Assessing swaption portfolios for prepayment risk mitigation: a parametric perspective.

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Abstract

We analyze the price behavior of Bermudan swaption portfolios used for hedging prepayment-driven interest rate risks in loan portfolios. We evaluate a variety of swaption portfolios across maturities and prepayment rates under various market conditions. Our findings reveal the existence of a parametric relation between swaption portfolio prices and the characteristics of the hedged loan. This relationship holds across different market conditions and valuation models, suggesting that one can swiftly adjust a swaption-based hedging strategy as loan portfolio characteristics evolve. This parametric approach allows financial institutions to reduce costs when assessing prepayment risks in their loan portfolios.

Keywords: swaptions; prepayment risk; option pricing; interest rates; parametric approach; model risk.

1 Introduction

In recent years, the use of machine learning (ML) techniques for pricing over the counter (OTC) instruments has gained popularity among financial institutions (Culkin and Das (2017), Dixon et al. (2020), Gan L. (2020), Huang et al. (2020), Hull (2020), Rundo et al. (2019)). This trend was driven by the need of pricing OTC instruments quickly and accurately to manage risk and make informed trading decisions.

OTC instruments, such as derivatives, are traditionally priced using complex mathematical models that require significant computational resources and can take a long time to run (Björk (2009), Brigo D. (2001), Glasserman (2004)). On the other hand, ML techniques can be trained on large sets of data to identify patterns and relations that can be used to predict prices under different scenarios. Several studies have shown that ML techniques can improve pricing accuracy and reduce pricing errors, especially for complex and illiquid instruments (Cao et al. (2020, 2021), Glorot and Bengio (2010), Hutchinson et al. (1994)). These techniques also offer the possibility to quickly assess the impact of different market scenarios on portfolios, enabling traders and risk managers to make informed decisions in a timely manner. Therefore, the application of ML techniques in OTC instrument pricing represents an exciting opportunity for financial institutions to improve their pricing accuracy and risk management capabilities.

Motivated by the success of these techniques, in this study we explore a new approach to evaluate portfolios of Bermudan swaptions. These instruments are widely used by financial institutions to assess interest rate risk arising by prepayments in loans portfolios. Precise valuation and effective risk management of OTC instruments, particularly portfolios of swaptions, are essential for hedging against prepayment risk (Andersen (1999), Bianchetti (2010), Rebonato (2012)). Financial institutions heavily rely on accurate estimation and hedging strategies to mitigate the impact of prepayment risk, as it directly influences the profitability and stability of their portfolios.

Valuing the impact on asset value of interest rate variations in presence of prepayment is a complex task (Schwartz and Torous (1989)). It involves estimate prepayment's rate with a fixed level of confidence, decomposing the loan's portfolio into a portfolio of swaptions, selecting an interest rate evolution model, calibrating it, and choosing a pricing methodology (Brigo D. (2001)). Furthermore, since multiple scenarios of interest rate variations must often be tested, the valuation process can be cumbersome. If one were to estimate the impact of using different models in terms of operational costs, the process becomes very expensive due to the time and the number of valuations required for each different hypothesis. The models involved encompass both hypotheses on the dynamics of interest rates and prepayment mechanisms. Therefore, developing methodologies that accelerate portfolio estimation is important in addressing this issue.

In this setting our approach has the potential to significantly reduce operational risks and time spent on valuation. Our study aims to evaluate portfolios of Bermudan swaptions using a parametric approach that enables pricing of an entire class of instruments based on the estimation of a few parameters. As a consequence, this approach allows for the quick revaluation of portfolios with different characteristics after an initial effort in model calibration.

This study stands out due to its novelty; no similar research is present in the literature, mainly because of the operational challenges involved in performing a large number of evaluations on real data, as accomplished in this study. This is particularly remarkable given the relevance of the topic and the importance of the evaluated risk.

In this study, we explore the pricing behaviour of portfolios of Bermudan swaptions with different characteristics, such as different prepayment profiles and maturities, using various pricing models and across multiple valuation dates. Our analysis reveals that the prices of these portfolios exhibit a parametric behaviour when plotted against standardized contract specifications units, and this behaviour remains consistent across different market conditions and pricing models.

This observation opens up new possibilities for pricing and model risk estimation, as the parametric behavior of prices enables us to evaluate portfolios with different characteristics using only a few parameters. Furthermore, our analysis demonstrates that different pricing models may exhibit the same parametric behaviors, but generate different parameter values.This provides valuable insights into the impact of model selection on portfolio valuation, offering significant time saving.

The parametric approach we introduce aligns with the literature on new pricing approaches based on machine learning techniques, where a wide range of financial instruments is priced with few assumptions on underlying dynamics. Our findings suggest that the parametric approach is a promising direction when applied to describe the collective dynamics of financial instruments, especially for the valuation of portfolios of Bermudan swaptions, regardless of the pricing model or market conditions at the valuation date.

This paper is organized as follows: in Section 2, we provide a brief introduction to the hedging strategies based on Bermudan swaptions. In Section 3, we address the problem of pricing portfolios of Bermudan swaptions. Moving on to Section 4, we analyze the results of our evaluations and present an application of the new parametric approach. Finally, in Section 5, we discuss the main conclusions of our work and offer insights into future perspectives.

2 Construction of hedging strategies via porfolios of swaptions

A swaption is a financial derivative that grants the holder the right to enter into a swap contract at a future date (Björk (2009), Brigo D. (2001)). The underlying of a swaption is a swap contract, which is an agreement between two parties to exchange cash flows based on two different interest rate variables. Swaptions are often used in conjunction with swap contracts as hedging tool against interest rate risk. While a European swaption can be exercised only at the expiry date, a Bermudan swaption works similarly to a standard European swaption, but gives to the holder the possibility to enter a swap contract on multiple predetermined dates (Brigo D. (2001)).

When considering a loan portfolio, in the absence of prepayment, banks typically convert payment streams from loans into floating rate payments to align the portfolio's value with the market. However, in the case of prepayment, hedging through an interest rate swap is ineffective. Once the hedged position, i.e., the individual loan, is prepaid, the cash flows derived from interest on the loan stop, while the hedging position, i.e., the swap, still continues to exist, requiring the payment of the fixed interest rate leg.

To maintain the effectiveness of the hedging strategy, adjustments can be made to the set of swaps in response to a prepayment event. This involves closing existing swaps and opening new ones that better align with the updated loan amortization profile. Alternatively, swaptions can be utilized to further enhance the efficiency of the hedging strategy. By exercising a swaption when prepayment occurs, it allows for the cancellation of the hedging derivative, i.e., the swap.

While using swaptions to construct hedging strategies can be challenging at times due to their limited liquidity, they are often priced with the purpose of estimating the cost of hedging. Ultimately, this cost enters in the fair pricing of the evaluated loans and it is often added as a spread to the fixed rate given by the loan contract.

To develop an effective hedging strategy, it is crucial to precisely estimate the prepayment rate and the associated amortization profile for each loan (Huang et al. (2020)). As this information is inherently probabilistic, it is typically expressed through two scenarios or bands that represent the maximum and minimum possible amortization profiles. Estimating a prepayment model involves establishing these bands for various amortization scenarios, serving as initial step in constructing a risk management strategy for interest rate fluctuations in case of prepayment. From the estimation of the two bands, it is possible to derive the number and characteristics of the Bermudan swaptions necessary to construct a hedging strategy using the decomposition proposed in Evers and Jamshidian (2005). This decomposition aims to build a *covering* of the area between the two amortization profiles filling the temporal dimension with the tenor of swap options and the capital dimension with their notional amounts.

In the present work, we evaluated hedging strategies that involve portfolios of loans with 7 different maturities (5, 10, 12, 15, 20, 25, and 30 years). For each of these portfolios, we considered amortization profiles corresponding to six different levels of prepayment rates ($1\%, 2\%, 3\%, 5\%, 7\%, 10\%$) on a French amortization profile, i.e., constant payments (Fabozzi et al. (1992)), see Figure 1a and Figure 1b. The prepayment rates were chosen constant across the entire amortization profile of the portfolio. We therefore constructed 5 different pairs of bands setting the upper band as the amortization profile corresponding to a 1% prepayment rate and the lower band as the amortization profile corresponding to prepayment rates of 2%, 3%, 5%, 7%, and 10%, respectively, as shown in Figure 1b.

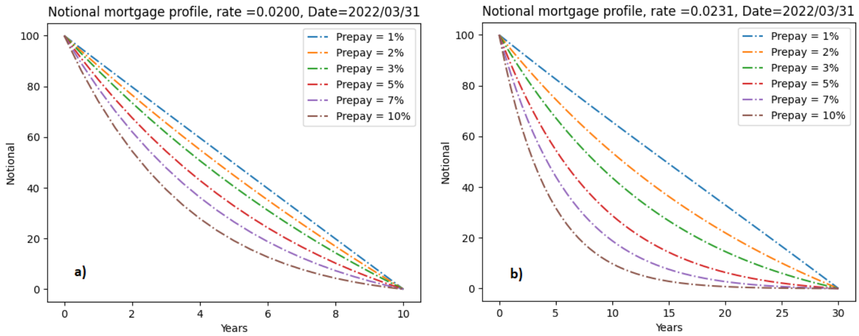

Fig. 1 In Figure a (left) and b (right), two amortization profiles of loans with different maturity periods (i.e., 10 years on the left and 30 years on the right) are shown. Various prepayment rates ranging from 1% to 10% have been applied to these loans. The loans are structured with fixed instalment payments, and the interest rate level is set equal to the swap rate corresponding to the average duration of the loan at that given period.

While opting for constant-rate prepayment scenarios, which yield a consistent and convex profile, might appear overly simplistic, it is crucial to acknowledge that these features —convexity and regularity— are in fact typical of real portfolios, in particular of those composed by a large number of individual loans.

Note that the bands represent a general approach to make a probabilistic projections of the future debt level of the loans portfolio. Even if in the present work we focus on prepayment events, the same approach can be used to account also for default events, as well as new debt issuance.

It should be noted that the prepayment rate level is the outcome obtained from the adoption of specific prepayment models and, for the reasons specified above, these can also be very complex. We did not make any assumptions about the nature of these models but directly simulated their output with a constant prepayment rate. The selected rate levels allowed us to explore a variety of prepayment scenarios consistent with those that banks face in their operations.

3 Pricing Swaption portfolios

The flexibility of a Bermudan swaption makes its valuation more challenging compared to its classical European-style counterpart, which can only be exercised at a single expiration date. This complexity arises because, at each of the predetermined exercise dates, the benefit of exercising the option immediately must be assessed in relation to the potential scenarios where the option is not exercised.

In our study, we adopted the Least Square Monte Carlo (LSM) pricing methodology for valuing Bermudan options, as it is considered the standard approach (Glasserman (2004), Moreno and Navas (2003)). This methodology evaluates parametrically the possible scenarios in case the option is not exercised (*continuation value*). By using backward induction on generated Monte Carlo scenarios, it maps the relationship between the continuation value of the option and the simulated market conditions in a parametric manner. For the regression, we chose the level of the swap rate and the annuity at the exercise dates; see (Brigo D., 2001, Chapter 1) for details on these quantities.

Valuing swaptions also requires making assumptions on the simulated interest rate dynamics. To this end, we used three widely recognized interest rate simulation models: the Libor Market Model (LMM), the two-factor Gaussian model, and the one-factor Hull-White model (HW1) (Brigo D. (2001)). In presence of market scenarios with negative interest rates values, the LMM has been replaced by its shifted version, where a market curve is adjusted by a constant equal to 3%.

These models differ mainly in the number of simulated risk factors, and therefore in their ability to accurately replicate the interest rate market scenario. We decided to perform the evaluations using these three different models to observe the impact, on portfolios price, of different assumptions about the underlying dynamics of interest rates.

The market data used to calibrate interest rate models were obtained from the Bloomberg data provider, including the ATM swaptions matrix and the interest rate curve at five different dates (i.e., 31 December 2015, 31 December 2017, 31 December 2019, 31 December 2021 and the 31 March 2022). Specifically, we calibrated the selected models to replicate market conditions, focusing on the most relevant parts of the swaption matrices for our portfolio simulation.

Given the interest rate simulation model, we performed a separate calibration for each portfolio at each valuation date. It's important to note that each portfolio consists of different options with varying strike prices, tenors, and notional amounts, and represents a hedging strategy for a different reference portfolio with specific prepayment rates and maturity characteristics.

Our study considered a total of 35 portfolios, defined by 7 different maturities and 5 different prepayment rate confidence intervals. This led to a total of 105 calibrations and evaluations (at fixed valuation date), corresponding to the 35 portfolios and 3 interest rate models adopted. These evaluations were performed at 5 different dates, characterized by significantly different market conditions in terms of interest rate levels and swaption volatilities (see Figure 5). The hedging strategies were evaluated using ATM swaptions, after which we explored the dependence of portfolio values on option moneyness.

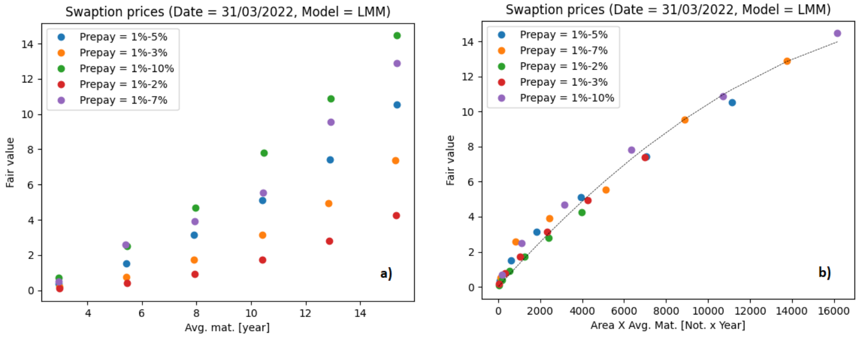

Fig. 2 In Figure a (left), the estimated portfolio values of swaptions used to hedge loans with different maturities are shown, considering various assumptions for the range of prepayment rates. The evaluation date is set as 31/03/2022 and the adopted model is the LMM (Libor Market Model). In Figure b (right), the same portfolio values as in the left panel are presented, where the x-axis represents the product of the average maturity of each portfolio and its notional amount.

4 Results and Discussion

In the next section, we discuss the results of our evaluations of Bermudan swaptions portfolios. In particular, we explore the possibility of parameterise the portfolios values with respect to their macro characteristics.

4.1 The parametric behaviour

Figure 2a shows the estimated prices of portfolios of swaptions hedging loans with different maturities and different assumptions about the prepayment rate ranges. All the valuations correspond to a single date (31 March 2022), hence to the same market conditions, and a single model, namely the LMM. It is possible to observe that the price level increases as the interval between the prepayment rates associated with the two extreme amortization profiles (i.e., upper and lower bands) increases and as the average maturity of the portfolio increases. This trend is in line with expectations, since both extending the mortgage duration and considering more severe prepayment scenarios (i.e., choosing wider bands) increase the probability of a prepayment event. The evaluations also show that these two variables, i.e., the maturity and the prepayment rate range, have a combined impact on the price of the individual portfolio. At higher maturities, the dispersion of prices due to the effect of the prepayment rate ranges alone is much more pronounced than at lower maturities.

Given the observed price trends, qualitatively in line with expectations, we normalized the data to understand the quantitative role played by the two variables (maturities and area between extreme amortization profiles) in determining the price.

Figure 2b depicts the portfolio values at the same date, plotted with respect to the new units: area between the extreme amortization profiles times average maturity of the portfolio. The values depicted correspond to a single model, the LMM. We observe that prices follow a regular parabolic trend. This trend is evident when considering portfolios in which the reference unit (i.e. the average maturity times area between amortization profiles) varies considerably from zero up to values of the order of 10,000. In the region where the reference unit goes to zero, the hedging cost also goes to zero, which is consistent. The deviation from a regular trend is rather limited and concentrated in the region where the reference unit takes smaller values. In the following, we will refer to all plots in these newly standardized units as *masterplots*.

The consistent pattern observed in the price of portfolios concerning new reference units reveals an unexpected regularity in the collective price dynamics of underlying swaptions. It is noteworthy because option prices are generally difficult to predict.

In particular, the price of a swaption is highly nonlinear with respect to the level of the underlying (the swap rate) and its volatility, which depends non linearly on the expiration date and on the moneyness of the option. In this particular case, the hedging portfolios contain many options (from 16 to 116) with varying relative weights. Each of the swaptions contained in the individual portfolio has a different swap rate as underlying, and the strike is at-the-money (ATM). The consistency in price observed across portfolios indicates that when multiple options are combined, specifically chosen according to the typical prepayment band structure, it leads to a more stable overall trend in volatility compared to considering individual options alone.

This is because the volatility of individual options within the portfolio can be quite diverse, as they correspond to different regions of the market's volatility. These regions change based on the expiration and exercise dates of the underlying swap rate.

Therefore, the observed consistent pattern may arise by combining two effects: analyzing the collective behavior of options (i.e., pricing portfolios with numerous options) and selecting them from regions of differing volatilities based on the range between prepayment bands.

This explanation is consistent with the data reported in Figure 3. In 3a, we present the fair value of portfolios of swaptions with fixed characteristics (i.e. the model is LMM, the valuation date is 31 December 2021, the prepayment bands correspond to 1%-10% prepayment rate) plotted in normalized units (area between the bands of each portfolio and its average maturity).

Figure 3b displays the prices of all swaptions that make up each portfolio. It can be observed that, although the prices of individual swaptions are plotted in normalized units before being aggregated into a portfolio, a significant non-linear dispersion still remains as the units on the x-axis increase.

This indicates that the parametric parabolic behavior of the portfolio of swaptions is a characteristic of the collective price dynamics of the swaptions composing the hedging strategies.

The parametric behaviour of prices with respect to the new reference units, does not seem to be specific of the particular market conditions. In fact, we

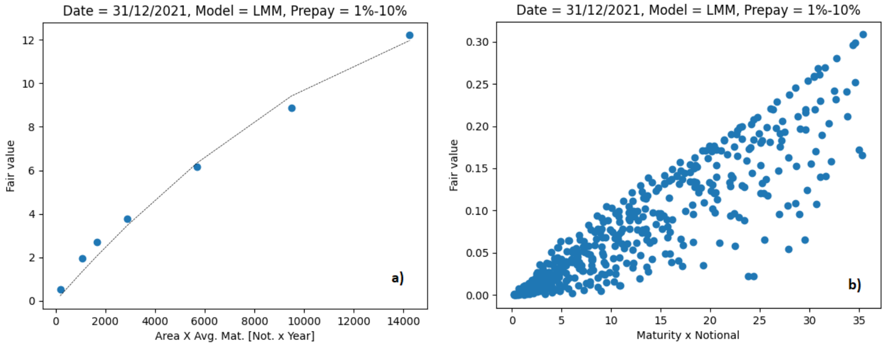

Fig. 3 The figure a (left) displays the value of Swaption portfolio with different maturities (5Y to 30Y) and at fixed date (31/12/2021), model (LMM) and bands of prepayment ($1\% - 10\%$). In figure b (right), the value of all single swaptions involved in the portfolios of the left panel are reported.

observe the same phenomenon at different dates (see Figure 4a and 4b), where market scenarios differ significantly in terms of interest rate levels and swaption volatilities (see also Figure 5b). We remark that have also examined scenarios where the yield curve is negative even for maturities beyond 2 years (e.g., on 31 December 2019 and 31 December 2021).

We also test the impact of different interest rate models on the prices of the selected portfolios of swaptions. We observed that, even changing the simulation model, the price trend of the portfolios remains parametric. The change of model alters the value of each portfolio but does not affect the regularity of price dependency on the new reference units.

Parametric trends associated with different interest rate models enable us to evaluate not only the value of each portfolio, but also to quantify the impact of the model on the evaluation. In other words, we can provide a parametric estimate of model risk (see Figures 6a and 6b).

As a final sensitivity test, we also analyse the dependence of the parametric trend on the moneyness of the option. Given a valuation date and a simulation model, we consider mortgages that pay an additional spread over the swap rate. The parametric trend remains consistent varying the level of option moneyness. As the spread increases, the dependence of the option value with respect to the reference unit also increases.

4.2 Applications of the parametric approach

The masterplots in Figures 2b, 4, and 6 illustrate the possibility of describing the value of a wide range of prepayment risk hedging strategies with only a few parameters (i.e., those required to describe a linear or parabolic trend). This applies to portfolios of loans with maturities ranging from 5 to 30 years and prepayment rates between 1% and 10%, with a certain level of confidence. This means that within the range of maturities and prepayment rates used to build the masterplots, the value of a portfolio, and therefore the cost of any hedging

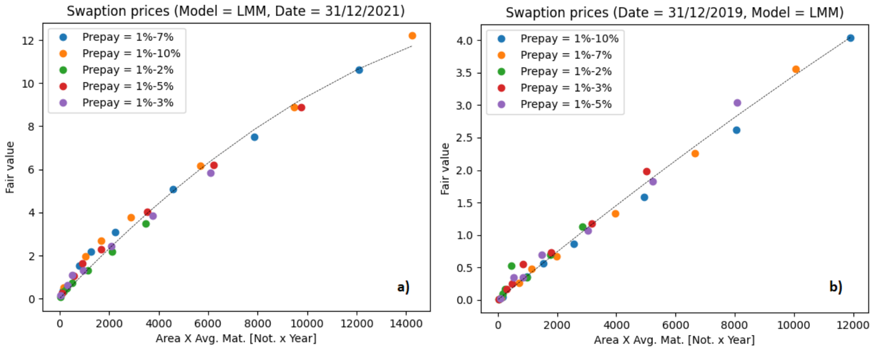

Fig. 4 Figures a (left) and b (right) display the prices of swaptions portfolio values used to hedge loans with different maturities and various assumptions for the range of prepayment rates on two different dates: 31/12/2021 and 31/12/2019. The evaluations were conducted using the same interest rate dynamics model, the LMM (Libor Market Model). The x-axis represents the product of the average maturity of each portfolio and its notional amount.

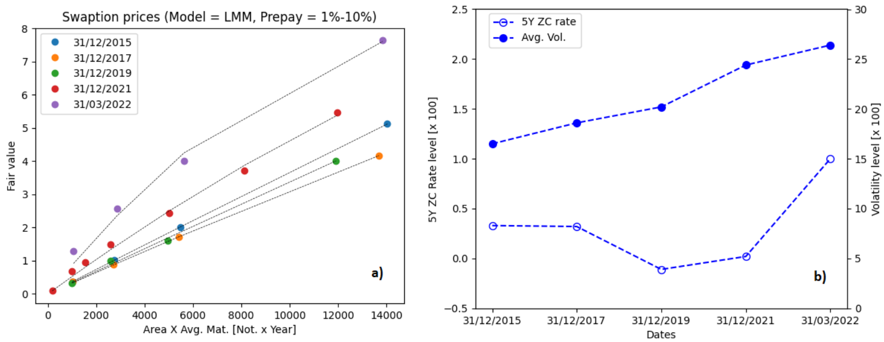

Fig. 5 The figure a (left) displays the value of Swaption portfolio at different Dates once fixed the adopted model (LMM) and the band interval ($1\% - 10\%$). The figure b (right) displays the level of ZC interest rate with maturity 5Y (full symbol) and the average level of the ATM swaption matrix at different dates (from 31/12/2015 till 31/03/2022).

strategy, can be immediately accessed at a fixed initial cost determined by the estimation of the masterplot. The cost of estimating the masterplot is the calibration and evaluation of the masterplot's reference portfolios.

The ability to immediately evaluate portfolios with very different characteristics is a crucial operational element when assessing the effectiveness of a hedging strategy. Due to the complex nature of prepayment events, prepayment rate assumptions are constructed using different approaches/models. Therefore, it is often desirable to test the impact of different prepayment rate assumptions on the value of the hedging strategy across multiple portfolios. This activity involves a large number of evaluations and it is a priori highly time-consuming. The use of a parametric approach dramatically reduces costs, allowing for testing a wide variety of hedging strategies.

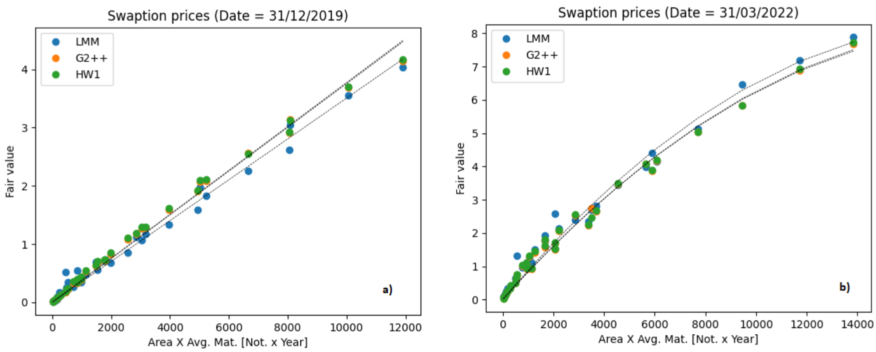

Fig. 6 Figures a (left) and b (right) depict, at two different dates (31/12/2019 and 31/03/2022), the prices of swaptions portfolio values with different maturities and various assumptions for the range of prepayment rates. These values are evaluated using three different models (HW1, G2++, LMM).

Since the parametric behaviour of swaption prices persists when using different interest rate models, a parametric pricing approach can be used to estimate the impact of different interest rate models on the price of a swaption portfolio. In other words, it's possible to monitor model risk. The masterplot can therefore serve not only as a pricing tool, but also as a risk management tool, allowing for an immediate estimation of hedging costs under different underlying model assumptions.

Our analysis, which explores the price behaviour of prices of portfolios of swaptions under a range of market conditions, could be enriched by exploring the relationship between market conditions and different parametric behaviours of swaption prices. This would align with new ML-based approaches to pricing, which evaluate entire classes of financial instruments with minimal or no assumptions about underlying dynamics.

5 Conclusions

The present paper illustrates the use of a parametric approach to estimate the prices of portfolios of Bermudan swaptions used for hedging interest rate risk in the presence of prepayment. We investigated a diverse range of portfolios of Bermudan swaptions designed to replicate the prepayment option associated with different loan portfolios. These loan portfolios vary based on two key parameters: different prepayment bands and the maturity of each portfolio. The evaluation process involved pricing a large number of individual portfolios and calibrating three different interest rate models on ATM swaption market data for each evaluation date. Our analysis shows that the price of different portfolios of swaptions can be parametrically described in terms of the maturity and notional value of each individual portfolio. In other words, the collective price dynamics of Bermudan swaptions exhibit a regular trend with respect to some portfolio's characteristics, even as the model and market conditions (such as interest rate levels and volatility) vary significantly (see Figure

5b). These characteristics, i.e. maturity and level of notional, are determined by the specific hedging strategy of each loan portfolio.

The regular pattern observed in the price dynamics of these hedging strategies strongly suggests the application of machine learning techniques in this evaluation. Our study highlights that the price of a portfolio swaptions can be accurately determined solely by information on portfolio characteristics and market conditions. The ability to describe price dynamics without relying on assumptions about the underlying dynamics represent a significant advantage of employing machine learning techniques for pricing. Moreover, our study suggests the effectiveness of machine learning in describing collective price dynamics.

Our observations also have significant implications for operational cost of interest rate risk assessment in presence of prepayment. The parametric approach we propose can quickly price a wide variety of portfolios of swaptions, with only an initial effort required to calibrate and price some benchmark instruments. This new pricing approach enables extensive testing of the effectiveness of prepayment risk hedging strategies for different portfolio characteristics. Without a parametric approach, such verification would be impractical due to the significant time required for testing.

In addition, the parametric approach also addresses a topic that is gaining growing attention from regulators: the estimation of model risk. In this context, model risk involves not only the description of interest rate dynamics but also the methodology used to estimate prepayment scenarios. This significantly increases the number of evaluations required to accurately estimate the impact of models on the cost of hedging prepayment risk. Therefore, estimating model risk using a traditional pricing approach in this context can be very costly. On the other hand, applying a parametric approach makes the estimation of model risk affordable.

The pricing approach we propose has interesting further future development. We plan to enrich the study investigating the correlation between the parametric behavior of portfolio prices and the specific market conditions at evaluation date. This would further generalize our approach, making this category of instruments excellent candidates for pricing approaches based on ML techniques.

Acknowledgments. Financial support through the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – SFB 1283/2 2021 – 317210226 is gratefully acknowledged by the third author.

Disclosure of potential conflicts of interest

Author Andrea Monaco declares that he has no conflict of interest. Author Adamaria Perrotta declares that she has no conflict of interest. Author Alessandro Sgarabottolo declares that he has no conflict of interest.

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