{3(\beta_1-\beta_2)}\left(\frac{c}{X(1-3\eta k_3)}\right)^{-\beta_2} + \frac{X\left(1-\eta K_L^{ff} - 2\eta k_3\right)}{r-\mu} - \frac{c}{r} - \delta \\ &-\frac{c}{6(\beta_1-\beta_2)}\frac{1-(\beta_2+1)3\eta k_3}{1-3\eta k_3}\left(\frac{c}{X(1-3\eta k_3)}\right)^{-\beta_2}\left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{r-\mu}\right) = 0 \ . \end{aligned}$$

For a given capacity size K, the follower's investment threshold $X_F^{ff}(K_L^{ff}, K)$ is such that the following equation holds,

$$(\beta_1 - \beta_2) N2_F(K) X^{\beta_2} + \frac{(\beta_1 - 1)XK\left(1 - \eta K_L^{ff} - \eta K\right)}{r-\mu} - \frac{\beta_1 K(c+r\delta)}{r} = 0 \ . \tag{53}$$

Combining $K_F^{ff}(K_L^{ff}, X)$ and $X_F^{ff}(K_L^{ff}, K)$ yield the follower's optimal investment decision $K_F^{ff^*}(K_L^{ff})$ and $X_F^{ff^*}(K_L^{ff})$.

Proof of Proposition 7:

Flexible follower not producing right after investment: $X < c$

Suppose the follower's value before and after investment is denoted as

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} AX^{\beta_1} & \text{if } X < X_F^{ff^*}(K_L^{ff}) \ , \\ L2_F(K_F^{ff})X^{\beta_1} - \delta K_F^{ff} & \text{if } X \geq X_F^{ff^*}(K_L^{ff}) \ . \end{cases}$$

The value matching and smoothing pasting at the moment of the follower's investment threshold requires that $K_F^{ff} = 0$, implying the follower does not invest in this region.

Flexible follower producing below capacity right after investment when the leader producing below capacity: $c \leq X < c/\left(1 - 3\eta K_L^{ff}\right)$

For a given leader's investment capacity size K_L^{ff}, assume the follower's value before and after investment is given by

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} AX^{\beta_1} & \text{if } X < X_F^{ff^*}(K_L^{ff}) \ , \\ M2_{F1}(K_F^{ff})X^{\beta_1} + M2_{F2}X^{\beta_2} & \\ +\frac{1}{9\eta}\left(\frac{X}{r-\mu} - \frac{2c}{r} + \frac{c^2}{X(r+\mu-\sigma^2)}\right) - \delta K_F^{ff} & \text{if } X \geq X_F^{ff^*}(K_L^{ff}) \ . \end{cases}$$

For a given GBM level X, taking the first order derivative of the follower firm's value right after investment with respect to K_F^{ff} and combining with the boundary condition that $K_L^{ff} \geq K_F^{ff}$ leads to (48). For a given a capacity level K, the follower's investment threshold $X_F^{ff}(K)$ satisfies the value matching and smooth pasting conditions at the threshold, which leads to (49). The solution to the two equations characterizing $K_F^{ff}(X)$ and $X_F^{ff}(K)$ is not influenced by K_L^{ff}.

Flexible follower producing up to capacity right after investment when the leader producing below capacity: $c/\left(1 - 3\eta K_L^{ff}\right) \leq X < c/\left(1 - \eta K_L^{ff} - 2\eta K_F^{ff}\right)$

With the leader's investment capacity K_L^{ff} the follower's value before and after investment is given by

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} AX^{\beta_1} & \text{if } X < X_F^{ff^*}(K_L^{ff}) \ , \\ N2_F(K_F^{ff})X^{\beta_2} + \frac{K_F^{ff}}{2}\left(\frac{X\left(1-\eta K_F^{ff}\right)}{r-\mu} - \frac{c}{r}\right) - \delta K_F^{ff} & \text{if } X \geq X_F^{ff^*}(K_L^{ff}) \ . \end{cases}$$

For a given X, maximizing the value at the moment of investment with respect to K_F^{ff} and taking into the boundary condition lead to (50). Given a capacity size K, the corresponding investment threshold, i.e., equation (51). The solution to the two equations characterizing $K_F^{ff}(X)$ and $X_F^{ff}(K)$ is also not influenced by K_L^{ff} because the leader is producing below capacity when the follower enters.

Flexible follower producing up to capacity right after investment when the leader producing up to capacity: $X \geq c/\left(1 - \eta K_L^{ff} - 2\eta K_F^{ff}\right)$

The follower's value before and after investment in this situation is given by

$$V_F^{ff}(K_L^{ff}, X, K_F^{ff}) = \begin{cases} AX^{\beta_1} & \text{if } X < X_F^{ff^*}(K_L^{ff}) \ , \\ N2_F(K_F^{ff})X^{\beta_2} + \frac{XK_F^{ff}(1-\eta K_L^{ff} - \eta K_F^{ff})}{r-\mu} - \frac{cK_F^{ff}}{r} - \delta K_F^{ff} & \text{if } X \geq X_F^{ff^*}(K_L^{ff}) \ . \end{cases}$$

Given a GBM level X, taking the first order derivative of the value at the moment of investment with respect to K_F^{ff} and combining with the boundary condition of $K_L^{ff} \geq K_F^{ff}$ yield the equation (52). For a given capacity size K, the follower's investment threshold $X_F^{ff}(K_L^{ff}, K)$ can be derived from the value matching and smooth pasting conditions, which leads to (53).

C.2 Flexible leader

For the flexible leader, it is possible that the leader adjusts its output at time τ_F, i.e., the follower's investment timing. This adjustment could cause a decrease in the leader's profit flow, and then leads to a decrease in the project value. Denote the leader's output as Q_L^{ff}, then at time τ_F^- and time τ_F^+, the leader's output equals to

$$\tau_F^- :\ Q_L^{ff} = \begin{cases} \frac{X(\tau_F^-)-c}{2\eta X(\tau_F^-)} & \text{LB}, \\ K_L^{ff} & \text{LU}, \end{cases} \quad \text{and} \quad \tau_F^+ : Q_L^{ff} = \begin{cases} \frac{X(\tau_F^+)-c}{3\eta X(\tau_F^+)} & \text{LB,FB}, \\ K_L^{ff} & \text{LU,FB}, \\ \frac{\left(1-\eta K_F^{ff*}(K_L^{ff})\right)X(\tau_F^+)-c}{2\eta X(\tau_F^+)} & \text{LB,FU}, \\ K_L^{ff} & \text{LU,FU}. \end{cases}$$

The corresponding leader's profit at time τ_F^- and time τ_F^+ is given by

$$\tau_F^- :\ \pi_L^{ff}(Q_L^{ff}, X(\tau_F^-)) = \begin{cases} \frac{\left(X(\tau_F^-)-c\right)^2}{4\eta X(\tau_F^-)} & \text{LB}, \\ K_L^{ff}\left(\left(1-\eta K_L^{ff}\right)X(\tau_F^-)-c\right) & \text{LU}, \end{cases}$$

and

$$\tau_F^+ : \pi_L^{ff}(Q_L^{ff}, X(\tau_F^+), K_F^{ff*}(K_L^{ff})) = \begin{cases} \frac{\left(X(\tau_F^+)-c\right)^2}{9\eta X(\tau_F^+)} & \text{LB,FB}, \\ \frac{K_L^{ff}}{2}\left(\left(1-\eta K_L^{ff}\right)X(\tau_F^+)-c\right) & \text{LU,FB}, \\ \frac{\left(\left(1-\eta K_F^{ff*}(K_L^{ff})\right)X(\tau_F^+)-c\right)^2}{4\eta X(\tau_F^+)} & \text{LB,FU}, \\ K_L^{ff}\left(\left(1-\eta K_L^{ff}-\eta K_F^{ff*}(K_L^{ff})\right)X(\tau_F^+)-c\right) & \text{LU,FU}. \end{cases}$$

In order to derive the leader's value function, we have to get the expressions for $\mathrm{ECP}^{ff}(X, K_L^{ff})$ as equation (35). To achieve this, we rewrite as follows

$$\mathbb{E}_{\tau_F}\left[\int_0^\infty \left(\pi_L^{ff}\left(Q_L^{ff}, X(t)\right) - \pi_L^{ff}\left(Q_L^{ff}, X(t), K_F^{ff*}\right)\right)\exp\left(-rt\right)\mathrm{d}t\right] =$$

$$\begin{cases} \frac{5}{36\eta}\left(\frac{X_F^{ff*}}{r-\mu} + \frac{c^2}{(r+\mu-\sigma^2)X_F^{ff*}} - \frac{c}{r}\right) & \text{if } \tau_F^- : \text{LB}; \tau_F^+ : \text{LB, FB} \\ \frac{K_L^{ff}\left(1-\eta K_L^{ff}\right)X_F^{ff*}}{r-\mu} - \frac{cK_L^{ff}}{r} - \frac{1}{9\eta}\left(\frac{X_F^{ff*}}{r-\mu} + \frac{c^2}{(r+\mu-\sigma^2)X_F^{ff*}} - \frac{c}{r}\right) & \text{if } \tau_F^- : \text{LU}; \tau_F^+ : \text{LB, FB} \\ \frac{K_F^{ff*}}{4}\left(\frac{\left(2-\eta K_F^{ff*}\right)X_F^{ff*}}{r-\mu} - \frac{2c}{r}\right) & \text{if } \tau_F^- : \text{LB}; \tau_F^+ : \text{LB, FU} \\ \frac{K_L^{ff}\left(1-\eta K_L^{ff}\right)X_F^{ff*}}{r-\mu} - \frac{cK_L^{ff}}{r} - \frac{1}{4\eta}\left(\frac{\left(1-\eta K_F^{ff*}\right)^2 X_F^{ff*}}{r-\mu} + \frac{c^2}{(r+\mu-\sigma^2)X_F^{ff*}} - \frac{2c\left(1-\eta K_F^{ff*}\right)}{r}\right) & \text{if } \tau_F^- : \text{LU}; \tau_F^+ : \text{LB, FU} \\ \frac{\eta K_L^{ff} X_F^{ff*}}{r-\mu} K_F^{ff*} & \text{if } \tau_F^- : \text{LU}; \tau_F^+ : \text{LU, FU} \\ \frac{\eta K_L^{ff} X_F^{ff*}}{r-\mu} \times \frac{\left(1-\eta K_L^{ff}\right)X_F^{ff*}-c}{2\eta X_F^{ff*}} & \text{if } \tau_F^- : \text{LU}; \tau_F^+ : \text{LU, FB} \end{cases} \tag{54}$$

Expression (54) combined with the leader's output levels right after its own investment at time τ_L^+ yields 9 value functions for the leader. We don't give the explicit investment decisions for the leader, but rather the idea to calculate the leader's optimal investment:

a. Given a GBM level X, taking the first order derivative of the leader's value at the moment of investment $V_L^{ff}(X, K_L^{ff}) - \delta K_L^{ff}$ with respect to K_L^{ff} yields the leader's investment size $K_L^{ff}(X)$.

b. Denote the leader's option value to invest as $A_L X^{\beta_1}$, then for a given capacity size K, the leader's investment threshold $X_L^{ff}(K)$ can be derived by the value matching and smooth pasting conditions.

c. Combining $K_L^{ff}(X)$ and $X_L^{ff}(K)$ leads to the leader's optimal investment decision $K_L^{ff^*}$ and $X_L^{ff^*}$, based on which, we can calculate the corresponding follower's investment decision $K_F^{ff^*}$ and $X_F^{ff^*}$.

d. Note that these decisions have to be checked against the corresponding boundaries for $X_L^{ff^*}$ and $X_F^{ff^*}$. So check whether it is just one firm's solution lies beyond the boundary of both firms' solutions lie beyond the boundaries.

e. If it is just one firm's solution is out of the boundary, then take the boundary solution for this firm and go to *c*.

f. If both firms' solutions are out of the boundary, then substitute the corresponding $K_F^{ff}(X)$ from the boundary expression and go to *c*.

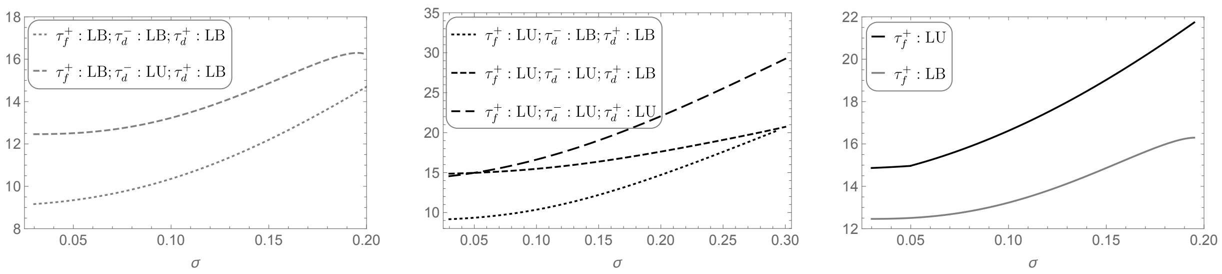
D Extra explanation for the equilibrium analysis

D.1 Asymmetric production technology

In sub-section 4.1, we present the dominance of the dedicated follower in figure 3b and 3c given that the leader is flexible. In particular, the numerical analysis includes the optimal investment size by the leader, rather than a representative investment size. In the following analysis, we show how the flexible leader's decides on its investment in both FD model and FF model.

FD Model

We first analyze the FD model and check which output possibility, i.e., characterized at the dedicated follower's investment timing τ_d, generates the largest value for the flexible leader.



(a) Flexible leader's value if τ_f^+ : LB (b) Flexible leader's value if τ_f^+ : LU (c) Flexible leader's NPV comparison

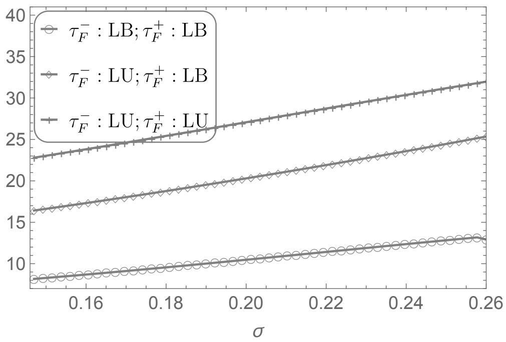
Figure 9: FD model flexible leader's value of investment. Parameter values are $r = 0.1$, $\mu = 0.03$, $\eta = 0.05$, $c = 2$, $\delta = 10$ and $X_0 = 3$.

Figure 9 shows the flexible leader's value as functions of σ in FD model. Recall that "τ_i^+" with $i \in \{f, d\}$ denotes the point in time right after the flexible firm's ($i = f$) or the dedicated firm's ($i = d$) investment, and "LB" ("LU") denotes the flexible leader produces below (up to) capacity. Subfigure 9a depicts that, if the flexible leader produces below capacity right after its own investment, it has larger value if it produces up to capacity before the follower's entry.[11] This is because if the flexible leader invests in a way such that it is producinng up to capacity right before the follower's entry, then the leader's expected profit change (decrease) is smaller, which is good for the flexible leader's. Similar reasoning also applies when the flexible leader produces up to capacity right after its own investment. As shown in subfigure 9b, where the flexible leader has the largest value for almost all σs if it produces up to capacity both before and after the follower's investment. In fact, the flexible leader behaves like a dedicated firm in the sense that it produces up to capacity at all the three time points τ_f^+, τ_d^- and τ_d^+, i.e., though the flexible leader cannot commit to a certain output, it can choose its investment so as to imitate a dedicated firm. Subfigure 9c compares the flexible leader's values for producing below and up to capacity right after its own investment, and shows that the latter generates a larger value for the leader.

FF Model

Because there are two cases in model FF, depending on the size of the flexible leader and follower's capacity, so we show in the following that in both cases, the dedicated follower has a larger value than the flexible follower.

*Case 1. Flexible follower installs a capacity **non-smaller** than the flexible leader in FF model*



(a) FF model flexible leader's value if τ_F^+ : FB

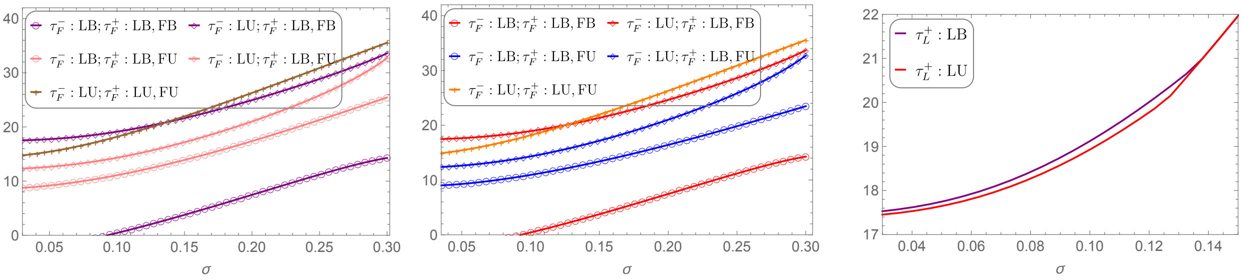
Figure 10: Flexible leader's value in FF model and the dominance of the dedicated follower in Case 1. Parameter values are $r = 0.1$, $\mu = 0.03$, $\eta = 0.05$, $c = 2$, $\delta = 10$ and $X_0 = 3$.

Figure 10 illustrates the flexible leader's value as functions of σ in FF model in figure 10a, and compares the dedicated follower and flexible follower's value in subfigure 3b. Note that 10a is under the condition that the flexible follower produces below capacity right after investment, i.e., τ_F^+ : FB, because the flexible follower does not produce up to capacity right after investment, as proved in Appendix C. Subfigure 10a reveals that the flexible leader also has the largest value when it produces up to capacity both before and after the follower's entry. The reason is similar as that for the flexible leader in FD model, i.e., the high utilization rate around time τ_F makes the follower invest much later and thus prolongs the leader's monopoly period. Subfigure 3b suggests the dominance of a dedicated follower for a given exogenous flexible leader. The dedicated follower corresponds to the flexible leader in FD model and the leader produces up to capacity right after its own investment. The flexible follower corresponds to the flexible leader in the FF model and

[11] Note that for τ_d^- : LU, τ_d^+ : LU, our numerical solution implies the dedicated follower invests in a monopolistic way, and the leader does not invest.

the leader produces up to capacity before the follower's investment.

*Case 2. Flexible follower installs a **smaller** capacity than the flexible leader in FF model*



(a) FF model flexible leader's value if τ_L^+ : LB (b) FF model flexible leader's value if τ_L^+ : LU (c) FF model flexible leader's NPV comparison

Figure 11: Flexible leader's value of investment in Case 2. Parameter values are $r = 0.1$, $\mu = 0.03$, $\eta = 0.05$, $c = 2$, $\delta = 10$ and $X_0 = 3$.

Figure 11 demonstrates the flexible leader's value in Case 2 and supports that the dedicated follower dominates a flexible follower.[12]

As displayed in subfigures 11a and 11b, the flexible leader has larger value if it produces up to capacity than below capacity right before the follower's investment, i.e., the three τ_F^- : LU lines are always higher than the two τ_F^- : LB lines. This is due to the smaller expected profit change for the leader. We compare the three lines characterized by τ_F^- : LU. Given that the leader produces up to capacity before the follower's entry and below capacity after the follower's entry, i.e., τ_F^- : LU and τ_F^+ : LB, the leader has larger value if the flexible follower produces below capacity right after investment. This is because the leader's instant profit decreases less, i.e., the line τ_F^- : LU; τ_F^+ : LB, FB is higher than the line τ_F^- : LU; τ_F^+ : LB, FU.

Between the line τ_F^- : LU; τ_F^+ : LB, LB and the line τ_F^- : LU; τ_F : LU, LU: when σ is relatively small, the leader has the larger value when both firms produce below capacity right after the follower's entry; when σ is relatively large, the leader has the larger value if both firms produce up to capacity right after the follower's investment. This is also due to the difference in the EPC, the expected profit change. Without this term, the leader's value just consists of the profit and the option value yielded by the output adjustment, the same as that for the monopolist, and the difference is small between these two cases. Whereas the difference in EPC is more significant and dominant. When σ is relatively small, EPC is larger in the case of τ_F^- : LU; τ_F : LU, LU, and vice versa when σ is relatively large.

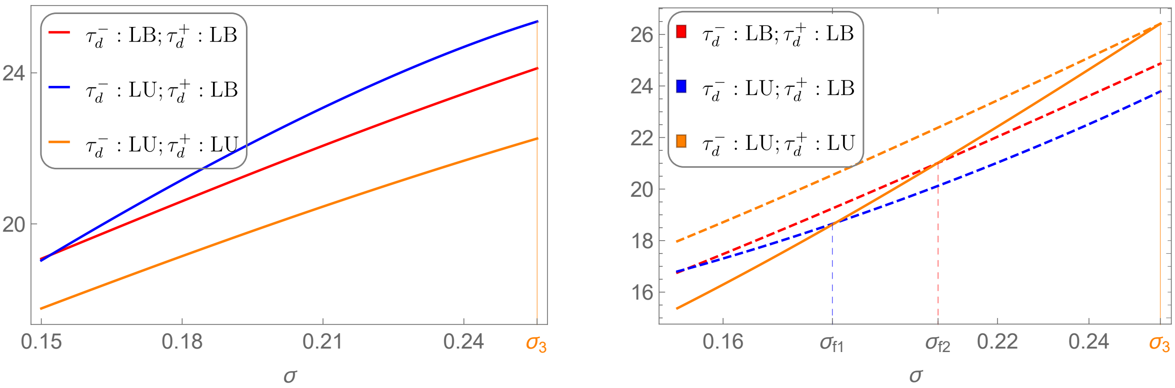
Subfigure 11c compares the flexible leader's value between producing below capacity and up to capacity right after its own investment. It reveals that producing below capacity yields a larger value for the flexible leader when $\sigma \leq 0.135$. For $\sigma > 0.135$, the flexible leader has slightly larger values when it produces up to capacity right after investment. However, this difference is too small to be visible in figure 11c.

D.2 Understanding the jumps in Figure 8

In order to understand the jumps in the flexible firm's value function as the follower, we need to understand how the dedicated firm invests as a leader in this asymmetric preemption game. We take the game depicted

[12] Note that it is possible that the leader's value is the same in subfigures 11a and 11b. This happens when the leader's investment decision is a boundary solution.

in Figure 5 as an example, where the dedicated firm preempts the flexible firm when $\sigma < \sigma_3$, and the flexible firm is consistent, i.e., if it produces below capacity right after investment as a follower, it also produces below capacity right after investment as a leader.



(a) Dedicated firm's value as leader

(b) Consistent flexible firm's value as follower (dashed lines), and as leader (real lines)

Figure 12: Consistent flexible firm produces up to capacity right after investment. Parameter values are $r = 0.1$, $\mu = 0.03$, $\sigma = 0.1$, $\eta = 0.05$, $c = 2$, $\delta = 10$ and $X(0) = 3$.

Subfigure 12a is cropped from subfigure 5a and the three lines represent the dedicated firm's value as the leader in the preemption game for the three cases described in Table 4. As a leader, the dedicated firm would like to invest at the flexible firm's preemption point in case τ_d^- : LU; τ_d^+ : LB, represented by the blue line in subfigure 12a, because it generates the largest value. The flexible firm would allow this to happen if $\sigma < \sigma_{f1}$ because being a follower leads to larger values than being the leader for all three cases. Note that the orange real line is the largest value for the flexible firm to be a leader in the three cases. When $\sigma > \sigma_{f1}$, the dedicated firm cannot get the blue line as the leader value, because the flexible firm has incentives to become a leader. Instead, the dedicated firm invests at the flexible firm's preemption points in the case τ_d^- : LB; τ_d^+ : LB and gets the leader value represented by the red line in subfigure 12a. The flexible firm allows this if $\sigma < \sigma_{f2}$ because being a follower generates larger value than being a leader, i.e., the red dashed line is above the orange real line. When $\sigma > \sigma_{f2}$, the dedicated firm cannot get the value represented by the red line in subfigure 12a anymore, because the flexible firm again has incentives to be a leader, i.e., the orange real line is above the red dashed line in subfigure 12b. Thus, the dedicated firm chooses the preemption point in the case τ_d^- : LU; τ_d^+ : LU and get the leader value represented by the orange line in subfigure 12a. Then the flexible firm gets the value represented by the orange dashed line in subfigure 12b. Overall, the dedicated firm has to switch among the preemption points for the three different cases. This results in the jumps not only in its own value function as a leader (which we do not show), but also the flexible follower's value function.