Observable Interpersonal Utility Comparisons

Lasse Mononen[†]

Abstract

Harsanyi's seminal aggregation theorem axiomatized weighted utilitarianism based on expected utility theory. However, the weights assigned to each individual cannot be separated from the individual's utility. We show that once we depart from the expected utility framework, it is often possible to uniquely identify the utilities and the weights.

1 Introduction

Harsanyi's seminal aggregation theorem (Harsanyi, 1955) axiomatized weighted utilitarianism based on expected utility theory. However, the result has been criticised since the weights attributed to individuals are not meaningful since they cannot be separated from the individuals' utilities (Sen, 1976; Broome, 1987; Weymark, 1991). To overcome this identification issue, Harsanyi (1977) used direct interpersonal utility comparisons. However, interpersonal utility comparisons are difficult to make and have remained controversial in the literature (Elster and Roemer, 1991; Greaves and Lederman, 2018). Additionally, the assumption of expected utility theory in Harsanyi's aggregation theorem has been criticised (e.g. Diamond, 1967; Sen, 1970; Broome, 1987).

We show that the lack of identification of the weights attributed to individuals and utilities in Harsanyi's aggregation theorem is only a knife-edge case. As soon as we depart from the expected utility theory, the weights and utilities are meaningful even without direct interpersonal utility comparisons. This shows that the fairness of the society and interpersonal utility comparisons are observable. This result formalizes Kaneko's (1984) suggestion for observing interpersonal utility comparisons from the social welfare function.

Here, we identify the interpersonal utility comparisons from the non-linearities of the social welfare function. For example, in the case of Rawlsian social welfare function (Rawls,

[†]Center for Mathematical Economics, University of Bielefeld, PO Box 10 01 31, 33 501 Bielefeld, Germany: lasse.mononen@uni-bielefeld.de
The author thanks Niels Boissonnet, Arthur Dolgopolov, Marc Fleurbaey, Dominik Karos, and Frank Riedel for useful comments and suggestions. This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) — Project-ID 317210226 — SFB 1283.

1971), the non-linearities capture the change of the worst-off individual that allows us to identify utilities across individuals. We generalize this identification strategy beyond Rawlsian social welfare function.

More specifically, we consider the min-of-means social welfare function. This has been considered as capturing the ignorant observer in Gajdos and Kandil (2008). Additionally, it has been considered in the context of income inequality in Ben-Porath et al. (1997), Gajdos and Maurin (2004), Crès et al. (2011), and recently in Mongin and Pivato (2021). This representation includes utilitarianism and Rawlsian social welfare function (Rawls, 1971) as special cases.

This min-of-means representation consists of a (von Neumann-Morgenstern) utility function u_i for each member $i \in \mathcal{I}$ and a set of weights for each member $\Lambda \subseteq \Delta(\mathcal{I})$ such that the societal value of an alternative x is

$$\min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(x).$$

We show that the set of weights and the utility functions are identified if and only if any redistribution from one member to another changes the welfare in some situation. That is if $v \in \mathbb{R}^{\mathcal{I}}$ is a utility redistribution such that there exist members i and j with $v_i > 0 > v_j$, then there exists a utility distribution $h \in \mathbb{R}^{\mathcal{I}}$ such that

$$\min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i h_i \neq \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i (h_i + v_i).$$

Technically, this condition is equivalent to the set of weights Λ having a non-empty interior. This result shows that generally interpersonal utility comparisons and weights assigned to individuals are observable as soon as there is uncertainty about the weights. However, utilitarianism is only a knife-edge case where they cannot be separated without direct interpersonal utility comparisons.

Our second contribution is that we characterize the existence of the min-of-means social welfare function by relaxing Harsanyi's assumption that the societal preferences satisfy expected utility theory. Instead, we allow for violations of expected utility theory when the alternatives involve trade-offs across the members and only assume that the societal preferences satisfy expected utility theory when there are no trade-offs across the members.

Our results are closely related to Gajdos and Kandil (2008). They study when an impartial observer's extended preferences have a min-of-means representation. In this extended setting, the observer is especially able to make direct interpersonal utility comparisons and Harsanyi's utilitarianism is fully identified. We instead study observable societal preferences that do not include direct interpersonal utility comparisons.

We follow the approach pioneered by Harsanyi (1955) that studied how preferences over fixed individuals are aggregated. This approach has been used for example in Mongin (1995), Gilboa et al. (2004), Chambers and Hayashi (2006), Gajdos et al. (2008). This approach is in contrast to the literature on studying preference aggregation over varying individuals as pioneered by Arrow (1951) and Sen (1970) and has been summarized in d'Aspremont and Gevers (2002).

The solution in the literature for the lack of identification in Harsanyi (1955) has been to consider non-observable extended lotteries that allow for direct interpersonal utility comparisons. This approach was pioneered in Harsanyi (1977) and used in Karni and Weymark (1998), Gajdos and Kandil (2008), Grant et al. (2010), and discussed in Adler (2014) and Greaves and Lederman (2018).

Technically, our results are related to the literature on income inequality measurement Weymark (1981), Yaari (1988), Ben-Porath et al. (1997). However, here we focus on the more general welfare inequality measurement with subjective utility for each member.

The remainder of the paper proceeds as follows: Section 2 studies the identifications of the min-of-means social welfare function and Section 3 concludes. The Appendix axiomatically characterizes the existence of the representations and proves all the results.

2 Identification

2.1 Preliminaries and Notation

We follow the setting from Harsanyi (1955; 1977). Society consists of members $\mathcal{I}=\{1,\dots,n\}$. X is a set of social-alternatives. Each member $i \in \mathcal{I}$ has preferences $\succsim_i$ over (simple) social-alternative lotteries $\Delta(X)$ and we observe societal preferences $\succsim_0$ over (simple) social-alternative lotteries $\Delta(X)$. (Normalized) weights for the members are probability distributions on the members $\Delta(\mathcal{I})$. $\Delta(\mathcal{I})$ is equipped with the Euclidean topology.

We consider the min-of-means social welfare function following (Ben-Porath et al., 1997; Gajdos and Kandil, 2008) over expected utility members as in Harsanyi (1955; 1977).

Definition Affine utilities $u_i : \Delta(X) \to \mathbb{R}$ for each $i \in \mathcal{I}$ and a convex and closed set of Pareto weights $\Lambda \subseteq \Delta(\mathcal{I})$ is a *min-of-means representation* for $((\succsim_i)_{i\in\mathcal{I}}, \succsim_0)$ if the following two conditions hold

(1) for each $i \in \mathcal{I}$ and $p, q \in \Delta(X)$, we have

$$p \succsim_i q \iff u_i(p) \geq u_i(q).$$

(2) for all $p, q \in \Delta(X)$, we have

$$p \succsim_0 q \iff \min_{\lambda\in\Lambda} \sum_{i\in\mathcal{I}} \lambda_i u_i(p) \geq \min_{\lambda\in\Lambda} \sum_{i\in\mathcal{I}} \lambda_i u_i(q).$$

We focus especially on min-of-means functions with the smallest possible set of Pareto weights as defined next.

Definition Affine utilities $u_i : \Delta(X) \to \mathbb{R}$ for each $i \in \mathcal{I}$ and a convex and closed set of Pareto weights $\Lambda \subseteq \Delta(\mathcal{I})$ is a *minimal min-of-means representation* for $((\succsim_i)_{i\in\mathcal{I}}, \succsim_0)$ if for any other min-of-means representation with the same utilities $(u_i)_{i\in\mathcal{I}}$ and a set of Pareto weights $\tilde{\Lambda}$, we have $\Lambda \subseteq \tilde{\Lambda}$.

The next example shows the significance of minimal representations since there can be weights that the social welfare function never uses. We connect general and minimal representations in the next section.

Remark (Non-minimal example) If $n = 2$ and for all $p \in \Delta(X)$, $u_1(p) < u_2(p)$, then the set of Pareto weights $\Lambda = \{(\lambda, 1-\lambda) | \lambda \in [0, \frac{1}{2}]\}$ is not minimal since for all $p \in \Delta(X)$

$$\min_{\lambda\in[0,\frac{1}{2}]} \lambda u_1(p) + (1-\lambda)u_2(p) = \frac{1}{2}u_1(p) + \frac{1}{2}u_2(p).$$

Next, we define when the minimal set of weights and the utility functions are identified.

Definition The set of weights in the minimal min-of-means representation is identified if for all minimal min-of-means representations $((u_i)_{i\in\mathcal{I}}, \Lambda)$ and $((\tilde{u}_i)_{i\in\mathcal{I}}, \tilde{\Lambda})$, we have

$$\Lambda = \tilde{\Lambda}.$$

Definition The utilities in the min-of-means representation are identified up to a common positive affine transformation if for all min-of-means representations $((u_i)_{i\in\mathcal{I}}, \Lambda)$ and $((\tilde{u}_i)_{i\in\mathcal{I}}, \tilde{\Lambda})$, there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that for each $i \in \mathcal{I}$ and $p \in \Delta(X)$

$$u_i(p) = \alpha\tilde{u}_i(p) + \beta.$$

2.2 Uniqueness

Our main result characterizes when the minimal min-of-means representation is fully identified. This identification is characterized by the following condition.

Axiom 1 If p and q are lotteries such that there exist $i, j \in \mathcal{I}$ with $p \succ_i q$ and $q \succ_j p$, then there exists a lottery r and $\alpha \in [0, 1]$ such that

$$\alpha p + (1-\alpha)r \not\sim \alpha q + (1-\alpha)r$$

Here, we consider redistributing the utility by $(u_k(q) - u_k(p))_{k\in\mathcal{I}}$ that benefits the member j and makes the member i worse off. Then the axiom assumes that there exists a utility distribution

$$\Big(u_k(\alpha p + (1-\alpha)r)\Big)_{k\in\mathcal{I}}$$

such that performing the redistribution to change the utility distribution to

$$\Big(u_k(\alpha p + (1-\alpha)r) + \alpha(u_k(q) - u_k(p))\Big)_{k\in\mathcal{I}} = \Big(u_k(\alpha q + (1-\alpha)r)\Big)_{k\in\mathcal{I}}$$

changes the welfare. That is, any utility redistribution changes the welfare in some situation.

The next result shows that Axiom 1 characterizes the identification of the minimal min-of-means representation.

Theorem 1 Assume that $((\succsim_i)_{i\in\mathcal{I}}, \succsim_0)$ has a minimal min-of-means representation $((u_i)_{i\in\mathcal{I}}, \Lambda)$ such that

$$\operatorname{int}\Big\{\Big(u_i(p)\Big)_{i\in\mathcal{I}}\Big| p \in \Delta(X)\Big\} \neq \varnothing.$$

Then the following four conditions are equivalent.

(1) $((\succsim_i)_{i\in\mathcal{I}}, \succsim_0)$ satisfy Axiom 1.

(2) $\operatorname{int}\Lambda \neq \varnothing$.

(3) The set of weights in the minimal min-of-means representation is identified.

(4) The utilities in the min-of-means representation are identified up to a common positive affine transformation.

This result shows the identification of weights assigned to members and interpersonal utility comparisons. This result shows that as soon as any redistribution changes welfare in some situation, then the weights assigned to members and interpersonal utility comparisons can be identified from the societal preferences.

The assumption that

$$\text{int}\left\{\left(u_i(p)\right)_{i\in\mathcal{I}}\middle|p\in\Delta(X)\right\}\neq\varnothing$$

is a standard identification condition in the literature. It has been used e.g. in Harsanyi (1955), Weymark (1991), and Fleurbaey and Mongin (2016). It is characterized by the independent prospects axiom assuming that for each $i\in\mathcal{I}$, there exists lotteries p and q such that $p\succ_i q$ and for each $j\neq i$, $p\sim_j q$.

The next example shows the identification for a convex combination of utilitarianism and Rawlsian social welfare function as proposed by Gajdos and Kandil (2008). Here, utilitarianism is the only case where the weights are not meaningful and interpersonal utility comparisons are not observed.

Example The social welfare function defined for all $p\in\Delta(X)$ by

$$(1-\theta)\min_{i\in\mathcal{I}}u_i(p)+\frac{\theta}{|I|}\sum_{i\in\mathcal{I}}u_i(p)$$

where $\theta\in[0,1]$ is fully identified and satisfies Axiom 1 iff $\theta\neq 1$.

The next result connects minimal and non-minimal representations and provides the general identification with Theorem 1.

Proposition 2 Assume that $((u_i)_{i\in\mathcal{I}},\Lambda)$ is a minimal min-of-means representation for $((\succsim_i)_{i\in\mathcal{I}},\succsim_0)$ and

$$\text{int}\left\{\left(u_i(p)\right)_{i\in\mathcal{I}}\middle|p\in\Delta(X)\right\}\neq\varnothing.$$

Denote $\mathcal{U}=\{(u_i(p))_{i\in\mathcal{I}}|p\in\Delta(X)\}$ and

$$\Lambda^*=\bigcap_{x\in\mathcal{U}}\{\lambda\in\Delta(\mathcal{I})|x\cdot\lambda\geq\min_{\delta\in\Lambda}x\cdot\delta\}.$$

Then $((u_i)_{i\in\mathcal{I}},\tilde{\Lambda})$ is a min-of-means representation for $((\succsim_i)_{i\in\mathcal{I}},\succsim_0)$ iff $\Lambda\subseteq\tilde{\Lambda}\subseteq\Lambda^*$.[1]

Especially, this shows that if the set of utilities contains a constant utility in the interior, then all representations are minimal and so all the weights can be identified.

Corollary 3 If $((u_i)_{i\in\mathcal{I}}, \Lambda)$ is a min-of-means representation for $((\succsim_i)_{i\in\mathcal{I}}, \succsim_0)$ and there exists $a \in \mathbb{R}$, such that

$$(a)_{i\in\mathcal{I}} \in \operatorname{int}\left\{\left(u_i(p)\right)_{i\in\mathcal{I}}\middle| p \in \Delta(X)\right\},$$

then $((u_i)_{i\in\mathcal{I}}, \Lambda)$ is a minimal min-of-means representation for $((\succsim_i)_{i\in\mathcal{I}}, \succsim_0)$.

3 Conclusion

Our identification result shows that as soon as we depart from the expected utility framework and have uncertainty about the weights for each individual, we can observe interpersonal utility comparisons and the set of weights assigned to individuals uniquely. This allows us to dispense with extended preferences for the identification that have been recently criticized in Greaves and Lederman (2018).

Our result follows as a direct application of the identification of subjective probabilities and state dependent utility from decision theory under uncertainty (Mononen, 2023). This highlights the close connection between state dependent utilities and social choice. Our result formalizes Kaneko's (1984) suggestion for observing the interpersonal utility comparisons from the social welfare function. It is an open question if this identification strategy generalizes to other social welfare functions.

Here, we have maintained Harsanyi's (1955) assumption that each member of the society is an expected utility maximizer. It is an open question on if the state dependent utility theory can be generalized without vNM-utility following the approach of Alon and Schmeidler (2014). Secondly, it is an open question on if the identification result generalizes beyond affine transformations using the approach from Fleurbaey and Mongin (2016). Alon and Schmeidler (2014) offer suggestive evidence that Fleurbaey and Mongin's (2016) identification result for utilitarianism generalizes to the maxmin approach.

[1] Geometrically, Λ^* is the convex hull of Λ using the half-spaces defined by $\mathcal{U}$.

Appendix to "Observable Interpersonal Utility Comparisons"

A Axioms and Existence

In this section, we axiomatize the existence of minimal min-of-means representation. Our Axioms 2-6 maintain Harsanyi's assumptions.

Following Harsanyi (1955), we first assume that each member $i \in \mathcal{I}$ is an expected utility maximizer. This is assumed with the following three standard assumptions.

Axiom 2.1 For each $i \in \mathcal{I}$, $\succsim_i$ is complete and transitive.

Axiom 2.2 For each $i \in \mathcal{I}$ and for all $p, q, r \in \Delta(X)$, the sets $\{\alpha \in [0,1] | \alpha p + (1-\alpha)q \succsim_i r\}$ and $\{\alpha \in [0,1] | r \succsim_i \alpha p + (1-\alpha)q\}$ are closed.

Axiom 2.3 For each $i \in \mathcal{I}$ and for all $p, q, r \in \Delta(X)$ and $\alpha \in (0,1)$,

$$p \succsim_i q \iff \alpha p + (1-\alpha)r \succsim_i \alpha q + (1-\alpha)r.$$

Our first two assumptions on societal preferences are that they are a continuous weak order.

Axiom 3 $\succsim_0$ is complete and transitive.

Axiom 4 For all $p, q, r \in \Delta(X)$, the sets $\{\alpha \in [0,1] | \alpha p + (1-\alpha)q \succsim_0 r\}$ and $\{\alpha \in [0,1] | r \succsim_0 \alpha p + (1-\alpha)q\}$ are closed.

Next, we maintain the standard weak Pareto monotonicity axiom.

Axiom 5 If for each $i \in \mathcal{I}$, $p \succ_i q$, then $p \succ_0 q$.

Next, we assume the independent prospects axiom that is a commonly used richness assumption on the set of lotteries (Harsanyi, 1955; Weymark, 1991; Fleurbaey and Mongin, 2016).

Axiom 6 For each $i \in \mathcal{I}$, there exist lotteries p and q such that $p \succ_i q$ and for each $j \in \mathcal{I}$, $j \neq i$, $p \sim_j q$.

The last two assumptions relax Harsanyi's assumption that the societal preferences satisfy expected utility theory by taking into account that the social-alternatives might involve trade-offs across members. Diamond (1967) criticized Harsanyi's assumption that the societal preferences satisfy the expected utility theory with the following example. Consider two members and two lotteries p and q such that $u_1(p) = 1 = u_2(q)$ and $u_1(q) = 0 = u_2(p)$. Then, under symmetry, $p \sim_0 q$. However, under linearity $p \sim \frac{1}{2}p + \frac{1}{2}q$, even though $u_1(\frac{1}{2}p + \frac{1}{2}q) = \frac{1}{2} = u_2(\frac{1}{2}p + \frac{1}{2}q)$ that is more fair than p or q. We relax the expected utility theory exactly for this example by only assuming that the societal preferences satisfy the independence axiom when there are no trade-offs across members.

First, we define a lottery as neutral if it satisfies the independence axiom. These neutral lotteries give the same utility to each member under Axiom 1.

Definition A lottery r is *neutral* if for all lotteries p, q and $\alpha \in (0, 1)$

$$p \succsim_0 q \iff \alpha p + (1-\alpha)r \succsim_0 \alpha q + (1-\alpha)r.$$

Our first relaxation of the independence axiom assumes that there exist two different neutral lotteries such that one is better for each member than the other. This assumes expected utility theory only when there are no trade-offs across members.

Axiom 7 There exist neutral lotteries p and q such that for each $i \in \mathcal{I}$, $p \succ_i q$.

Secondly, we relax the independence axiom by assuming convexity of the societal preferences.

Axiom 8 For all lotteries p, q and $\alpha \in (0, 1)$, if $p \succsim_0 q$, then $\alpha p + (1-\alpha)q \succsim_0 q$.

These axioms characterize the existence of the min-of-means representation.

Theorem 4 (Existence) The following two conditions are equivalent:

(1) $((\succsim_i)_{i\in\mathcal{I}}, \succsim_0)$ satisfies Axioms 2-8.

(2) There exist $((u_i)_{i\in\mathcal{I}}, \Lambda)$ that is a minimal min-of-means representation for $((\succsim_i)_{i\in\mathcal{I}}, \succsim_0)$ such that

$$\left\{ \left(u_i(p)\right)_{i\in\mathcal{I}} \middle| p \in \Delta(X) \right\} \supseteq [0,1]^{\mathcal{I}}.$$

This result shows how relaxing the independence axiom for the societal preferences by convexity and independence axiom only when there are no trade-offs across members while maintaining the other axioms from Harsanyi (1955) characterizes the min-of-means representation.

B Proofs

Proof of Theorem 1. Theorem 1 follows directly from Mononen (2023). Mononen uses $\mathcal{U} = [a_i, b_i]^{\mathcal{I}}$ where $a_i, b_i \in \mathbb{R} \cup \{\infty, -\infty\}$ and $a_i < b_i$. However, the proof directly generalizes to Theorem 1 where $\mathcal{U}$ is a convex set with $\text{int}\,\mathcal{U} \neq \varnothing$. □

Proof of Proposition 2. We show first the "if" direction. Assume that $\Lambda \subseteq \tilde{\Lambda} \subseteq \Lambda^*$. Now for all $p \in \Delta(X)$,

$$\min_{\lambda \in \tilde{\Lambda}} \sum_{i \in \mathcal{I}} \lambda_i u_i(p) = \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(p)$$

that shows the claim.

Next, we show the "only if" direction. Since (u, Λ) is a minimal representation, we have $\tilde{\Lambda} \supseteq \Lambda$. By Mononen (2023), since $(u, \tilde{\Lambda})$ and (u, Λ) represent $\succsim_0$, we have for all $p \in \Delta(X)$,

$$\min_{\lambda \in \tilde{\Lambda}} \sum_{i \in \mathcal{I}} \lambda_i u_i(p) = \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(p).$$

Thus for each $p \in \Delta(X)$ and for all $\theta \in \tilde{\Lambda}$,

$$\sum_{i \in \mathcal{I}} \theta_i u_i(p) \geq \min_{\lambda \in \Lambda} \sum_{i \in \mathcal{I}} \lambda_i u_i(p).$$

So for each $p \in \Delta(X)$,

$$\{\lambda \in \Delta(\mathcal{I}) | \sum_{i \in \mathcal{I}} \lambda_i u_i(p) \geq \min_{\delta \in \Lambda} \sum_{i \in \mathcal{I}} \delta_i u_i(p)\} \supseteq \tilde{\Lambda}.$$

Thus $\Lambda^* \supseteq \tilde{\Lambda}$. □

The standard proofs for the following lemmas are omitted.

Lemma 5 Assume that $U \subset \mathbb{R}^n$ is a convex set such that for each $1 \leq i \leq n$, there exist $p, q \in U$ such that $p_i > q_i$ and for each $j \neq i$, $p_j = q_j$. Then $\text{int}\, U \neq \varnothing$.

Lemma 6 Assume that $\succsim_0$ is a weak order and p and q are neutral such that $p \succ_0 q$. Then

$$\alpha \geq \beta \iff \alpha p + (1-\alpha) q \succsim_0 \beta p + (1-\beta) q.$$

The last standard lemma is that weak Pareto monotonicity implies semi-strong Pareto monotonicity under continuity and vNM-individuals.

Lemma 7 Assume that $((\succsim_i)_{i\in\mathcal{I}},\succsim_0)$ satisfies Axioms 2-5 and 8. If for each $i\in\mathcal{I}$, $p\succsim_i q$, then $p\succsim_0 q$.

Proof of Theorem 4. The necessity of the axioms is standard and omitted. By the mixture space theorem (Herstein and Milnor, 1953), for each $i\in\mathcal{I}$, there exists an affine $u_i:\Delta(X)\to\mathbb{R}$ that represents $\succsim_i$ such that $u_i(p^*)=1$ and $u_i(p_*)=-1$. Denote $\mathcal{U}=\{(u_i(p))_{i\in\mathcal{I}}|p\in\Delta(X)\}$. Since each u_i is affine, $\mathcal{U}$ is convex. By Lemma 5 and Axiom 6, $\mathrm{int}\,\mathcal{U}\neq\varnothing$. Thus $\bar{0}\in\mathrm{int}\,\mathcal{U}$.

Define the function $I:\mathbb{R}^{\mathcal{I}}\to\mathbb{R}$ by the following: Let $\varphi\in\mathbb{R}^{\mathcal{I}}$. Now there exists $\alpha>0$ such that $\alpha\varphi\in(-1,1)^{\mathcal{I}}\subseteq\mathcal{U}$. Thus there exists $p\in\Delta(X)$ such that $(u_i(p))_{i\in\mathcal{I}}=\alpha\varphi$. By Axioms 4, 5, and 7 and Lemma 6, there exists a unique $\beta\in(0,1)$ such that $\beta p^*+(1-\beta)p_*\sim p$. Define $I(\varphi)=\frac{2}{\alpha}(\beta-\frac{1}{2})$.

By Axioms 7 and 8 and Lemma 7, it is standard to show that for all $p,q\in\Delta(X)$,

$$p\succsim_0 q \iff I(u_i(p)_{i\in\mathcal{I}})\geq I(u_i(q)_{i\in\mathcal{I}})$$

and I is well-defined, monotonic, C-additive, positively homogeneous, and concave. Thus the existence follows from Gilboa and Schmeidler (1989). □

References

Adler, Matthew D. (2014). Extended Preferences and Interpersonal Comparisons: A New Account. *Economics & Philosophy* 30(2), pp. 123–162.

Alon, Shiri and Schmeidler, David (2014). Purely subjective Maxmin Expected Utility. *Journal of Economic Theory* 152, pp. 382–412.

Arrow, Kenneth J. (1951). *Social Choice and Individual Values*. John Wiley & Sons.

Ben-Porath, Elchanan; Gilboa, Itzhak, and Schmeidler, David (1997). On the Measurement of Inequality under Uncertainty. *Journal of Economic Theory* 75(1), pp. 194–204.

Broome, John (1987). Utilitarianism and Expected Utility. *The Journal of Philosophy* 84(8), pp. 405–422.

Chambers, Christopher P. and Hayashi, Takashi (2006). Preference aggregation under uncertainty: Savage vs. Pareto. *Games and Economic Behavior* 54(2), pp. 430–440.

Crès, Hervé; Gilboa, Itzhak, and Vieille, Nicolas (2011). Aggregation of multiple prior opinions. *Journal of Economic Theory* 146(6), pp. 2563–2582.

d'Aspremont, Claude and Gevers, Louis (2002). Social Welfare Functionals and Interpersonal Comparability. *Handbook of Social Choice and Welfare*. Vol. 1. Handbook of Social Choice and Welfare, pp. 459–541.

Diamond, Peter A. (1967). Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparison of Utility: Comment. *Journal of Political Economy* 75(5), pp. 765–766.

Elster, Jon and John E. Roemer, eds. (1991). *Interpersonal Comparisons of Well-Being*. Cambridge University Press.

Fleurbaey, Marc and Mongin, Philippe (2016). The Utilitarian Relevance of the Aggregation Theorem. *American Economic Journal: Microeconomics* 8(3), pp. 289–306.

Gajdos, Thibault and Kandil, Feriel (2008). The ignorant observer. *Social Choice and Welfare* 31(2), pp. 193–232.

Gajdos, Thibault and Maurin, Eric (2004). Unequal uncertainties and uncertain inequalities: an axiomatic approach. *Journal of Economic Theory* 116(1), pp. 93–118.

Gajdos, Thibault; Tallon, Jean-Marc, and Vergnaud, Jean-Christophe (2008). Representation and aggregation of preferences under uncertainty. *Journal of Economic Theory* 141(1), pp. 68–99.

Gilboa, Itzhak; Samet, Dov, and Schmeidler, David (2004). Utilitarian Aggregation of Beliefs and Tastes. *Journal of Political Economy* 112(4), pp. 932–938.

Gilboa, Itzhak and Schmeidler, David (1989). Maxmin expected utility with non-unique prior. *Journal of mathematical economics* 18(2), pp. 141–153.

Grant, Simon; Kajii, Atsushi; Polak, Ben, and Safra, Zvi (2010). Generalized Utilitarianism and Harsanyi's Impartial Observer Theorem. *Econometrica* 78(6), pp. 1939–1971.

Greaves, Hilary and Lederman, Harvey (2018). Extended Preferences and Interpersonal Comparisons of Well-being. *Philosophy and Phenomenological Research* 96(3), pp. 636–667.

Harsanyi, John C. (1955). Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility. *Journal of Political Economy* 63(4), pp. 309–321.

— (1977). *Rational Behaviour and Bargaining Equilibrium in Games and Social Situations*. Cambridge University Press.

Herstein, Israel N. and Milnor, John (1953). An Axiomatic Approach to Measurable Utility. *Econometrica* 21(2), pp. 291–297.

Kaneko, Mamoru (1984). On Interpersonal Utility Comparisons. *Social Choice and Welfare* 1(3), pp. 165–175.

Karni, Edi and Weymark, John A. (1998). An informationally parsimonious impartial observer theorem. *Social Choice and Welfare* 15(3), pp. 321–332.

Mongin, Philippe (1995). Consistent Bayesian Aggregation. *Journal of Economic Theory* 66(2), pp. 313–351.

Mongin, Philippe and Pivato, Marcus (2021). Rawls's difference principle and maximin rule of allocation: a new analysis. *Economic Theory* 71(4), pp. 1499–1525.

Mononen, Lasse (2023). State Dependent Utility and Ambiguity.

Rawls, John (1971). *A Theory of Justice*. Harvard University Press.

Sen, Amartya (1970). *Collective Choice and Social Welfare*. Harvard University Press.

— (1976). Welfare Inequalities and Rawlsian Axiomatics. *Theory and Decision* 7(4), pp. 243–262.

Weymark, John A. (1981). Generalized Gini inequality indices. *Mathematical Social Sciences* 1(4), pp. 409–430.

— (1991). A Reconsideration of the Harsanyi-Sen Debate on Utilitarianism. *Interpersonal Comparisons of Well-Being*. Ed. by Jon Elster and John E. Roemer. Cambridge University Press, pp. 255–320.

Yaari, Menahem (1988). A Controversial Proposal Concerning Inequality Measurement. *Journal of Economic Theory* 44(2), pp. 381–397.