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Abstract

We consider a dynamic duopoly game where firms choose both the timing and size of their investments. 12 The default is that firms have a single option to invest, which is generally seen as an unrealistic feature of 13 the real options literature. This paper relaxes this assumption by giving Firm A multiple options to invest 14 while Firm B just keeps one option. In this asymmetric setting we get the surprising result that Firm B15 invests first. If Firm A would invest first it invests inefficiently early as a result of the preemption game. 16 Moreover, Firms A and B keep on being involved in preemption games for the subsequent investments 17 moments until B undertakes the investment. The intuition why Firm B invests first in equilibrium is 18 that then only one preemption game is played. Afterwards, Firm A is free to choose its unrestricted 19 optimal investment moments. This implies that Firm A's first investment takes place relatively late, 20 which gives Firm B a long monopoly period, generating Firm B's high incentive to invest first. 21

Keywords: Uncertainty; Investment Options; Preemption Analysis; Capacity Choice; Dynamic Games
 JEL classification: C73, D81, L13

²⁴ 1 Introduction

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Firms operating in an uncertain economic environment can build up capacity at once or by undertaking several investments over time. The operational flexibility of multiple investments has advantages. On the one hand, more could be learned about the demand evolution and the firm adjusts its sequential investments

²⁸ with updated anticipation of future demand. On the other hand, the incumbent firm also preempts its

²⁹ potential rivals entering the market (see e.g. Masson and Shaanan (1986), Swinney et al. (2011), and

³⁰ Huberts *et al.* (2019)). Most of the literature about competitive capacity expansion under uncertainty is

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³¹ under a static framework, and distinguishes the stages before and after the uncertainty is resolved (see e.g.

³² Maggi (1996)).

More recent contributions consider the competitive capacity accumulation also in a dynamic setting. 33 By assuming symmetric firms and endogenous investment orders, Boyer et al. (2012) investigate duopoly 34 firms with multiple investments and find a preemption pattern in Markov Perfect Equilibrium (MPE), i.e., 35 firms invest at different times but have equal values. However, Boyer et al. (2012) consider incremental 36 investments, whereas we consider lumpy investments. In that light, Huberts et al. (2019) analyze a scenario 37 where a market incumbent and a potential entrant are the players. They show that the incumbent invests 38 a small amount to preempt the potential entrant and prolong its monopoly period. Their model is related 39 to ours in the sense that the framework is asymmetric. However, where in our model one firm has more 40 investment options than the other, in Huberts et al. (2019) one of the firms is an active producer from the 41 start and the other one is a potential entrant. These studies build on Fudenberg and Tirole (1985) who 42 establish the rent equalization mechanism for preemption games. 43

The contribution of this research to the literature is to address, the impact of multiple investment options 44 on the endogenous investment order. We study the preemption game between two firms: one firm has 45 multiple investment options and the other firm has only one option. Both firms choose their investment 46 timing and size for every investment in a market with uncertain future demand. An intuitive equilibrium 47 outcome would be, given the equilibrium in the incumbent-entrant game by Huberts et al. (2019) and due 48 to the attractiveness of monopoly profits, that the multi-option firm invests first in MPE and prolongs its 49 monopoly privilege by preempting the one-option firm. However, this paper shows that, on the contrary, the 50 one-option firm wins the preemption game and invests first. The intuition is as follows. If the one-option firm 51 is not the first investor, the firms will also compete to be the first in subsequent investment races, which will 52 continue as long as the one-option firm has not undertaken its investment. The implication is that multiple 53 preemption games are played, and investments resulting from such games occur inefficiently early with 54 correspondingly small capacity expansions. Alternatively, if the one-option firm invests first, afterwards the 55 multi-option firm is able to choose its unrestricted optimal investment times and corresponding investment 56 sizes. This explains why the multi-option firm's incentive to be the first investor is not so high, implying 57 that the one-option firm can win the race. Moreover, since the second investment then is not the result of 58 a preemption game, after its investment the one-option firm enjoys monopoly profits for a long time, which 59 makes it especially attractive for this firm to be the first investor. 60

The structure of this paper is as follows. Section 2 builds up the theoretical model, whereas Section 3 determines the preemption equilibrium for the scenario where one firm has one investment option and the other two or more investment options. Section 4 concludes.

$_{^{64}}$ 2 Model

By allowing one of the firms to undertake multiple investments, this paper extends the recent stream of papers that studies oligopoly games from a real options perspective where both investment timing and size are chosen. Our set-up is close to Huisman and Kort (2015) and Huberts *et al.* (2019). Since the exact specification of strategy profiles is elaborately described in previous literature and is identical to the profiles considered in our set-up, we refer to the extensive literature for in-detail description of profiles and the technical description of equilibrium strategies in dynamic games.¹ In this section, we will outline the

71 fundamentals of our model, necessary to build our set-up.

¹Most notably Huberts *et al.* (2019) give a full specification of the profiles for the set-up very similar to ours, based on Riedel and Steg (2017), Thijssen *et al.* (2012), and Fudenberg and Tirole (1985).

72 2.1 Set-up Fundamentals

Following Huisman and Kort (2015), consider a framework with two risk-neutral, rational firms i = A, B, that can undertake lump-sum irreversible investments to enter a market. At the outset both firms are entrants and have therefore yet to build any capacity. Time is assumed to be continuous and the considered horizon is infinite. Both firms aim to maximize their payoff through investment at discount rate r > 0.

The inverse demand at time t > 0 is given by

$$p(t) = x(t) \left(1 - \eta Q(t)\right),$$

where p(t) is the market-clearing price, Q(t) is the market output and $\eta > 0$ is a constant elasticity parameter.

Firms are assumed to operate using full capacity. $x = (x(t))_{t \ge 0}$ is an exogenous stochastic process and follows

⁸⁰ a geometric Brownian motion,

$$dx(t) = \mu x(t)dt + \sigma x(t)dw(t),$$

in which μ is the drift rate, $\sigma > 0$ is the uncertainty parameter, and dw(t) is the increment of a Wiener 81 process. The usual assumption is made that $r > \mu$ to ensure investments to happen in finite time. Through-82 out, we also assume that x(0) is sufficiently low such that it is not optimal for firms to invest at t = 0. The 83 corresponding natural filtration is denoted by $\mathbf{F} = (\mathcal{F}_t^x)_{t \ge 0}$ and we denote by \mathcal{M} the set of \mathbf{F} -stopping times. 84 Denote the conditional expectation operator by \mathbb{E}_X , i.e., $\mathbb{E}_X\{\cdot\} = \mathbb{E}\{\cdot|\mathcal{F}_t^x\}, t \geq 0$. Each firm can build up 85 or expand its current production in the market by investing in productive capacity. The new characteristic 86 of our model is that, although otherwise identical, the firms differ in one aspect: Firm A has a series of 87 options to build capacity, whereas Firm B has only one option. The unit cost of capacity investment is δ , 88 so that a firm investing with capacity q incurs sunk investment costs δq . The firms engage in a game where 89 the investment order is endogenously determined. 90

91 2.2 Pay-off Functions

Assume that Firm $i \in \{A, B\}$ has N_i (compound) options to undertake investment. Specifically we assume that $N_B = 1$ and $N_A \in \mathbb{N}, 2 \leq N_A < \infty$. The order of investment is determined endogenously and therefore $N_A + 1$ investment orders are possible. Figure 1 visualizes all potential outcomes of the game through the presentation of the sequential-move game in extensive form. As such we will be considering Markovian strategies. At the outset both firms can exercise their investment option(s), which facilitates the preemptive behavior of the firms so that investment is undertaken applying the rent equalization mechanism $\hat{a} \ la$ Fudenberg and Tirole (1985). For subsequent investments the same principle applies as long as both



Figure 1: Stylized game.

⁹⁹ firms hold an option. From the moment Firm *B* has chosen to exercise its option, this no longer applies and ¹⁰⁰ successive future investments are undertaken by Firm *A* only. Firm *A*'s exercise strategy then involves the ¹⁰¹ maximization of an American style perpetual (real) option (see, e.g, Dixit and Pindyck (1994)).

In order to solve the sequential game illustrated in Figure 1, at each node it is not only decided which of the two firms undertakes investment but also the investment timing and size are decided. Thereto, let $\{\tau, \xi\}$ denote a set of controls, where τ denotes the moment of investment and ξ the amount by which the firm installs (additional) capacity. To formulate the value functions that correspond to each node and to each subgame, as illustrated in Figure 1, we distinguish between segments of the tree where Firm *B* has previously exercised its investment option and segments of the tree where *B* has not yet undertaken investment. In what follows, the optimization problems are subject to $\{\tau, \xi\} \in \mathcal{M} \times \mathbb{R}_+$. Figure 2 features an example with

 $N_A = 3$ to illustrate the following 3 cases.

Case I: *B* has invested Assume Firm *A* has invested k - 1 times, $1 \le k \le N_A$, and has built up a capacity of size $q_0 \ge 0$ and assume Firm *B* has invested 1 time with capacity $q_B = \xi_B$. In this case, Firm *B* has no options left and the next investment is undertaken by Firm *A*. This means that Firm *A* can choose its investment strategy without being disturbed by potential actions of Firm *B*. By the property of dynamic programming, we reset time at the start of the subgame to t = 0 and let X = x(0). Then Firm *A*'s problem is to decide the stochastic investment timing τ_k and its corresponding size ξ_k^A to maximize the expected value for its *k*-th investment, which is defined as follows. If $k = N_A$, then

$$V_{N_{A}}^{A}(X,q_{0},q_{B}) = \sup_{\tau_{N_{A}},\xi_{N_{A}}^{A}} \mathbb{E}_{X} \bigg\{ \int_{0}^{\tau_{N_{A}}} e^{-rt} x(t)(1-\eta(q_{B}+q_{0}))q_{0} dt + \int_{\tau_{N_{A}}}^{\infty} e^{-rt} x(t)(1-\eta(q_{B}+q_{0}+\xi_{N_{A}}^{A}))(q_{0}+\xi_{N_{A}}^{A}) dt - \delta\xi_{N_{A}}^{A} \bigg\}.$$

110 If $k < N_A$, then

$$V_k^A(X, q_0, q_B) = \sup_{\tau_k, \xi_k^A} \mathbb{E}_X \left\{ \int_0^{\tau_k} e^{-rt} x(t) (1 - \eta(q_B + q_0)) q_0 dt + e^{-r\tau_k} (V_{k+1}^A(x(\tau_k), q_0 + \xi_k^A, q_B) - \delta \xi_k^A) \right\}.$$

To formally write down Firm B's value for the node under consideration, let $Q_A(t, q_0)$ denote Firm A's total output at each time t, where at time t = 0, $Q_A(t, q_0) = q_0$. Then,

$$Q_A(t, q_0) = q_0 + \sum_{m=k}^{N_A} \xi_m^A \mathbf{1}_{\{\tau_m \le t\}}$$

where $\mathbf{1}_{\{\nu\}} = 1$ if ν is true and 0 otherwise, and where ξ_m^A and τ_m follow from Firm A's *m*-th investment problem, $m \ge k$, so that

$$V_k^B(X, q_0, q_B) = \mathbb{E}_X \left\{ \int_0^\infty e^{-rt} x(t) (1 - \eta(q_B + Q_A(t, q_0))) q_B dt \right\}$$

is Firm B's value when Firm A has already undertaken k-1 investments and is now facing the option of the k-th investment.

¹¹⁷ **Case II:** *B* has not invested Assume Firm *A* has already installed capacity k - 1 times, $k < N_A$, and ¹¹⁸ has built up a capacity of size $q_0 \ge 0$ and assume Firm *B* has not invested yet. For this part of the tree, the



Figure 2: Example, with $N_A = 3$, labeling all subgames.

¹¹⁹ preemption mechanism comes in play with rent equalization. Thereto, we denote the value functions for the

- leader and follower roles by L_k^i and F_k^i , i = A, B, respectively (see Fudenberg and Tirole (1985)). Denote the subgame-perfect equilibrium timing that follows from the principle of rent equalization for preemption
- games by τ_k^P . If, in equilibrium, Firm A is the investment leader, then its value at this stage of the game is
- 123 given by

$$V_k^{P,A}(X,q_0) = \mathbb{E}_X \left\{ \int_0^{\tau_k^P} e^{-rt} x(t) (1 - \eta q_0) q_0 dt + e^{-r\tau_k^P} L_k^A(x(\tau_k^P),q_0) \right\}.$$

¹²⁴ The value for Firm *B* as follower in this scenario is given by $V_k^{P,B}(X,q_0) = \mathbb{E}_X \left\{ e^{-\tau \tau_k^P} F_k^B(x(\tau_k^P),q_0) \right\}$. If, ¹²⁵ in equilibrium, Firm *A* takes the follower role, then the value for Firm *A* is given by

$$V_k^{P,A}(X,q_0) = \mathbb{E}_X \left\{ \int_0^{\tau_k^P} e^{-rt} x(t) (1 - \eta q_0) q_0 dt + e^{-r\tau_k^P} F_k^A(x(\tau_k^P),q_0) \right\}$$

and Firm B's value can be found analogously.

For the scenario where Firm B assumes the leader role, if it installs $\xi^B \ge 0$ all subsequent investments are made by Firm A. The optimal timings of those investments are determined as described in Case I and thus the stopping time of the next investment by Firm A is known and is equal to $\tau_k(\xi^B)$ for a given ξ^B . Then the value function of Firm B in the leader role for investing at the start of the subgame is given by

$$L_k^B(X,q_0) = \sup_{\xi^B} \mathbb{E}_X \left\{ \int_0^{\tau_k(\xi^B)} e^{-rt} x(t) (1 - \eta(q_0 + \xi^B)) \xi^B \mathrm{d}t - \delta\xi^B + e^{-r\tau_k(\xi^B)} V_k^B(x(\tau_k(\xi^B)), q_0, \xi^B) \right\},$$

where V_k^B is defined in Case I. The value of ξ^B also appears in the value of Firm A in the follower role,

$$F_k^A(X,q_0) = \mathbb{E}_X \left\{ \int_0^{\tau_k(\xi^{B^*}(X,q_0))} e^{-rt} x(t) (1 - \eta(q_0 + \xi^{B^*}(X,q_0))) q_0 dt + e^{-r\tau_k(\xi^{B^*}(X,q_0))} V_k^A(x(\tau_k(\xi^{B^*}(X,q_0))), q_0, \xi^{B^*}(X,q_0)) \right\}.$$

 $_{127}$ The alternative case where Firm A assumes the leader role and B the follower role, leads in the next stage to

a new preemption game. Therefore, this stage lasts until τ_{k+1}^P , which denotes the timing of the subsequent investment, so that $L_k^A(X, q_0) = \sup_{\xi_k^A} V_{k+1}^{P,A}(X, q_0 + \xi_k^A)$ and $F_k^B(X, q_0) = V_{k+1}^{P,B}(X, q_0 + \xi_k^A)$. ¹³⁰ **Case III:** A has invested N_A times The only case remaining is the scenario where A has exercised all ¹³¹ its options and B still has its option left unexercised. In this scenario, Firm B's optimization problem is ¹³² analogous to Firm A's optimization problem as described in Case I, $k = N_A$ and Firm B has not invested.

133 2.2.1 Stopping Times

The (stochastic) optimal investment time is related to the level of the underlying demand shock process $(x(t))_t$. As customarily done for the investment strategies under consideration, the optimal investment time can be characterized as the first hitting time of a threshold. For the preemption games (Case II) these thresholds are determined as the maximum of the two preemption points, where the preemption point for each firm is such that its value as leader equals its value as follower, i.e.

$$X_P^i = \inf\{X > 0 \mid L^i(X, q_0) \ge F^i(X, q_0)\}$$

for any k and for a given $q_0 \ge 0$. Investment is then undertaken by the firm with the smallest preemption point

and will do so at the time corresponding to the maximum of the two preemption points (see also Pawlina and

¹³⁶ Kort (2006)). For Cases I and III, the threshold is derived using sufficient conditions determining a trigger

 X^* such that for $X < X^*$ investment is delayed and for $X \ge X^*$ investment is undertaken immediately.²

Throughout, we will assume that x(0) is sufficiently low, i.e., it is below any threshold.

¹³⁹ **3** Equilibrium Analysis

The following two sections describe the outcome when $N_A = 2$ and $N_A > 2$. Key in these results will be that the preemption mechanism accelerates investment such that the investment is undertaken at a relatively low level of X, which in turn implies a low output price. Therefore, the firm invests in a relatively low capacity size. The firms only engage in a preemption run if both have options left unexercised.

¹⁴⁴ **3.1** The Case of $N_A = 2$

In the scenario where $N_A = 2$, only two preemption games need to be studied, as illustrated by Figure 3. For the subgame where the first investment was undertaken by A, we refer to Huberts *et al.* (2019). They found that in equilibrium Firm A preempts Firm B, and thus the outcome of A-B-A as a subgame-perfect equilibrium investment order is ruled out. Therefore, in the subgame-perfect equilibrium for $N_A = 2$, either Firm B will undertake the first investment and become a monopolist temporarily, or Firm B's singleinvestment option is the last option to be exercised.

The intuition behind Huberts et al.'s result that Firm A preempts Firm B in the subgame is the following. 151 An investment by Firm A or B will reduce the output price and thus Firm A's revenue of its already installed 152 capacity. What Firm A will do therefore is to delay Firm B's investment by investing a small amount. This 153 will not reduce the revenue associated with Firm A's first investment too much. In addition it will delay 154 the investment of Firm B so that Firm A will be a monopolist for a longer time. Moreover, the advantage 155 for Firm B to invest last is that it is not bothered by a further action of Firm A. This implies that Firm B156 invests at its unrestricted optimal investment time and therefore can choose a considerable capacity size. 157 The following example illustrates what happens for the first preemption game, i.e. for the stage where 158

¹⁵⁹ both firms are entrants. For the investment in the first stage neither firm suffers from cannibalization, which

²For proofs showing optimality and uniqueness, see McDonald and Siegel (1986), Smets (1991), Dixit and Pindyck (1994), and Huberts *et al.* (2019). For more technical details see, e.g., Peskir and Shiryaev (2006).



Figure 3: Equilibrium in the subgame with $N_A = 2$.

 $_{160}$ thus bring a different dynamic to this preemption game.³

Example 1 As an illustrative example, we consider the same baseline parametrization as in Huberts et al. 161 (2019), i.e., r=0.1, $\mu=0.02$, $\eta=0.1$, $\sigma=0.1$, and $\delta=1000$. For the preemption game where no firm has 162 invested yet, it holds that $X_P^A = 122.23$ and $X_P^B = 118.97$. This means that for X < 118.97 both firms prefer 163 to wait, for $118.97 \le X < 122.23$ Firm B prefers to be leader and Firm A prefers to wait, and for $X \ge 122.23$ 164 both firms prefer to be the first to exercise their option. As a result, assuming X < 122.23, Firm B will 165 undertake investment first when x(t) hits the preemption point of Firm A, i.e., when x(t) = 122.23, and 166 sets quantity $q_B = 1.53$. The first timeline in Figure 4 summarizes the investment thresholds and quantities 167 for the Markov Perfect Equilibrium and gives Firm A's and Firm B's value. The second timeline shows 168 the alternative case where A is assumed to undertake the first investment and the third timeline provides a 169 summary of the optimal investment strategy for a scenario where 2 firms (the leader L and the follower F) 170 have only 1 option each as in Huisman and Kort (2015). 171



Figure 4: Timelines.

In the example, it is Firm B that preempts Firm A. For the subgame we saw that Firm A installs a small capacity level: small in order not to make the cannibalization effect too large, but large enough to delay investment of Firm B. For the MPE, Firm B's preemption point is below Firm A's preemption point so that Firm B undertakes the first investment.

 $^{^{3}}$ Appendix A provides detailed description of solutions where the solution to the integrals are studied.

In the alternative scenario where A exercises its option first, A will also undertake the second investment. This scenario has disadvantages for both firms. From Firm B's perspective, if Firm A undertakes investment first, Firm B's investment will be delayed twice. As illustrated by Figure 4, the monopoly period for Firm Ais longer, which leads to a relatively low present value for Firm B so that B has a stronger incentive to undertake investment first.

Firm B's early investment comes at a cost of a lower capacity being installed but this is compensated 181 by the early exercise. From Firm A's perspective, a long monopoly period in A-A-B might be interesting, 182 however, delaying Firm B's entry comes at a high cost. In order to preempt Firm B twice, A would have to 183 install a small capacity *twice* since both investments are taking place early: as a result of potential erosion of 184 revenue resulting from the initial installment, in the subgame, A is forced to preempt B again and "sacrifices" 185 its second option. The latter would not happen if A chooses to undertake investment secondly and thirdly. 186 By comparing B-A-A and A-A-B with L-F, we gain further insight into the impact of Firm A's extra 187 investment option. The extra investment opportunity gives Firm A the flexibility to expand its production 188 after the initial installment, which thus yields more value for A. Being able to expand capacity in the future 189 allows the firm to initially set a lower capacity, only incorporating expected demand in the short run. This 190 makes the first investment cheaper. A larger value and a smaller size prompts the firm to invest earlier, which 191 is illustrated in Figure 4 by comparing F's investment timing in L-F with A's first investment threshold in 192 B-A-A. 193



Figure 5: Influence of the level of uncertainty σ on the equilibrium of the preemption game. Default parameter values are r=0.1, $\mu=0.02$, $\eta=0.1$, $\sigma=0.1$, $\delta_A = \delta_B = 1000$ and $X_0 = 100$.

To identify which firm acts as leader in the first preemption game for a wider range of parameterizations, 194 Figure 5 depicts the preemption points of Firm A and Firm B for different values of σ as well as the resulting 195 quantities. This figure shows that the result that Firm B is the first investor stays in all these cases. We also 196 conduct a robustness study where for wide ranges of parameter values of other parameters the investment 197 order is determined. Table 1 summarizes our robustness analysis and illustrates that our result of having 198 the investment order B-A-A holds in (at least) all the checked intervals for r, η , and the symmetric unit 199 investment cost δ . The only exception is for μ when μ is close to r. The intuition is that if μ has a relatively 200 high value, firms set large quantities and the period between investments is long, which erodes A's incentive 201 to invest secondly. 202

Parameter	Description	Baseline	Tested Interval	Robustness
r	Discount rate	0.1	[0.05, 0.3]	\checkmark
μ	Drift rate	0.02	[0.002, 0.09]	[0.002, 0.077]
η	Elasticity parameter	0.1	[0.01, 0.15]	\checkmark
δ	Unit investment cost	1000	[100, 1500]	\checkmark

Table 1: Range of parameter values for which one-option firm preempts two-option firm. A checkmark indicates this result is robust.

The Case of $N_A > 2$ 3.2203

Without further elaborate numerical exploration we present the result for $N_A = 3$. In this case, the subgame 204 where A has undertaken investment once, leads to the result, similar to the subgame for $N_A = 2$, that for q_0 205 sufficiently different from $q_0 = 0$, Firm A preempts Firm B's investment. Figure 6a illustrates the order of 206 the preemption points X_P^A and X_P^B for the case k = 2. Note that for our parametrization q_0 typically exceeds 207 $q_0 = 1$. Figure 6a is in line with what was found for the subgame where A undertakes the first investment 208 with $N_A = 2$: only if Firm A has no capacity installed, or a capacity level sufficiently close to zero to make 209 the cannibalization effect insignificant, Firm B will take the leader role in the preemption game. Otherwise, 210 the incumbent delays Firm B's entry. This is, consequently, exactly what we find as a MPE when both firms 211 are entrants: Firm B undertakes investment first and becomes a monopolist ahead of Firm A's entry on the 212

market, as shown in Figure 6b. 213



second investment (incumbent A with capacity q_0)

(b) Preemption points for Firm B and Firm A

Figure 6: Three investment options for Firm A. Default parameter values are r=0.1, $\mu=0.02$, $\eta=0.1$, $\sigma=0.1$, and $\delta_A = \delta_B = 1000.$

Panel (b) illustrates the effect of the level of uncertainty on the preemption points and shows that the 214 investment order is robust again for different values of σ . Similarly, a robustness check can be performed for 215 other parameter constellations and the same conclusion is drawn: Firm B undertakes investment first. 216

In a similar fashion one can analyze cases where $N_A \ge 4$. 217

Conclusion 4 218

This paper studies the preemption game between a one-option firm and a multi-option firm and concludes 219 that, ceteris paribus, the one-option firm preempts the multi-option firm and is the first investor in the 220

market. By allowing firms to invest more than once, we detect the strategic implications of having additional

- 222 options, thus flexibility in investment. Our intuition follows that the multi-option firm does not see its
- ²²³ capacity negatively affected from the preemption run and the single-option firm has the advantage of a large
- ²²⁴ monopoly period due to the fact that the timing of the second investment in the market is not determined

²²⁵ by the outcome of a preemption game. Our result extends and contrasts Huberts et al. (2019) in a scenario

²²⁶ in which one firm is already an active producer and both firms have one investment option, the incumbent

227 invests before the entrant.

Future extensions of this work include the study of economies of scale and/or learning curves, differentiation of products, and firms' potential to innovate.

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Appendix: Preemption Analysis and Calculation

In this appendix, we analyze the Markov Perfect Equilibria of the preemption type by illustrating the 255 concept to analyze firms' preemption points, especially for the first investment. Please note that calculations 256 for the complete game and to determine whether Firm A or Firm B carries out the first investment are 257 in general complex. This is because by backward induction, the input when analyzing a subgame depends 258 on the result (subgame equilibrium) from the subsequent subgame analysis.⁴ We illustrate the complexity 259 by Figure 7 with the game tree for $N_A = 4$. All the possible game outcomes shown in the figure can be 260 characterized into two groups. One group is defined as the strategies above the blue line, where B is the 261 first investor, and the other group is all strategies given below the blue line, where A is the first investor. In 262 order to conduct the preemption analysis for the first investment in this example, it is necessary to derive 263 players' values as leader and as follower, and the details are as follows. 264

According to the backward induction mechanism, our analysis starts with the subgame SA_{N_A-1} and is to determine whether *B* preempts *A*'s N_A -th investment, given that *A* has already exercised $N_A - 1$ investment options. Subgame SA_{N_A-1} is analogous to the model in Huberts *et al.* (2019), and they find that if *A* is already an incumbent in the market, Firm *A* would rather cannibalize than to be preempted by Firm *B* for its last investment. So the investment order A-A-B-A can be ruled out as an equilibrium.

The result from analyzing subgame SA_{N_A-1} , i.e., the investment order A-A-A-B, is then used when analyzing the subgame SA_{N_A-2} , where Firm A is still an incumbent in the market. This analysis is also to decide whether Firm A prefers cannibalization over being preempted by Firm B. If A prefers cannibalization, then the result is the same as in the subgame SA_{N_A-1} , i.e., the investment order A-A-A-B. If not, then

the result in subgame SA_{N_A-2} is the investment order A-A-B-A-A. The result then enters the analysis of the subgame SA_{N_A-3} .

Again, the analysis in subgame SA_{N_A-3} is to determine whether Firm A as an incumbent prefers cannibalization to being preempted by Firm B, and the result of the subgame enters the next round of analysis. Preference for cannibalization leads to the same result as in subgame SA_{N_A-2} , i.e., the order A-A-B-A-A

²⁷⁹ in case it is also the result in subgame SA_{N_A-2} , or A-A-A-B in case it is the result in subgame SA_{N_A-2} .

Preference for being preempted by Firm B leads to the result of investment order A-B-A-A in the analysis

of subgame SA_{N_A-3} .

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Figure 7: Example case of $N_A = 4$.

In the end for our example of $N_A = 4$, the result of subgame SA_{N_A-3} is used to determine whether A's first

⁴The result from analyzing a subgame is an investment order that cannot be ruled out as an equilibrium for the whole game.

investment is preempted by Firm B. For the implementation, it is necessary to derive explicit expressions

²⁸⁴ for both firms' value functions as leader and as follower. It is easy to calculate the value functions when Firm

B takes the leader role for the first investment and Firm A takes the follower role, i.e., the investment order

 $_{286}$ is B-A-A-A. However, it is complicated to specify Firm A's value as leader and B's as the corresponding

follower. There are three possible scenarios (results from subgame SA_{N_A-3}), i.e., the investment order A-B-

A-A-A, A-A-B-A-A, and A-A-A-B. Without knowledge of specific parameter values, we cannot rule out

any investment order as a possible equilibrium. Furthermore, the preemption analysis for the first investor becomes more challenging as N_A increases because there will be more possible investment orders resulted

becomes more challenging as N_A increases because there will be more possible investment orders resulted from the subgame analysis. For the rest of the appendix, we abstract from showcasing for larger N_A , but

focus on the example case of $N_A = 2$ in the main text. Cases with $N_A \ge 3$ can be analyzed and calculated

²⁹³ by similar techniques.

²⁹⁴ A Example Case of $N_A = 2$

²⁹⁵ When $N_A = 2$, Huberts *et al.* (2019) rule out A-B-A as an equilibrium outcome for the whole game. Thus, ²⁹⁶ we only have the investment order A-A-B as the outcome for subgame SA_{N_A-1}. Note that we need to specify ²⁹⁷ Firm A's and B's value functions in the investment order A-A-B and B-A-A to compute and compare their ²⁹⁸ preemption points for the first investment. In the exogenously given investment order A-A-B, A's second ²⁹⁹ investment, denoted as A_2 , happens at Firm B's preemption point in the subgame. Denote this preemption ³⁰⁰ point as X_B^{p2} , which can be derived by comparing B's values in the exogenous investment orders A-A-B and ³⁰¹ A-B-A, where Firm A is an incumbent.

³⁰² A.1 Derivation of X_B^{p2}

Assume the incumbent Firm A is active with a capacity size q_{A1} . In the investment order A-B-A, according to the backward induction, we need to derive the investment decisions for A_2 in order to calculate the value function for B. Firm A's instantaneous profit after A_2 depends on the overall production capacity in the market and the value of the realized stochastic process x(t),

$$\pi_A(q_{A1}, q_B^{aba}, x(t), q_{A2}) = (q_{A1} + q_{A2}) x(t) \left(1 - \eta \left(q_B^{baa} + q_{A1} + q_{A2} \right) \right),$$

where q_B^{aba} and q_{A2} represent Firm B's and A_2 's capacity sizes. Let $X_{A2}^{aba^*}$ be A_2 's investment threshold where Firm A is indifferent between carrying out A_2 and waiting to invest, and denote the corresponding investment size as $q_{A2}^{aba}(q_{A1}, q_B^{aba}, X_{A2}^{aba^*})$. Given X = x(0), then if $X \ge X_{A2}^{aba^*}$, Firm A is in the stopping region and it is optimal to carry out A_2 immediately. If $X < X_{A2}^{aba^*}$, then Firm A is in the continuation region and it is optimal to wait with the investment option for A_2 . Similar as Huisman and Kort (2015), the derivation of investment decisions has two steps. First, reset t = 0 to a time point in the stopping region and derive A_2 's investment size $q_{A2}(q_{A1}, q_B^{aba}, X)$, which is the solution of

$$SF_{A2}^{aba}(q_{A1}, q_B^{aba}, X) = \max_{q_{A2} > 0} \mathbb{E}_X \left[\int_0^\infty \pi_A(q_{A1}, q_B^{aba}, x(t), q_{A2}) \exp(-rt) dt - \delta q_{A2} \right]$$
$$= \max_{q_{A2} > 0} \left[\frac{X}{r - \mu} \left(1 - \eta q_{A1} - \eta q_B^{baa} - \eta q_{A2} \right) \left(q_{A1} + q_{A2} \right) - \delta q_{A2} \right].$$

Thus, it holds that

$$q_{A2}^{aba}(q_{A1}, q_B^{aba}, X) = \frac{1}{2\eta} - \frac{1}{2}q_B^{aba} - q_{A1} - \frac{\delta(r-\mu)}{2\eta X}$$

Secondly, reset t = 0 to a point in time between investments B and A_2 , i.e., the continuation region, and derive A_2 's investment threshold $X_{A2}^{aba}(q_{A1}, q_B^{aba}, q_{A2})$ for a given q_{A2} . Note that Firm A's expected value function in this region is given by

$$\operatorname{CF}_{A2}^{aba}(q_{A1}, q_B^{aba}, X) = C_{A2}(q_{A1}, q_B^{aba}) \left(\frac{X}{X_{A2}^{aba}}\right)^{\beta} + \frac{X}{r - u} \left(1 - \eta q_B^{aba} - \eta q_{A1}\right) q_{A1},$$

where the first term corrects for the change in A's net present value because of the second investment option A_2 , and the second term is the expected payoff generated by the profit flow of its first investment from time

 $_{\rm 312}$ –0 and on. β is the positive root of a quadratic equation

$$\frac{\sigma^2 \beta^2}{2} + \left(\mu - \frac{\sigma^2}{2}\right)\beta - r = 0.$$

³¹³ The value matching and smooth pasting conditions at X_{A2}^{aba} imply that

$$\begin{split} & \mathrm{SF}_{A2}^{aba}(q_{A1}, q_B^{aba}, X_{A2}^{aba}) &= \mathrm{CF}_{A2}^{aba}(q_{A1}, q_B^{aba}, X_{A2}^{aba}), \\ & \frac{\partial \mathrm{SF}_{A2}^{aba}(q_{A1}, q_B^{aba}, X)}{\partial X} \Big|_{X = X_{A2}^{aba}} &= \frac{\partial \mathrm{CF}_{A2}^{aba}(q_{A1}, q_B^{aba}, X)}{\partial X} \Big|_{X = X_{A2}^{aba}} \end{split}$$

which yield the threshold $X_{A2}^{aba}(q_{A1}, q_B^{aba}, q_{A2})$ and the expression for $C_{A2}(q_{A1}, q_B^{aba})$. Combining $X_{A2}^{aba}(\cdot)$ and $q_{A2}(\cdot)$ gives the optimal investment decisions for A_2 as

$$q_{A2}^{aba^{*}}(q_{A1}, q_{B}^{aba}) = \frac{1 - \eta q_{B}^{aba} - 2\eta q_{A1}}{(1 + \beta) \eta},$$

$$X_{A2}^{aba^{*}}(q_{A1}, q_{B}^{aba}) = \frac{\delta (1 + \beta) (r - \mu)}{(\beta - 1) (1 - 2\eta q_{A1} - \eta q_{B}^{aba})}$$

In this continuation region of A_2 , B's leader value function is given by

$$\mathcal{L}_{B}^{aba}(q_{A1}, X) = \max_{q_{B}^{aba}} \left\{ \frac{X}{r - \mu} \Big(1 - \eta \big(q_{A1} + q_{B}^{aba} \big) \Big) q_{B}^{aba} - \delta q_{B}^{aba} - \left(\frac{X}{X_{A2}^{aba^{*}}(\cdot)} \right)^{\beta} \frac{\eta X_{A2}^{aba^{*}}(\cdot) q_{A2}^{aba^{*}}(\cdot)}{r - \mu} q_{B}^{aba} \right\}.$$

Substituting $X_{A2}^{aba^*}\left(q_{A1}, q_B^{aba}\right)$ and $q_{A2}^{aba^*}\left(q_{A1}, q_B^{aba}\right)$ into $\mathcal{L}_B^{aba}(\cdot)$ yields

$$\mathcal{L}_{B}^{aba}(q_{A1}, X) = \max_{q_{B}^{aba}} \left\{ \frac{X}{r - \mu} \Big(1 - \eta \big(q_{A1} + q_{B}^{aba} \big) \Big) q_{B}^{aba} - \delta q_{B}^{aba} - \left(\frac{X(\beta - 1)(1 - 2\eta q_{A1} - \eta q_{B}^{aba})}{(r - \mu)\delta(\beta + 1)} \right)^{\beta} \frac{\delta q_{B}^{aba}}{\beta - 1} \right\}.$$

In the exogenous investment order A-A-B, we analyze B's investment decision in a similar way as analyzing for A_2 in A-B-A. Suppose that the incumbent Firm A is active in the market with a capacity size $q_{A1} + q_{A2}^{aab}$, where q_{A2}^{aab} is the investment size for A's second investment, denoted also as A_2 . Reset t = 0 to a time point in B's stopping region, then Firm B's value is

$$SF_{B}^{aab}(q_{A1}, q_{A2}^{aab}, X) = \max_{q_{B}} \left\{ \frac{X}{r - \mu} \left(1 - \eta q_{A1} - \eta q_{A2}^{aab} - \eta q_{B} \right) q_{B} - \delta q_{B} \right\}.$$

Thus, it can be derived that for a given X, Firm B's investment size is given by

$$q_B^{aab}\left(q_{A1}, q_{A2}^{aab}, X\right) = \frac{1}{2\eta} - \frac{1}{2}q_{A1} - \frac{1}{2}q_{A2}^{aab} - \frac{\delta\left(r - \mu\right)}{2\eta X}$$

Reset t = 0 to a time point between A_2 and B's investments, then B's value in its continuation region is assumed to be $CF_B^{aab}(q_{A1}, q_{A2}^{aab}, X) = C_B(q_{A1}, q_{A2}^{aab})X^{\beta}$. By the value matching and smooth pasting conditions at the investment threshold X_B^{aab} we could derive $X_B^{aab}(q_{A1}, q_{A2}^{aab}, q_B)$ for a given q_B and $C_B(\cdot)$ as well. Combining $q_B^{aab}(\cdot)$ and $X_B^{aab}(\cdot)$ yields Firm B's optimal investment decision that reads

$$q_{B}^{aab^{*}}\left(q_{A1}, q_{A2}^{aab}\right) = \frac{1 - \eta q_{A1} - \eta q_{A2}^{aab}}{\eta \left(1 + \beta\right)} \text{ and } X_{B}^{aab^{*}}\left(q_{A1}, q_{A2}^{aab}\right) = \frac{\delta \left(1 + \beta\right) \left(r - \mu\right)}{\left(\beta - 1\right) \left(1 - \eta q_{A1} - \eta q_{A2}^{aab}\right)}$$

Thus, Firm B's value as follower in its continuation region equals

$$CF_{B}^{aab}(q_{A1}, q_{A2}^{aab}, X) = \left(\frac{X\left(\beta - 1\right)\left(1 - \eta q_{A1} - \eta q_{A2}^{aab}\right)}{\left(\beta + 1\right)\delta\left(r - \mu\right)}\right)^{\beta} \frac{1 - \eta q_{A1} - \eta q_{A2}^{aab}}{\eta\left(\beta + 1\right)} \frac{\delta}{\beta - 1}$$

Furthermore, analogous to the analysis for $L_B^{aba}(\cdot)$, the incumbent Firm A's expected value in B's continuation region is equal to

$$L_{A2}^{aab}(q_{A1}, X) = \max_{q_{A2}} \left\{ \frac{X(q_{A1} + q_{A2}) \left(1 - \eta(q_{A1} + q_{A2})\right)}{r - \mu} - \delta q_{A2} - \left(\frac{X}{X_B^{aab^*}(\cdot)}\right)^{\beta} \frac{\eta X_B^{aab^*}(\cdot) q_B^{aab^*}(\cdot)}{r - \mu} \left(q_{A1} + q_{A2}\right) \right\}.$$

Substituting $X_B^{aab^*}(q_{A1}, q_{A2})$ and $q_B^{aab^*}(q_{A1}, q_{A2})$ into $L_{A2}^{aab}(\cdot)$ leads to

$$L_{A2}^{aab}(q_{A1}, X) = \max_{q_{A2}} \left\{ \frac{X(q_{A1} + q_{A2})(1 - \eta(q_{A1} + q_{A2}))}{r - \mu} - \delta q_{A2} - \left(\frac{X(\beta - 1)(1 - \eta(q_{A1} + q_{A2}))}{(r - \mu)(\beta + 1)\delta}\right)^{\beta} \frac{(q_{A1} + q_{A2})\delta}{\beta - 1} \right\}$$

So the preemption point $X_B^{p2}(q_{A1})$ can be derived by solving $CF_B^{aab}(q_{A1}, q_{A2}^{aab}(q_{A1}, X), X) = L_B^{aba}(q_{A1}, X)$, with $q_{A2}^{aab}(q_{A1}, X)$ implicitly given in $L_{A2}^{aab}(q_{A1}, X)$.

³²⁸ A.2 Preemption Analysis for the First Investment

Denote Firm A's first investment as A_1 . The preemption analysis of the first investment requires to calculate Firm A's and B's value functions as the leader and the follower in A-A-B, and as the follower and the leader in B-A-A, respectively.

Investment order B-A-A: First note that for the given capacity q_B^{baa} of B and q_{A1}^{baa} of A_1 , the optimal investment decisions of A_2 in B-A-A are such that

$$q_{A2}^{baa^*}\left(q_B^{baa}, q_{A1}^{baa}\right) = q_{A2}^{aba^*}\left(q_{A1}^{baa}, q_B^{baa}\right) \quad \text{and} \quad X_{A2}^{baa^*}\left(q_B^{baa}, q_{A1}^{baa}\right) = X_{A2}^{aba^*}\left(q_{A1}^{baa}, q_B^{baa}\right).$$

In what follows, we derive A's value as follower and B's value as leader. Reset t = 0 to a point in time

between investments A_1 and A_2 , i.e., the stopping region of A_1 . Firm A's value is assumed to be

$$SF_{A1}^{baa}(q_B^{baa}, X) = \max_{q_{A1} \ge 0} CF_{A2}^{aba}(q_{A1}, q_B^{baa}, X),$$

which further implies that the investment capacity $q_{A1}^{baa}(q_B^{baa}, X)$ satisfies the following implicit equation

$$\frac{X\left(1 - 2\eta q_{A1} - \eta q_B^{baa}\right)}{r - \mu} - \delta - \frac{2\delta}{\beta - 1} \left(\frac{X(\beta - 1)\left(1 - 2\eta q_{A1} - \eta q_B^{baa}\right)}{(\beta + 1)\delta(r - \mu)}\right)^{\beta} = 0.$$

Reset t = 0 to a time point between B's investment and investment A_1 , i.e., A_1 's continuation region. Then Firm A's value in this region is assumed to be given by

$$CF_{A1}^{baa}(q_B^{baa}, X) = C_{A1}(q_B^{baa}) \times X^{\beta}$$

The value matching and smooth pasting conditions at A_1 's investment threshold X_{A1}^{baa} lead to the expression of $C_{A1}(\cdot)$ and

$$X_{A1}^{baa^{*}}(q_{B}^{baa}) = \frac{\beta \delta(r-\mu)}{(\beta-1) \left(1 - \eta q_{A1}^{baa}(\cdot) - \eta q_{B}^{baa}\right)}$$

Substituting $X_{A1}^{baa^*}(q_B^{baa})$ into $q_{A1}^{baa}(\cdot)$ yields A_1 's optimal investment capacity size $q_{A1}^{baa^*}(q_B^{baa})$. Therefore, Firm B's value equals

$$\begin{split} \mathcal{L}_{B}^{baa}(X) &= \max_{q_{B}} \left\{ \frac{X\left(1 - \eta q_{B}\right)q_{B}}{r - \mu} - \delta q_{B} - \left(\frac{X}{X_{A1}^{baa^{*}}(q_{B})}\right)^{\beta} \frac{\eta q_{B} X_{A1}^{baa^{*}}(q_{B}) \times q_{A1}^{baa^{*}}(q_{B})}{r - \mu} \\ &- \left(\frac{X}{X_{A2}^{baa^{*}}(q_{B}, q_{A1}^{baa^{*}}(q_{B}))}\right)^{\beta} \frac{\eta q_{B} X_{A2}^{baa^{*}}(q_{B}, q_{A1}^{baa^{*}}(q_{B})) \times q_{A2}^{baa^{*}}(q_{B}, q_{A1}^{baa^{*}}(q_{B}))}{r - \mu} \right\}. \end{split}$$

Investment order A-A-B: We have calculated in Subsection A.1 Firm *B*'s optimal investment decision $q_B^{aab^*}(q_{A1}^{aab}, q_{A2}^{aab})$ and $X_B^{aab^*}(q_{A1}^{aab}, q_{A2}^{aab})$ for a given q_{A1}^{aab} and q_{A2}^{aab} . The optimal investment threshold for investment A_2 is given by $X_{A2}^{aab^*}(q_{A1}^{aab}) := X_B^{p2}(q_{A1}^{aab})$, and the corresponding optimal capacity is $q_{A2}^{aab^*}(q_{A1}^{aab}) := q_{A2}^{aab}(q_{A1}^{aab}, X_B^{p2}(q_{A1}^{aab}))$. Reset t = 0 to a point between Firm *A*'s first investment A_1 and second investment A_2 , then Firm *A*'s value equals

$$\begin{aligned} \mathcal{L}_{A1}^{aab}(X) &= \max_{q_{A1}} \left\{ \frac{X \left(1 - \eta q_{A1}\right) q_{A1}}{r - \mu} - \left(\frac{X (\beta - 1) \left(1 - \eta q_{A1} - \eta q_{A2}^{aab^*}(q_{A1})\right)}{\delta (\beta + 1) (r - \mu)} \right)^{\beta} \frac{\delta \left(q_{A1} + q_{A2}^{aab^*}(q_{A1})\right)}{\beta - 1} \\ &+ \left(\frac{X}{X_{A2}^{aab^*}(q_{A1})} \right)^{\beta} \left(\frac{\left(1 - 2\eta q_{A1} - \eta q_{A2}^{aab^*}(q_{A1})\right) X_{A2}^{aab^*}(q_{A1})}{r - \mu} - \delta \right) q_{A2}^{aab^*}(q_{A1}) - \delta q_{A1} \right\}. \end{aligned}$$

Preemption results: Based on the calculation in the exogenous investment orders B-A-A and A-A-B, we can derive Firm A's preemption point $X_A^{p_1}$ by solving $L_{A1}^{aab}(X) = CF_{A1}^{baa}(q_B^{baa}(X), X)$ with $q_B^{baa}(X)$ implicitly given in $L_B^{baa}(X)$, and Firm B's preemption point $X_B^{p_1}$ by solving $L_B^{baa}(X) = CF_B^{aab}(q_{A1}^{aab}(X), q_{A2}^{aab^*}(q_{A1}^{aab}(X)), X)$ with $q_{A1}^{aab}(X)$ implicitly given in $L_{A1}^{aab}(X)$. In case $X_A^{p_1} \ge X_B^{p_1}$, it can be concluded that Firm B preempts Firm A in the first investment. Otherwise, it can be concluded the Firm A preempts Firm B.