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WILLEM VAN ZWET'S RESEARCH

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Willem van Zwet made deep and influential contributions to probability and statistics which we review in this paper. Bickel and Götze collaborated with him on his major contributions to higher order asymptotics of non-linear statistics and on resampling and the bootstrap. We relate this work to his remarkable development of the properties of the Hoeffding expansion for symmetric statistics as well as Fourier analytic tools. Fiocco and De Gunst were his students. We describe how in their theses and subsequent papers with him they developed statistical inference in two subtle stochastic models, the contact process and cell development under a plausible regime. We also touch on his solutions of intriguing problems not related directly to his main interests.

The beginnings. Willem (Bill) van Zwet's first significant piece of research was his thesis "Convex transformations of random variables", carried out under the supervision of Jan Hemelrijk, but quite unlike any other work that Hemelrijk had done previously. The thesis was published as a Math. Center tract [Zwe64] and has been cited some 500 times. It is a completely original analysis of various stochastic orderings of distributions and their consequences in terms of inequalities for various functions of the data. The main type of ordering introduced, which was motivated by consideration of probability plots is $F \leq G$ if and only if $G^{-1}(F(x))$ is convex. This ordering implies stochastic ordering and much else.

As a consequence of this work, he was appointed as an Associate Professor of Mathematics at the University of Leiden in 1965, first going abroad as a visiting faculty member at the University of Oregon in 1965 and as a visiting researcher at UC, Berkeley in 1967. In Berkeley he worked with Richard Barlow and made substantial contributions to isotonic regression which were published between 1969 and 1971.

Second order inference. Bill was drawn to a problem posed in Hodges and Lehmann's paper on Deficiency [HL70]. In it, Hodges and Lehmann studied a novel aspect of the old measure of Pitman efficiency of test A, based on n observations, to test B having the same level and power, but based on $n' > n$ observations. This efficiency is (when it exists) the limit of n'/n as $n \to \infty$. Hodges and Lehmann noted in various examples where $n'/n \to 1$, that $n' - n$ tended to a finite limit which they called the deficiency of A to B. For instance, they showed that this happened if one samples $N(\mu, \sigma^2)$ and the null hypothesis is $\mu = 0$, when A is the Z-test requiring knowledge of σ and B is the t-test which does not. They studied this idea in a number of other examples, extending it also to estimation and confidence bounds. At the end, they asked if this could be done comparing parametric and rank tests under the

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appropriate assumptions. For instance, could the normal scores test be compared in this way to the t-test for Gaussian data? In general, they asked could one compare optimal parametric tests, with hypothesis and alternative in the simple vs. simple form, and the alternatives converging to the hypothesis at rate $n^{-1/2}$ to rank based tests known to have Pitman efficiency 1 under the same circumstances. They noted that this would require going beyond Gaussian limit theorems to asymptotic expansions to order n^{-1} for rank test statistics. For these statistics such expansions were quite unknown.

Bill was drawn to this challenge.

At the same time, one of us, Peter Bickel, in his second year on the faculty of University of California Berkeley, was independently drawn to the same problem. Van Zwet and Bickel discovered their common interest and started a collaboration on this and other circles of problems which lasted 30 years or more. The first two major papers that resulted from this collaboration were [ABZ76] (1976) and [BZ78] (1978), in which they studied asymptotic expansions of the power of one- and two-sample rank statistics and more generally permutation test statistics under contiguous shift alternatives and compared them to the more classical expansions for most powerful simple vs simple tests. The results in the first paper were statistically interesting: for instance, the permutation t-test had 0 deficiency with respect to the t-test under the Gaussian model, which in turn had been shown in [HL70] to have bounded deficiency with respect to the most powerful Z-test. The normal scores test had deficiency tending to infinity in the same situation, but the deficiency was of the order of $\log(n)$. The rank test results corresponded to similar comparisons of estimates based on rank tests, as proposed in [HL70]. This work in part originated in the thesis of W. Albers who was a coauthor of [ABZ76]. The second paper [BZ78] obtained similar results for two-sample tests and related these, in part unexpected results, to the general work of J. Pfanzagl [Pfa79]. Both papers were long and technical, requiring some novel Fourier analysis using and extending results of Berry and Esseen and, in the second paper, of Erdős and Rényi [Rén70]. Both papers showed that the deficiency of estimates defined by such tests, using the ideas of Hodges and Lehmann [HL70], coincided with that of the tests, though the property for tests was defined through power and for estimates through mean squared error. A phenomenon observed in the second paper was that, unlike in the one-sample case, where only terms of order n^{-1} needed to be compared, power expansions could have terms of order $n^{-1/2}$, but these always agreed when one compared first order efficient procedures. This phenomenon, under the name "first order efficiency implies second order efficiency", has been extensively studied, in the parametric context by J. Pfanzagl and his students in [Pfa82].

During the period 1965-1972, before the work on deficiency was in gestation, Bill worked with students and other collaborators on a number of unrelated but highly interesting questions. An example was [ZO67] with J. Oosterhoff, a delicate and rigorous examination of methods proposed for the combination of test statistics (now called meta-analysis), due to Fisher, Pearson and other writers. Others included work on the likelihood ratio test for the multinomial distribution, nonparametric tests for independence and topics in reliability theory.

At an Oberwolfach meeting in 1977, Van Zwet and Bickel met F. Götze, who had just completed his Ph.D. thesis with Pfanzagl in which he established the possibility of Edgeworth-like expansions for the expectations of very general statistics. This meeting led to collaborations on a variety of topics which we shall now discuss.

There had been growing interest in asymptotic expansions of Edgeworth type for a variety of non-linear statistics, estimates or test statistics, which are asymptotically normal, and of classes of goodness of fit and likelihood tests of weighted χ^2 type limit as described in a 1974 review by one of us [Bic74].

Centered versions of such statistics for i.i.d. samples, when arising in parametric models, can in many cases be approximated by a Taylor series in variables that are averages of i.i.d. vectors. In that case, as shown by Linnik and Mitrofanova, see [Bic74] for a review, expansions of the distributions of the statistics may be obtained using multivariate Edgeworth expansions for the distributions of the vector averages. Even if the statistics are not smooth functions of such averages, such as the L^1 or L^∞ norms, the results in the monograph of Bhattacharya and Ranga Rao [BR10] enable us to get expansions. However, statistics arising from i.i.d. nonparametric models can only be expressed as functions of the empirical distribution, which can be thought of as an average of i.i.d. variables in a function space. The empirical distribution, when suitably centered and scaled, obeys a central limit theorem and converges weakly to a Gaussian process. The limiting process, like all genuinely high-dimensional Gaussian processes (with fixed trace of covariance), has marginal distributions, which are extremely singular in some directions with a non-existent or bad joint Lebesgue density. This still allows for expansion for expectations of very smooth functions but not for distribution functions, see [Göt81]. Thus for approximations of distribution functions one is led to the hard approach of Fourier inversion of an expansion of the characteristic function of a non-linear statistic as used in the papers [ABZ76] and [BZ78], and for general classes of statistics we describe now.

Statistics which are functions of n independent random variables and have second moments, can be represented by a stochastic expansion into a sum of linear, quadratic and higher-order functions of the observations which are orthogonal in L^2. The series and its terms called U-statistics were introduced by Hoeffding in 1948 [Hoe48]. This expansion, usually called Hoeffding's expansion, and occasionally ANOVA expansion (Efron-Stein), bears the same relation to the von Mises expansion [Mis47] as Ito integrals do to Stratonovich integrals. The von Mises expansion may be thought of as a form of Taylor series for smooth functions of the empirical distribution. For asymptotically normal statistics, the first term is usually determinative, as shown by Hajek [Háj68] while for goodness-of-fit statistics under the null hypothesis the second term leading to weighted χ^2 statistics is asymptotically dominant. Bill was fascinated by Hoeffding's insight that symmetric statistics could be decomposed in this way and the uses of this expansion for understanding normal and higher approximations to their distribution. In [HZ82], with Helmers, he proved that Berry-Esseen type error bounds of order $O(n^{-1/2})$ were valid for asymptotically normal statistics with a two-term expansion (U-statistics of order 2). This followed a more restrictive result in [Bic74]. Then, in a seminal paper, [Zwe84], Bill proved the validity of this bound for arbitrary symmetric statistics, and hence approximate normality, under appropriate conditions on the terms of the Hoeffding expansion.

To study statistics at the deficiency level, as in the previous work on rank tests and estimates, Edgeworth expansions up to an error $o(n^{-1})$, rather than just Berry-Esseen bounds are needed. Van Zwet, Bickel and Götze in [BGZ86] derived such Edgeworth expansions up to an error of $o(n^{-1})$ for U-statistics of degree two under the condition of a strong non-lattice assumption for the first-order term. This classical smoothness condition had been introduced by Cramér. Moment conditions on Hoeffding expansion terms combined with the requirement of a minimum number of non-zero eigenvalues for the bivariate U-statistic kernel (second term in the Hoeffding expansion) had to be added. A decade later Bill showed in a joint paper [BGZ97a] with Bentkus and Götze that this result could be extended to arbitrary symmetric and asymptotically normal statistics. By very intricate Fourier arguments it was shown that the two-term Hoeffding expansion condition could be dropped with error bounds of order $O(n^{-1})$ still retained. Simple examples showed that this was best possible. The interaction of a smooth first-order term with an appropriate second-order term in the U-statistic could indeed create lattice-type jumps of the distribution of order $O(n^{-1})$ for the distribution function. The final answer as to minimal conditions for errors of order $o(n^{-1})$ requires even deeper analytic arguments and appeared in a recent preprint of Bloznelis and Götze [BG21].

Revisiting the one-sample statistic [ABZ76], where an expansion up to an error $O(n^{-3/2})$ was derived for the lattice-valued Wilcoxon statistic, Van Zwet and Götze studied Student's statistic for symmetric Rademacher-type observations and found in [GZ06] that, apart from the center, the distribution function had jumps of order $O(n^{-1})$ only. The papers [ABZ76], [BZ78] (1978) discussed above for the one- and two-sample problem led Bill to study in detail related validity problems for Edgeworth expansions for linear combinations of order and rank statistics and likelihood ratios. This resulted in a series of papers from 1977 till 1985 with various coauthors including several students and Chibisov, Bickel and Götze, see [Zwe77], [Zwe79], [Zwe80],[Zwe82], [CZ84a] and [CZ84b]. In addition to a revisit with Götze in [GZ06], looking at studentization in the one-sample problem, Bill, Götze, and Bickel turned to another area where the Hoeffding expansion and Edgeworth expansions also proved to be of great value.

Bootstrap and resampling methods. In [PZ96] Van Zwet and his student Putter studied the consistency of estimates of parameters $\tau(P)$ obtained by plugging in estimates of P based on resampling methods. They concluded that choosing the estimate successfully is extremely model dependent. In [BGZ97b] Van Zwet, Bickel and Götze were trying to understand the effectiveness and the limitations of resampling methods compared to Efron's bootstrap. This was done in light of the observation independently made by Politis and Romano [PR94] and Götze [Göt93] that if one used resampling distributions based on samples of size $m < n$ without replacement, one could obtain consistent estimates of limiting distributions of many statistics, such as the properly centered and scaled maximum of an i.i.d. sample from a distribution on an interval, for which Efron's bootstrap failed.

In [BGZ97b] they qualitatively addressed the following question:
Given $X_1,\dots,X_n$, an i.i.d. sample from a distribution P, let P_n be the empirical distribution of the sample and $T_n(P_n,P)$ be a function of both the data and P. Let us call such a T_n a mixed statistic. Let P_m^* be the empirical distribution of a resample of size m from P_n. They asked: when is the distribution of $T_m(P_m^*,P_n)$ an approximation to first order, to that of $T_n(P_n,P)$ for m,n large? They showed that such an approximation was valid for many situations (including the ones for which Efron's method worked, such as $\bar{X}-\mu(P)$, where $\mu(P)$ is the expectation of $\bar{X}$.) Specifically, they started with a mixed statistic $S_n(P_n,P)$ which converges to 0 in probability and then considered a rescaling, say $T_n(P_n,P) := \sigma_n\, S_n(P_n,P)$, such that T_n converges in law to a non-trivial limit law. For all P in the model of interest they assumed:
1. The renormalization scale, σ_n, at which convergence to a non-trivial limiting distribution occurs, is known or consistently estimable.
For instance $\bar{X}-\mu(P)$ renormalized by $\sigma_n=\sqrt{n}$, gives a Gaussian limit for P which have a finite second moment. For convenience, from now on assume T_n are of this type.
It was shown that if the following further conditions hold
2. $m\to\infty$ and $m/n\to 0$ and for such sequences
3. $\mathcal{D}(T_m(P_m^*,P_n)) = \mathcal{D}(T_m(P_m^*,P)) + o_P(1)$,
then,

$$\mathcal{D}(T_m(P_m^*,P_n)) = \mathcal{D}(T_n(P_n,P)) + o_P(1).$$

That is, m-out-of-n-resampling works. $\mathcal{D}$ here means distribution and the convergence is in Prokhorov or some other distance for weak convergence.

Van Zwet, Bickel and Götze also noted the (usual but not inevitable) connection between the situations where Efron's method worked and mixed statistics being a smooth function of the empirical process. Finally, they showed that bootstrapping (resampling with replacement) essentially would do as well as sampling without replacement as long as $m = o(\sqrt{n})$. This partly qualitative work, including extensive comparisons using simulations of various methods, including the jacknife, led Bill and Putter to consider resampling further in the framework of Hoeffding and Edgeworth expansions. They introduced what they called called empirical Edgeworth expansions for such mixed statistics in [PZ98]. These were based on cumulants estimated using the jacknife. Using an unpublished extensive analysis of the Hoeffding expansion which Bill presented in part in his 1997 Wald lectures and [Zwe84], they proved that these expansions gave higher-order Edgeworth approximations under natural conditions on the terms of this Hoeffding expansion.

Other questions. A persistent feature of Bill's work was solving hard problems only distantly related to his main interest, often with students or collaborators. One example is [BZ80] in which he and Bickel asked the question: for what class of functions ϕ is $\mathbf{E}(\phi(X_1 + \ldots + X_n))$ with $X_1, .., X_n$ independent with distributions $F_1, \ldots, F_n$ maximized when the X_i are identically distributed with common distribution $\bar{F} = (F_1 + \ldots + F_n)/n$? The class of convex ϕ was shown by Hoeffding [Hoe56] to have this property for Bernoulli variables. Bill and Bickel showed among other results, that for general variables this holds only for the class of Laplace transforms of non-negative distributions. The result was picked up by members of the algebraic community and pushed in several general directions [CR81].

Other examples include [Zwe94], [DZTD96], [LZ04] and [DHKZ04]. Bill often returned and picked up threads left in his early papers. There is, for instance, the case of the Kakutani conjecture about which he learnt from Dudley who challenged the audience of a joint Statistics and Probability meeting in 1976 at Oberwolfach for a proof. Bill solved it shortly after in [Zwe78] only to return to it 26 years later in a joint paper with Ron Pyke [PZ04] where they studied variants of this phenomenon via weak convergence.

During the 1986/87 visits of David Mason in Leiden, Bill worked with him on improved versions of the KMT strong approximation methods as well as on strong approximations for renewal processes in [MZ87b] and [MZ87a].

Inference for cell population growth. In the late 1980s with his student Mathisca de Gunst, Bill turned to a series of problems that originated from biology.

The phenomenon of interest was the growth of a population of plant cells in a liquid medium. In collaboration with biologist Kees Libbenga, Bill and De Gunst developed a stochastic model for the population growth that takes into account the influence of two specific medium components. A key ingredient of the model is that the cell population consists of two types of cells: type-A cells, which are actively cycling and finally divide, and type-B cells, which are differentiating and don't divide. At time $t = 0$, the population is assumed to consist of type-A cells only. After a certain time period, the first cell divisions occur and produce either two A-cells or two B-cells, or one of each type. Depletion of one of the medium components decreases the division rate, while depletion of the other decreases the birth rate of the A-cells. As a consequence of such depletion the population will eventually stop growing. Because the cells compete for the components whose concentration decreases at random times, there is a complicated type of dependence between the division times for different cells.

The total cell number $N_n(t)$ as predicted by the model is a non-Markovian counting process. In [GZ92] Bill and De Gunst investigated the asymptotic behavior of the population growth when the initial cell number n tends to infinity. Conditioning on the number of A-cells born at each division and making use of a random time change based on the integrated conditional intensity process of $N_n(t) - n$, they proved that the relative growth of the population, $n^{-1}(N_n(t) - n)$ converges almost surely to a nonrandom function X, and proved a central limit theorem. The uniform convergence depends heavily on the boundedness of $n^{-1}(N_n(t) - n)$, and the representation of the limit distribution in terms of two independent Wiener processes and its covariance structure were obtained by going into the special structure of the underlying model. A statistical analysis of experimental data showed that for appropriate parameter values, X describes actual population growth quite well [GZ93].

In [DZHVL90] Bill and De Gunst determined the order of magnitude of the duration of the growth process and the tail behavior of the limit process $X(t)$ as t tends to infinity. A striking discontinuity was found in the tail behavior of the two processes. As there are two possible causes for the process N_n to stop growing, and, correspondingly, the limit process $X(t)$ has a derivative $X'(t)$ which is the product of two factors, there are three different cases: one of the two or both factors may tend to zero as t tends to infinity. It turned out that if only one factor of $X'(t)$ tends to zero, then

$$X(\infty) - X(t) \sim A\mathrm{e}^{-at} \quad \text{as } t \to \infty, \quad \frac{T_n}{\log n} \to \frac{1}{a} \quad \text{in prob. as } n \to \infty.$$

If both factors tend to zero, this happens when the two medium components are exhausted at approximately the same time, a very different tail behavior occurs: X

$$X(\infty) - X(t) \sim \frac{1}{at} \quad \text{as } t \to \infty, \quad \frac{T_n}{n^{1/2}\log n} = 1 + o_P(1), \quad \text{as} \quad n \to \infty.$$

The expressions for a in the three cases are different.

Inference for the contact process. This major interest started in the late 1990's with Bill's student Marta Fiocco. A d-dimensional contact process is a model for the spread of an infection on the lattice Z^d. At each time $t \geq 0$, each site can be in one of two possible states: infected or healthy. The state of the site $x \in Z^d$ at time t will be indicated by a random variable $\xi_t(x)$ given by

$$\xi_t(x) = \begin{cases} 1 \text{ if } x \text{ is infected,} \\ 0 \text{ if } x \text{ is healthy.} \end{cases}$$

The function $\xi_t : Z^d \to \{0, 1\}$ gives the state of the process at time t. It is a $\{0, 1\}$-valued random field over Z^d. The evolution of this random field in time is described by the following dynamics. A healthy site is infected at rate λ by each of its $2d$ immediate neighbours which is itself infected; an infected site recovers at rate $\mu > 0$. Given the configuration at time t, the processes involved are independent until a change occurs. If λ is sufficiently small, infection dies out (subcritical process), whereas if λ is sufficiently large infection tends to be permanent (supercritical process).

Bill and Fiocco provided in [FZ98] a generalization for the shape theorem for the supercritical contact process ξ_t^A starting with an arbitrary, possibly random, set A of infected sites. Before this work, the shape theorem for the supercritical contact process was generally presented for the process that starts with a single infected site at the origin. This process is denoted by $\xi_t^{\{0\}}$. In [FZ00] Bill and Fiocco studied the estimation problem for the parameter λ of the supercritical contact process starting with a single infected site at the origin, and given that the process survives forever. Based on an observation of this process at a single time t, they obtained an estimator for the parameter λ which is consistent and asymptotically normal as $t \to \infty$. The probabilistic results needed to establish these facts were taken from a companion paper [FZ03a] where the authors studied the convex hull of the set of infected sites for the conditional $\xi_t^{\{0\}}$ process as well as its spatial correlation. They found that under some restrictions this correlation decays faster that any negative power of the distance. Results were extended in [FZ03b] to the process ξ_t^A which starts with a set $A \subset Z^d$ of infected sites

at time $t = 0$. Consistency and asymptotic normality were proved.
Maximum likelihood estimation for the case where the process is supercritical, starts with a single infected site at the origin and is observed during a long time interval $[0, 1]$, is discussed in [FZ04] and [FZ06]. The estimators are constructed and their consistency and asymptotic normality as $t \to \infty$ are proven. The relation with the estimation problem for the process observed at a single large time is also discussed.

The Figure below shows that process started with a single infected site at the origin with $\lambda = 0.7$ and $\mu = 1$ after 50000 steps, i.e. 50000 infections and recoveries. Infected sites are indicated by gray 1×1 squares. An additional feature of this figure is that for each infected site, the number of steps since it was last infected is recorded. It is indicated by the gray level at that site: the darker the gray level, the older the present infection at a site.

The process $\xi_t^{\{0\}}$ for $\lambda = 0.7$ and $\mu = 1$, after 50000 steps.

To conclude. Bill's research life was as rich and varied as his contributions to the profession. All of us remember the pleasure of our collaborations with him and mourn his passing.

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